Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/36-1.2.1.5-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [123]. This is test number [36].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (123)	0.00 (0)
Rubi	99.19 (122)	0.81 (1)
Maple	98.37 (121)	1.63 (2)
Fricas	90.24 (111)	9.76 (12)
Giac	73.17 (90)	26.83 (33)
Maxima	54.47 (67)	45.53 (56)
Sympy	54.47 (67)	45.53 (56)
Mupad	43.09 (53)	56.91 (70)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

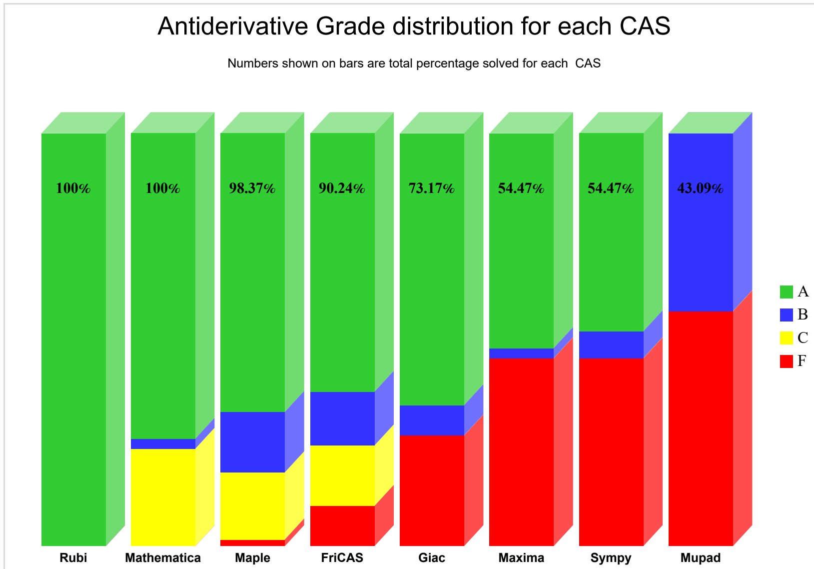
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

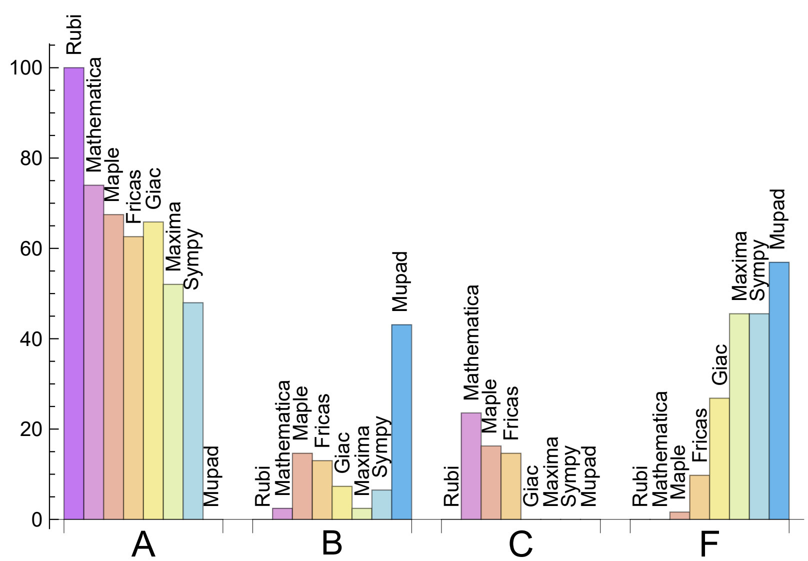
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.187	0.000	0.000	0.813
Mathematica	73.984	2.439	23.577	0.000
Maple	67.480	14.634	16.260	1.626
Giac	65.854	7.317	0.000	26.829
Fricas	62.602	13.008	14.634	9.756
Maxima	52.033	2.439	0.000	45.528
Sympy	47.967	6.504	0.000	45.528
Mupad	0.000	43.089	0.000	56.911

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Fricas	12	41.67	58.33	0.00
Giac	33	15.15	12.12	72.73
Maxima	56	64.29	0.00	35.71
Sympy	56	76.79	23.21	0.00
Mupad	70	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Sympy	0.24
Maxima	0.26
Giac	0.36
Rubi	0.53
Fricas	0.57
Maple	1.21
Mathematica	1.40
Mupad	6.08

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	90.12	0.90	72.00	0.83
Mupad	105.40	0.96	64.00	0.83
Rubi	210.64	1.05	136.50	1.05
Sympy	249.70	1.23	82.00	0.97
Mathematica	329.48	1.38	95.00	1.00
Giac	511.94	2.19	72.50	0.79
Fricas	541.48	2.43	112.00	1.24
Maple	584.38	1.84	76.00	0.80

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

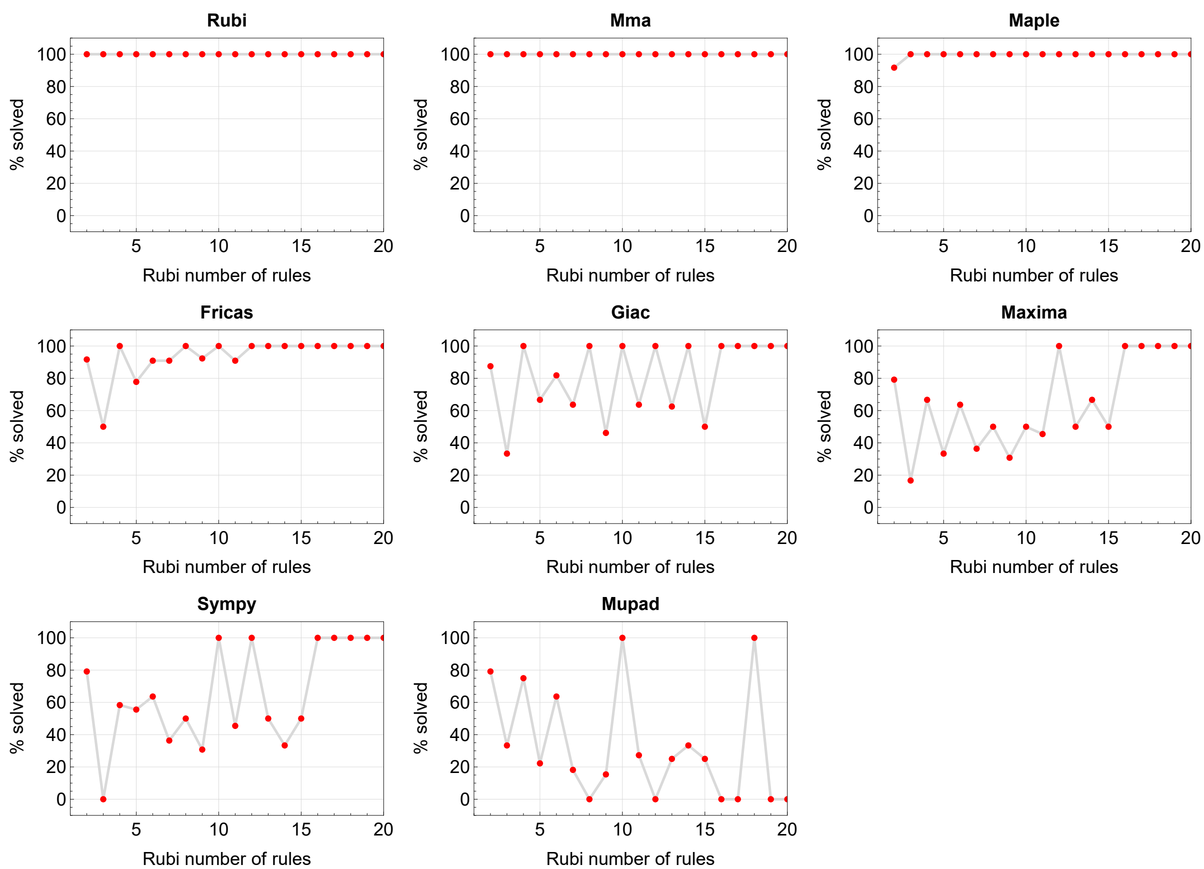


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

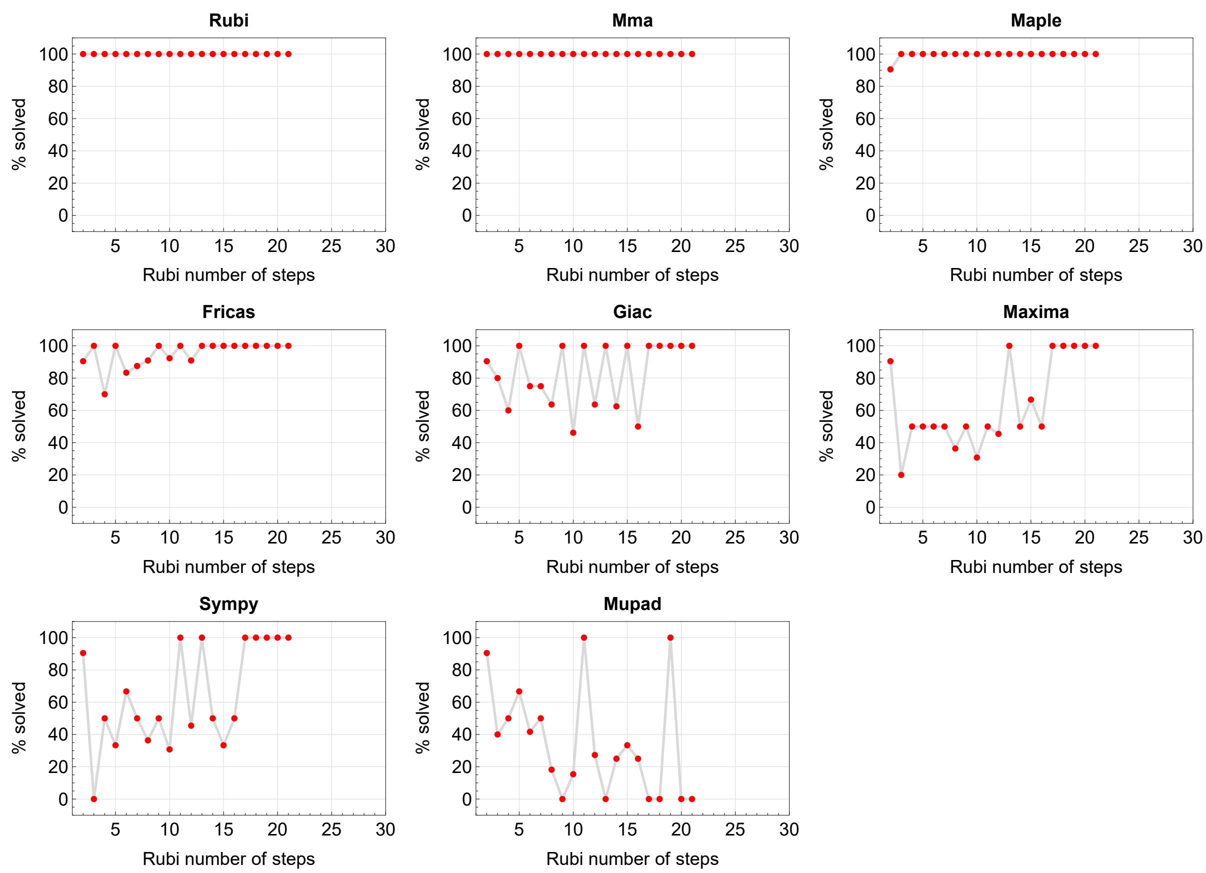


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

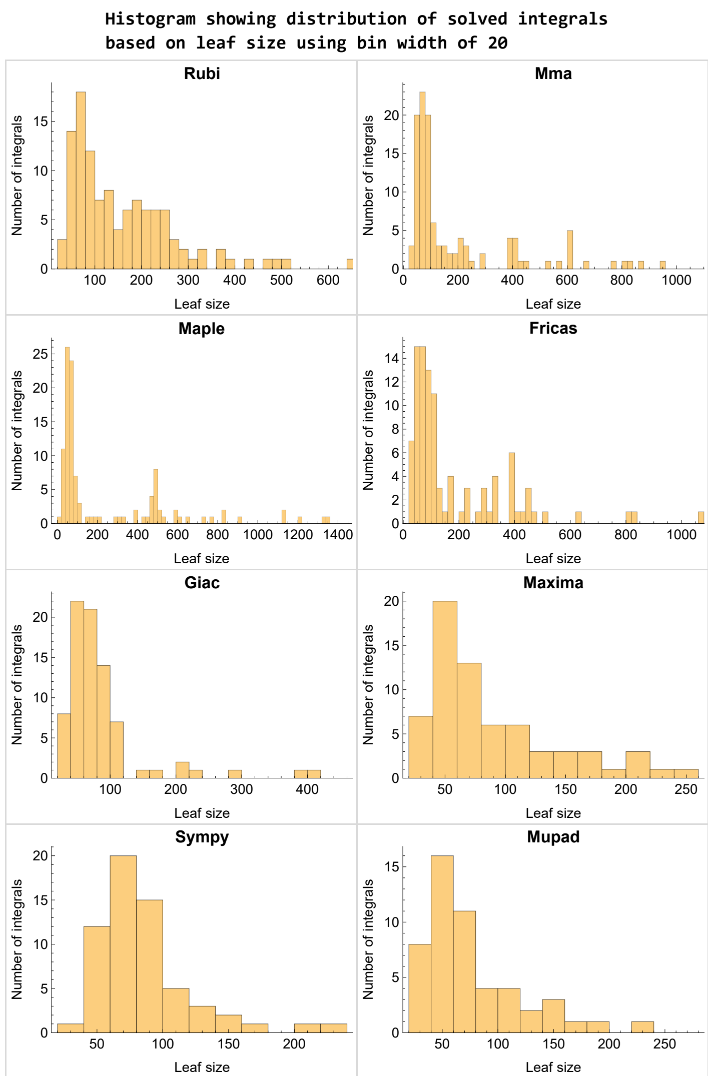


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

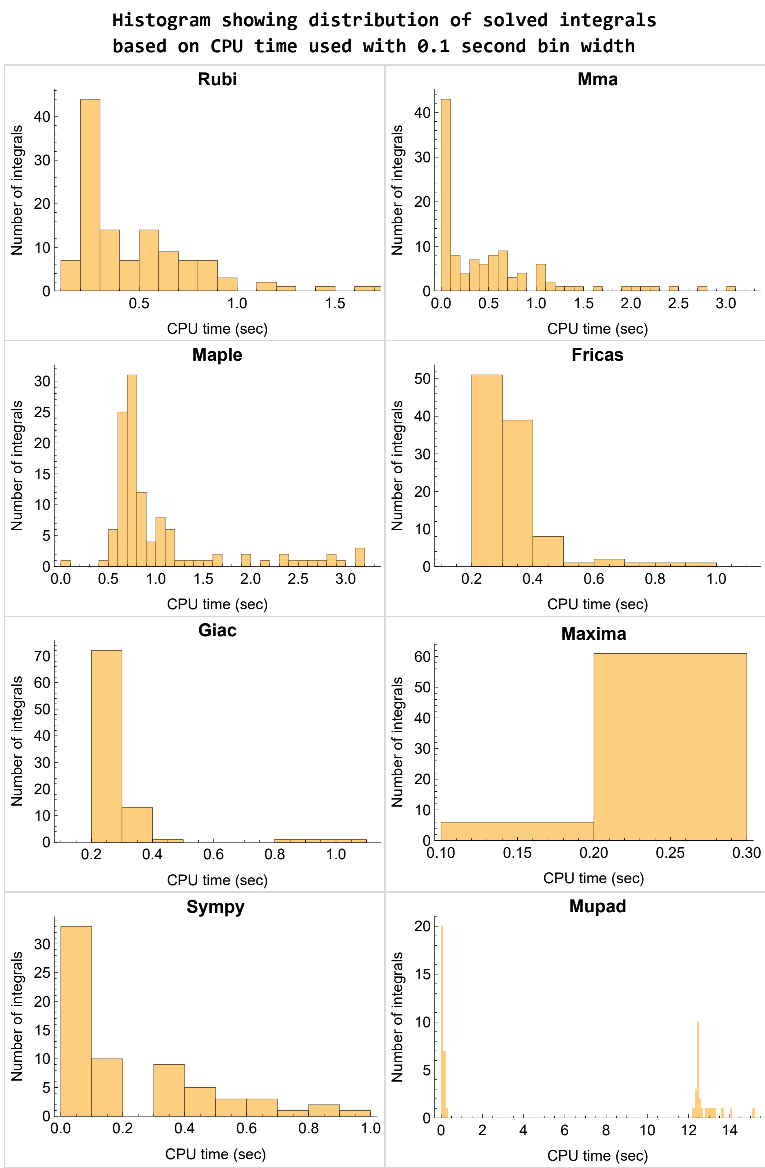


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

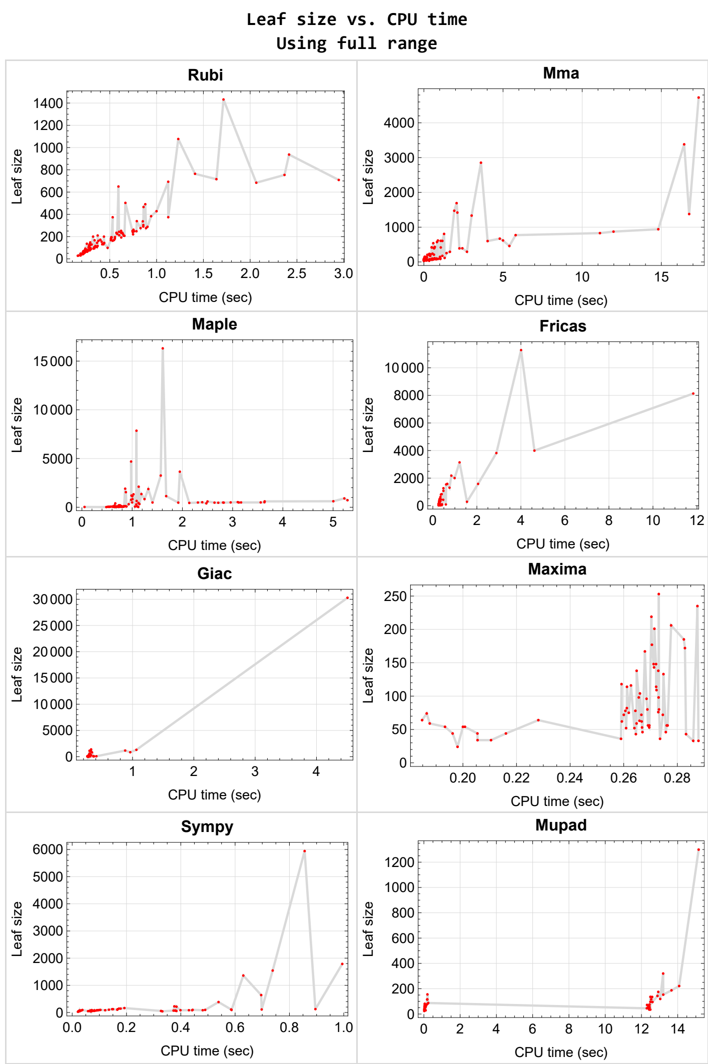


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{13, 14, 107, 108, 113, 122\}$

Mathematica $\{108, 113, 122, 123\}$

Maple $\{11, 13, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 92, 97, 98, 107, 108, 122\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

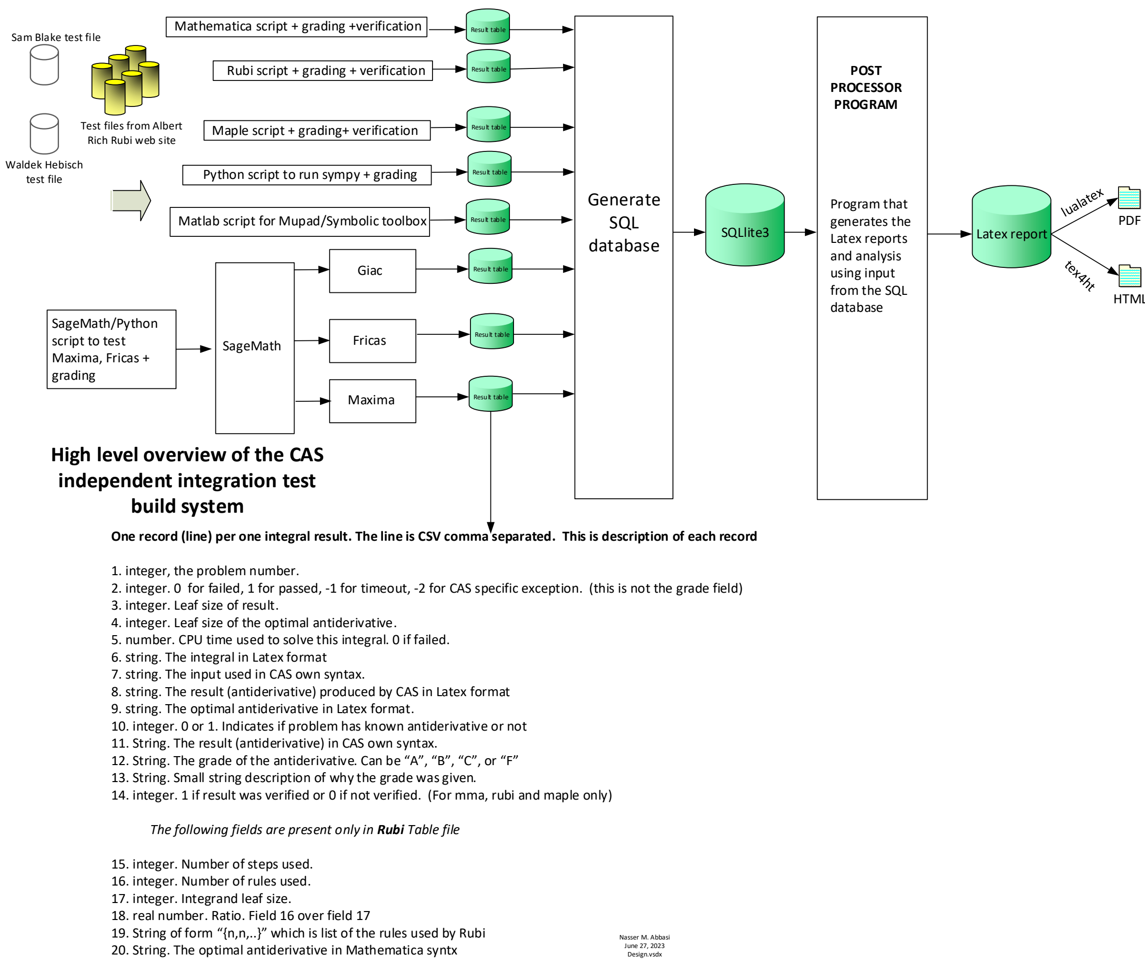
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123 }

B grade { }

C grade { }

F normal fail { 117 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 104, 105, 109, 110, 111, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123 }

B grade { 5, 6, 108 }

C grade { 2, 3, 4, 7, 13, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 102, 103, 106, 107, 112, 117 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 8, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 105, 109, 110, 111, 114, 115, 116, 120, 122, 123 }

B grade { 2, 3, 4, 5, 6, 7, 102, 103, 104, 106, 107, 108, 112, 113, 117, 118, 119, 121 }

C grade { 11, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99 }

F normal fail { 9, 10 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 8, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 93, 94, 96, 100, 101, 105, 109, 110, 111, 119 }

B grade { 2, 3, 4, 5, 6, 7, 89, 95, 104, 112, 114, 115, 116, 118, 120, 121 }

C grade { 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99 }

F normal fail { 9, 10, 13, 122, 123 }

F(-1) timedout fail { 102, 103, 106, 107, 108, 113, 117 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 96 }

B grade { 93, 94, 95 }

C grade { }

F normal fail { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 103, 107, 108, 113, 121, 122, 123 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 100, 101, 102, 104, 105, 106, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120 }

2.1.6 Giac

A grade { 1, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 105, 109, 110, 111, 114, 115, 116, 118, 119, 120 }

B grade { 2, 4, 6, 7, 8, 12, 14, 104, 121 }

C grade { }

F normal fail { 9, 10, 13, 122, 123 }

F(-1) timedout fail { 107, 108, 112, 113 }

F(-2) exception fail { 3, 5, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 102, 103, 106, 117 }

2.1.7 Mupad

A grade { }

B grade { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 89, 96, 100, 101, 116, 120 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 121, 122, 123 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82 }

B grade { 1, 100, 101, 104, 105, 109, 110, 111 }

C grade { }

F normal fail { 2, 3, 4, 8, 10, 11, 12, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 102, 112, 114, 115, 116, 121, 122, 123 }

F(-1) timedout fail { 5, 6, 7, 9, 103, 106, 107, 108, 113, 117, 118, 119, 120 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	103	87	79	0	205	214	81	0
N.S.	1	1.01	0.85	0.77	0.00	2.01	2.10	0.79	0.00
time (sec)	N/A	0.270	0.700	1.101	0.000	0.287	0.384	0.313	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	204	491	0	1079	0	851	0
N.S.	1	1.00	2.49	5.99	0.00	13.16	0.00	10.38	0.00
time (sec)	N/A	0.239	0.407	1.000	0.000	0.481	0.000	0.963	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	189	307	0	813	0	0	0
N.S.	1	1.00	2.86	4.65	0.00	12.32	0.00	0.00	0.00
time (sec)	N/A	0.216	0.394	0.931	0.000	0.367	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	1333	827	0	1544	0	1170	0
N.S.	1	1.00	10.33	6.41	0.00	11.97	0.00	9.07	0.00
time (sec)	N/A	0.295	3.017	1.006	0.000	0.604	0.000	0.880	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	243	940	1868	0	3818	0	0	0
N.S.	1	1.08	4.20	8.34	0.00	17.04	0.00	0.00	0.00
time (sec)	N/A	0.578	14.808	1.322	0.000	2.887	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	374	3382	3649	0	8134	0	30280	0
N.S.	1	1.14	10.31	11.12	0.00	24.80	0.00	92.32	0.00
time (sec)	N/A	1.089	16.443	1.949	0.000	11.832	0.000	4.506	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	1420	1352	0	2005	0	1305	0
N.S.	1	1.00	8.77	8.35	0.00	12.38	0.00	8.06	0.00
time (sec)	N/A	0.364	2.116	1.184	0.000	0.989	0.000	1.063	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	27	0	38	0	52	0
N.S.	1	1.00	1.39	0.96	0.00	1.36	0.00	1.86	0.00
time (sec)	N/A	0.161	0.166	1.121	0.000	0.267	0.000	0.275	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	142	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	172	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.402	0.000	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	48	30	41	16	0	22	0	49	0
N.S.	1	0.62	0.85	0.33	0.00	0.46	0.00	1.02	0.00
time (sec)	N/A	0.157	0.072	0.485	0.000	0.294	0.000	0.290	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	75	62	63	0	82	0	143	0
N.S.	1	1.07	0.89	0.90	0.00	1.17	0.00	2.04	0.00
time (sec)	N/A	0.227	0.173	0.617	0.000	0.328	0.000	0.299	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	1077	1077	600	721	0	0	0	0	0
N.S.	1	1.00	0.56	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.219	4.027	5.287	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	99	75	322	0	161	0	171	0
N.S.	1	1.01	0.77	3.29	0.00	1.64	0.00	1.74	0.00
time (sec)	N/A	0.470	0.162	1.145	0.000	0.296	0.000	0.303	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	65	54	54
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79
time (sec)	N/A	0.241	0.003	0.541	0.193	0.401	0.031	0.283	0.086

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	44	44	53	44	44
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79
time (sec)	N/A	0.231	0.002	0.515	0.196	0.369	0.028	0.294	0.034

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	34	34	41	34	34
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.93	0.77	0.77
time (sec)	N/A	0.211	0.002	0.525	0.210	0.340	0.025	0.279	0.026

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.186	0.002	0.052	0.198	0.342	0.021	0.276	0.022

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	49	33	35
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.17	0.79	0.83
time (sec)	N/A	0.211	0.013	0.688	0.286	0.348	0.059	0.279	12.427

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	35
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.81
time (sec)	N/A	0.198	0.013	0.633	0.274	0.279	0.067	0.283	12.489

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	69	53	47	56	75	63	46	55
N.S.	1	1.08	0.83	0.73	0.88	1.17	0.98	0.72	0.86
time (sec)	N/A	0.212	0.038	0.633	0.276	0.359	0.079	0.282	0.053

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	64	64	76	64	64
N.S.	1	1.00	1.00	0.81	0.80	0.80	0.95	0.80	0.80
time (sec)	N/A	0.257	0.003	0.578	0.228	0.362	0.034	0.263	0.110

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	55	54	54	63	54	54
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.95	0.82	0.82
time (sec)	N/A	0.249	0.005	0.612	0.200	0.386	0.029	0.271	0.079

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	44	44
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.81	0.81
time (sec)	N/A	0.228	0.002	0.632	0.216	0.352	0.024	0.263	0.035

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	34	34	41	34	34
N.S.	1	1.00	1.00	0.76	0.74	0.74	0.89	0.74	0.74
time (sec)	N/A	0.204	0.001	0.552	0.205	0.319	0.022	0.273	0.025

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	53	44	43	43	63	43	45
N.S.	1	1.00	0.95	0.79	0.77	0.77	1.12	0.77	0.80
time (sec)	N/A	0.208	0.015	0.710	0.283	0.312	0.065	0.279	12.295

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	66	59	50	52	78	65	52	51
N.S.	1	1.05	0.94	0.79	0.83	1.24	1.03	0.83	0.81
time (sec)	N/A	0.250	0.022	0.707	0.264	0.316	0.084	0.281	0.052

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	69	53	47	56	75	63	46	55
N.S.	1	1.08	0.83	0.73	0.88	1.17	0.98	0.72	0.86
time (sec)	N/A	0.251	0.017	0.631	0.276	0.422	0.080	0.269	12.438

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	95	63	57	76	105	83	56	75
N.S.	1	1.12	0.74	0.67	0.89	1.24	0.98	0.66	0.88
time (sec)	N/A	0.277	0.035	0.717	0.273	0.413	0.094	0.275	12.441

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	75	74	74	92	74	74
N.S.	1	1.00	1.00	0.78	0.77	0.77	0.96	0.77	0.77
time (sec)	N/A	0.297	0.004	0.615	0.186	0.371	0.038	0.280	0.145

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	65	64	64	78	64	64
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.95	0.78	0.78
time (sec)	N/A	0.252	0.005	0.635	0.185	0.362	0.028	0.278	0.105

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	55	54	54	65	54	54
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.96	0.79	0.79
time (sec)	N/A	0.247	0.005	0.613	0.201	0.361	0.026	0.279	0.079

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	44	44	53	44	44
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.95	0.79	0.79
time (sec)	N/A	0.227	0.001	0.557	0.205	0.353	0.026	0.275	0.035

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	76	53	55
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.09	0.76	0.79
time (sec)	N/A	0.226	0.016	0.702	0.267	0.339	0.072	0.284	0.045

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	78	77	60	62	88	78	62	61
N.S.	1	1.01	1.00	0.78	0.81	1.14	1.01	0.81	0.79
time (sec)	N/A	0.274	0.019	0.704	0.267	0.592	0.082	0.267	0.044

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	92	78	63	72	118	85	62	71
N.S.	1	1.10	0.93	0.75	0.86	1.40	1.01	0.74	0.85
time (sec)	N/A	0.318	0.025	0.713	0.275	0.257	0.092	0.277	0.052

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	72	64	63	63	87	63	65
N.S.	1	1.00	0.86	0.76	0.75	0.75	1.04	0.75	0.77
time (sec)	N/A	0.236	0.023	1.056	0.266	0.264	0.072	0.277	12.425

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	63	54	53	53	73	53	55
N.S.	1	1.00	0.90	0.77	0.76	0.76	1.04	0.76	0.79
time (sec)	N/A	0.218	0.017	0.711	0.270	0.255	0.067	0.269	12.373

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	44	43	43	60	43	45
N.S.	1	1.00	0.93	0.79	0.77	0.77	1.07	0.77	0.80
time (sec)	N/A	0.218	0.015	0.648	0.265	0.262	0.071	0.267	12.381

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	33	46	33	35
N.S.	1	1.00	1.00	0.81	0.79	0.79	1.10	0.79	0.83
time (sec)	N/A	0.199	0.009	0.648	0.288	0.265	0.067	0.274	0.041

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	79	73	60	59	59	83	59	79
N.S.	1	1.08	1.00	0.82	0.81	0.81	1.14	0.81	1.08
time (sec)	N/A	0.249	0.026	0.831	0.265	0.264	0.122	0.279	0.191

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	105	94	76	78	117	102	78	95
N.S.	1	1.12	1.00	0.81	0.83	1.24	1.09	0.83	1.01
time (sec)	N/A	0.335	0.055	0.835	0.264	0.285	0.166	0.268	12.608

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	131	104	89	98	177	119	88	115
N.S.	1	1.14	0.90	0.77	0.85	1.54	1.03	0.77	1.00
time (sec)	N/A	0.427	0.099	0.836	0.273	0.293	0.167	0.278	0.186

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	86	91	70	72	98	90	72	72
N.S.	1	0.95	1.00	0.77	0.79	1.08	0.99	0.79	0.79
time (sec)	N/A	0.290	0.037	0.680	0.260	0.264	0.099	0.272	12.308

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	76	77	60	62	88	75	62	61
N.S.	1	0.99	1.00	0.78	0.81	1.14	0.97	0.81	0.79
time (sec)	N/A	0.287	0.022	0.629	0.259	0.263	0.090	0.281	0.043

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	64	63	50	52	78	61	52	52
N.S.	1	1.02	1.00	0.79	0.83	1.24	0.97	0.83	0.83
time (sec)	N/A	0.248	0.023	0.694	0.261	0.301	0.082	0.273	12.444

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	34	36	45	42	36	36
N.S.	1	1.00	1.00	0.79	0.84	1.05	0.98	0.84	0.84
time (sec)	N/A	0.184	0.011	0.650	0.259	0.271	0.071	0.280	0.041

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	105	94	76	78	117	102	78	96
N.S.	1	1.12	1.00	0.81	0.83	1.24	1.09	0.83	1.02
time (sec)	N/A	0.344	0.040	0.887	0.261	0.260	0.135	0.274	12.494

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	143	106	94	96	167	122	96	115
N.S.	1	1.13	0.83	0.74	0.76	1.31	0.96	0.76	0.91
time (sec)	N/A	0.425	0.038	0.794	0.269	0.289	0.147	0.266	0.187

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	169	136	106	118	237	143	110	135
N.S.	1	1.14	0.92	0.72	0.80	1.60	0.97	0.74	0.91
time (sec)	N/A	0.535	0.043	0.834	0.259	0.306	0.172	0.277	12.589

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	106	98	73	82	128	95	72	81
N.S.	1	1.08	1.00	0.74	0.84	1.31	0.97	0.73	0.83
time (sec)	N/A	0.365	0.027	0.724	0.261	0.284	0.104	0.282	0.048

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	94	84	63	72	118	82	62	72
N.S.	1	1.12	1.00	0.75	0.86	1.40	0.98	0.74	0.86
time (sec)	N/A	0.328	0.028	0.690	0.267	0.269	0.095	0.275	12.437

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	69	51	47	56	75	63	46	56
N.S.	1	1.08	0.80	0.73	0.88	1.17	0.98	0.72	0.88
time (sec)	N/A	0.252	0.022	0.702	0.269	0.293	0.076	0.279	12.428

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	69	51	47	56	75	61	46	55
N.S.	1	1.08	0.80	0.73	0.88	1.17	0.95	0.72	0.86
time (sec)	N/A	0.218	0.020	0.645	0.270	0.282	0.078	0.271	0.047

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	131	99	89	98	177	122	88	116
N.S.	1	1.14	0.86	0.77	0.85	1.54	1.06	0.77	1.01
time (sec)	N/A	0.426	0.105	0.836	0.266	0.285	0.150	0.275	12.537

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	181	136	106	116	227	143	110	136
N.S.	1	1.13	0.85	0.66	0.72	1.42	0.89	0.69	0.85
time (sec)	N/A	0.550	0.072	0.838	0.263	0.283	0.178	0.270	12.493

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	207	151	118	138	297	163	116	155
N.S.	1	1.14	0.83	0.65	0.76	1.64	0.90	0.64	0.86
time (sec)	N/A	0.656	0.057	0.862	0.265	0.289	0.192	0.276	0.196

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	248	95	75	177	98	97	93	221
N.S.	1	1.19	0.46	0.36	0.85	0.47	0.47	0.45	1.06
time (sec)	N/A	0.759	0.809	0.766	0.271	0.289	0.444	0.293	14.079

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	196	85	65	143	88	83	83	187
N.S.	1	1.18	0.51	0.39	0.86	0.53	0.50	0.50	1.13
time (sec)	N/A	0.532	0.589	0.737	0.271	0.279	0.399	0.280	13.656

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	144	75	55	109	78	70	73	153
N.S.	1	1.16	0.60	0.44	0.88	0.63	0.56	0.59	1.23
time (sec)	N/A	0.365	0.371	0.711	0.272	0.297	0.379	0.286	13.202

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	92	65	45	75	68	56	63	119
N.S.	1	1.12	0.79	0.55	0.91	0.83	0.68	0.77	1.45
time (sec)	N/A	0.230	0.232	0.641	0.262	0.278	0.327	0.279	13.035

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	174	180	206	484	0	285	0	0	0
N.S.	1	1.03	1.18	2.78	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.502	0.220	1.405	0.000	0.311	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	188	194	419	473	0	336	0	0	0
N.S.	1	1.03	2.23	2.52	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.500	0.491	1.918	0.000	0.331	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	223	234	392	483	0	396	0	0	0
N.S.	1	1.05	1.76	2.17	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.637	0.650	2.309	0.000	0.303	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	276	105	85	206	108	110	103	0
N.S.	1	1.19	0.45	0.37	0.89	0.47	0.48	0.45	0.00
time (sec)	N/A	0.817	1.039	0.741	0.278	0.292	0.585	0.282	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	224	95	75	172	98	95	93	0
N.S.	1	1.19	0.50	0.40	0.91	0.52	0.50	0.49	0.00
time (sec)	N/A	0.573	0.788	0.764	0.283	0.293	0.490	0.285	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	172	85	65	138	88	83	83	0
N.S.	1	1.17	0.58	0.44	0.94	0.60	0.56	0.56	0.00
time (sec)	N/A	0.395	0.575	0.730	0.273	0.297	0.442	0.290	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	120	75	55	104	78	70	73	0
N.S.	1	1.14	0.71	0.52	0.99	0.74	0.67	0.70	0.00
time (sec)	N/A	0.267	0.359	0.686	0.266	0.280	0.373	0.275	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	197	208	228	590	0	316	0	0	0
N.S.	1	1.06	1.16	2.99	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.635	0.338	2.500	0.000	0.320	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	232	248	416	511	0	384	0	0	0
N.S.	1	1.07	1.79	2.20	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.775	0.508	2.395	0.000	0.301	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	223	234	572	483	0	396	0	0	0
N.S.	1	1.05	2.57	2.17	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.570	0.828	2.810	0.000	0.296	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	304	115	95	235	118	124	113	0
N.S.	1	1.20	0.45	0.37	0.93	0.46	0.49	0.44	0.00
time (sec)	N/A	0.862	1.313	0.839	0.287	0.300	0.896	0.287	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	252	105	85	201	108	110	103	0
N.S.	1	1.19	0.50	0.40	0.95	0.51	0.52	0.49	0.00
time (sec)	N/A	0.609	1.007	0.767	0.271	0.281	0.698	0.287	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	200	95	75	167	98	95	93	0
N.S.	1	1.18	0.56	0.44	0.98	0.58	0.56	0.55	0.00
time (sec)	N/A	0.428	0.770	0.754	0.268	0.273	0.587	0.287	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	148	85	65	133	88	83	83	0
N.S.	1	1.16	0.66	0.51	1.04	0.69	0.65	0.65	0.00
time (sec)	N/A	0.289	0.547	0.701	0.275	0.270	0.481	0.288	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	222	240	238	599	0	322	0	0	0
N.S.	1	1.08	1.07	2.70	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.732	0.544	3.638	0.000	0.341	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	255	276	433	522	0	404	0	0	0
N.S.	1	1.08	1.70	2.05	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.858	0.689	3.102	0.000	0.310	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	281	288	616	613	0	454	0	0	0
N.S.	1	1.02	2.19	2.18	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.894	0.886	5.005	0.000	0.375	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	220	85	65	148	88	83	83	0
N.S.	1	1.19	0.46	0.35	0.80	0.48	0.45	0.45	0.00
time (sec)	N/A	0.753	0.657	0.732	0.271	0.275	0.430	0.411	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	168	75	55	114	78	70	73	0
N.S.	1	1.17	0.52	0.38	0.80	0.55	0.49	0.51	0.00
time (sec)	N/A	0.500	0.446	0.727	0.272	0.405	0.380	0.361	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	116	65	45	80	68	56	63	0
N.S.	1	1.15	0.64	0.45	0.79	0.67	0.55	0.62	0.00
time (sec)	N/A	0.333	0.287	0.743	0.273	0.317	0.384	0.371	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	64	55	35	46	58	42	53	0
N.S.	1	1.08	0.93	0.59	0.78	0.98	0.71	0.90	0.00
time (sec)	N/A	0.210	0.148	0.727	0.267	0.295	0.332	0.362	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	154	135	441	0	265	0	0	0
N.S.	1	1.04	0.91	2.98	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.416	0.195	2.137	0.000	0.295	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	188	194	230	473	0	336	0	0	0
N.S.	1	1.03	1.22	2.52	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.524	0.365	2.824	0.000	0.320	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	223	234	396	483	0	396	0	0	0
N.S.	1	1.05	1.78	2.17	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.620	0.636	3.123	0.000	0.307	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	192	85	65	148	112	0	82	0
N.S.	1	1.16	0.51	0.39	0.89	0.67	0.00	0.49	0.00
time (sec)	N/A	0.619	0.890	0.762	0.272	0.279	0.000	0.274	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	137	75	55	114	102	0	72	0
N.S.	1	1.10	0.60	0.44	0.92	0.82	0.00	0.58	0.00
time (sec)	N/A	0.421	0.614	0.815	0.261	0.290	0.000	0.279	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	90	65	45	80	92	0	62	0
N.S.	1	1.10	0.79	0.55	0.98	1.12	0.00	0.76	0.00
time (sec)	N/A	0.285	0.467	0.793	0.269	0.303	0.000	0.274	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	55	35	46	82	0	53	87
N.S.	1	1.00	1.22	0.78	1.02	1.82	0.00	1.18	1.93
time (sec)	N/A	0.195	0.255	0.679	0.276	0.270	0.000	0.278	0.242

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	176	182	199	460	0	338	0	0	0
N.S.	1	1.03	1.13	2.61	0.00	1.92	0.00	0.00	0.00
time (sec)	N/A	0.504	0.342	2.714	0.000	0.291	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	222	414	491	0	388	0	0	0
N.S.	1	1.05	1.96	2.33	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	0.627	0.679	3.168	0.000	0.332	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	246	262	607	493	0	458	0	0	0
N.S.	1	1.07	2.47	2.00	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.760	1.064	3.559	0.000	0.316	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	163	85	65	253	132	0	81	0
N.S.	1	1.11	0.58	0.44	1.72	0.90	0.00	0.55	0.00
time (sec)	N/A	0.517	1.043	0.690	0.273	0.281	0.000	0.276	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	116	75	55	219	122	0	72	0
N.S.	1	1.10	0.71	0.52	2.09	1.16	0.00	0.69	0.00
time (sec)	N/A	0.364	0.774	0.797	0.270	0.292	0.000	0.288	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	71	65	45	185	112	0	61	0
N.S.	1	1.04	0.96	0.66	2.72	1.65	0.00	0.90	0.00
time (sec)	N/A	0.259	0.562	0.733	0.282	0.291	0.000	0.288	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	59	51	0	29	29
N.S.	1	1.00	0.70	0.64	1.26	1.09	0.00	0.62	0.62
time (sec)	N/A	0.187	0.338	0.670	0.188	0.275	0.000	0.282	0.098

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	199	212	209	472	0	398	0	0	0
N.S.	1	1.07	1.05	2.37	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.604	0.637	2.639	0.000	0.315	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	250	416	493	0	448	0	0	0
N.S.	1	1.07	1.78	2.11	0.00	1.91	0.00	0.00	0.00
time (sec)	N/A	0.742	1.067	2.961	0.000	0.301	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	290	605	511	0	518	0	0	0
N.S.	1	1.08	2.25	1.90	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	0.865	1.169	3.634	0.000	0.317	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	383	459	645	0	1269	1544	627	1299
N.S.	1	0.88	1.05	1.48	0.00	2.91	3.54	1.44	2.98
time (sec)	N/A	0.935	5.401	1.089	0.000	0.480	0.738	0.306	15.152

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	167	174	197	0	465	384	204	320
N.S.	1	0.95	0.99	1.13	0.00	2.66	2.19	1.17	1.83
time (sec)	N/A	0.343	1.169	0.733	0.000	0.326	0.539	0.299	13.195

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	491	539	1547	0	0	0	0	0
N.S.	1	1.14	1.25	3.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.897	0.594	0.875	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	488	503	1691	4691	0	0	0	0	0
N.S.	1	1.03	3.47	9.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.669	2.068	0.979	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	564	428	829	1200	0	2179	5942	1132	0
N.S.	1	0.76	1.47	2.13	0.00	3.86	10.54	2.01	0.00
time (sec)	N/A	0.976	11.122	0.986	0.000	0.840	0.856	0.298	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	210	293	398	0	839	1360	403	0
N.S.	1	0.89	1.24	1.69	0.00	3.56	5.76	1.71	0.00
time (sec)	N/A	0.363	2.722	0.664	0.000	0.403	0.630	0.291	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	684	1472	1328	0	0	0	0	0
N.S.	1	1.01	2.17	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.066	1.920	1.027	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	704	716	2854	7856	0	0	0	0	0
N.S.	1	1.02	4.05	11.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.641	3.605	1.086	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	671	693	4727	16309	0	0	0	0	0
N.S.	1	1.03	7.04	24.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.127	17.357	1.609	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	717	754	618	837	0	1583	1787	818	0
N.S.	1	1.05	0.86	1.17	0.00	2.21	2.49	1.14	0.00
time (sec)	N/A	2.354	4.998	1.246	0.000	0.655	0.995	0.339	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	337	254	286	0	637	641	296	0
N.S.	1	1.07	0.80	0.91	0.00	2.02	2.03	0.94	0.00
time (sec)	N/A	0.852	1.430	1.082	0.000	0.346	0.696	0.325	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	121	98	86	0	227	226	93	0
N.S.	1	1.04	0.84	0.74	0.00	1.96	1.95	0.80	0.00
time (sec)	N/A	0.284	0.525	0.764	0.000	0.300	0.376	0.304	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	218	761	0	11287	0	0	0
N.S.	1	1.00	0.58	2.03	0.00	30.18	0.00	0.00	0.00
time (sec)	N/A	0.533	0.415	0.979	0.000	4.009	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	789	764	1377	2108	0	0	0	0	0
N.S.	1	0.97	1.75	2.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.407	16.758	1.130	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	649	709	771	1139	0	3143	0	1093	0
N.S.	1	1.09	1.19	1.76	0.00	4.84	0.00	1.68	0.00
time (sec)	N/A	2.884	5.797	1.677	0.000	1.214	0.000	0.320	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	339	291	439	0	1305	0	399	0
N.S.	1	1.10	0.94	1.42	0.00	4.22	0.00	1.29	0.00
time (sec)	N/A	0.781	1.644	1.126	0.000	0.758	0.000	0.305	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	106	201	0	429	0	118	143
N.S.	1	1.00	0.95	1.81	0.00	3.86	0.00	1.06	1.29
time (sec)	N/A	0.272	0.671	0.756	0.000	0.472	0.000	0.309	12.881

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	0	806	1906	0	0	0	0	0
N.S.	1	0.00	1.21	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.277	0.861	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	891	937	872	3251	0	3995	0	1392	0
N.S.	1	1.05	0.98	3.65	0.00	4.48	0.00	1.56	0.00
time (sec)	N/A	2.377	11.980	1.569	0.000	4.616	0.000	0.325	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	466	392	1129	0	1581	0	579	0
N.S.	1	1.05	0.88	2.54	0.00	3.56	0.00	1.30	0.00
time (sec)	N/A	0.867	2.432	1.003	0.000	2.056	0.000	0.299	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	147	176	0	286	0	234	175
N.S.	1	1.00	1.12	1.34	0.00	2.18	0.00	1.79	1.34
time (sec)	N/A	0.284	1.020	0.649	0.000	1.551	0.000	0.295	12.927

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	144	0	154	0	205	0
N.S.	1	1.00	1.04	2.82	0.00	3.02	0.00	4.02	0.00
time (sec)	N/A	0.219	0.157	0.773	0.000	0.268	0.000	0.287	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD
size	1432	1432	670	905	0	0	0	0	0
N.S.	1	1.00	0.47	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.719	4.796	5.224	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	652	650	390	395	0	0	0	0	0
N.S.	1	1.00	0.60	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.596	2.245	2.477	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [72] had the largest ratio of [.7407409999999999983]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.01	29	0.172
2	A	3	2	1.00	31	0.065
3	A	3	2	1.00	27	0.074
4	A	5	4	1.00	27	0.148
5	A	7	6	1.08	27	0.222
6	A	9	8	1.14	27	0.296
7	A	5	4	1.00	31	0.129
8	A	3	2	1.00	23	0.087
9	A	2	2	1.00	31	0.065
10	A	2	2	1.00	34	0.059
11	A	4	4	0.62	22	0.182
12	A	8	7	1.07	18	0.389
13	A	4	3	1.00	26	0.115
14	A	15	14	1.01	27	0.519
15	A	2	2	1.00	23	0.087
16	A	2	2	1.00	23	0.087
17	A	2	2	1.00	23	0.087
18	A	2	2	1.00	21	0.095
19	A	2	2	1.00	23	0.087
20	A	5	4	1.00	23	0.174
21	A	6	5	1.08	23	0.217
22	A	2	2	1.00	25	0.080

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	2	2	1.00	25	0.080
24	A	2	2	1.00	25	0.080
25	A	2	2	1.00	23	0.087
26	A	2	2	1.00	25	0.080
27	A	4	4	1.05	25	0.160
28	A	7	6	1.08	25	0.240
29	A	8	7	1.12	25	0.280
30	A	2	2	1.00	25	0.080
31	A	2	2	1.00	25	0.080
32	A	2	2	1.00	25	0.080
33	A	2	2	1.00	23	0.087
34	A	2	2	1.00	25	0.080
35	A	4	4	1.01	25	0.160
36	A	6	6	1.10	25	0.240
37	A	2	2	1.00	25	0.080
38	A	2	2	1.00	25	0.080
39	A	2	2	1.00	25	0.080
40	A	2	2	1.00	23	0.087
41	A	8	7	1.08	25	0.280
42	A	10	9	1.12	25	0.360
43	A	12	11	1.14	25	0.440
44	A	4	4	0.95	25	0.160
45	A	4	4	0.99	25	0.160
46	A	4	4	1.02	25	0.160
47	A	5	4	1.00	23	0.174
48	A	10	9	1.12	25	0.360
49	A	12	11	1.13	25	0.440
50	A	14	13	1.14	25	0.520
51	A	6	6	1.08	25	0.240
52	A	6	6	1.12	25	0.240
53	A	7	6	1.08	25	0.240
54	A	6	5	1.08	23	0.217
55	A	12	11	1.14	25	0.440
56	A	14	13	1.13	25	0.520

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	16	15	1.14	25	0.600
58	A	19	18	1.19	27	0.667
59	A	15	14	1.18	27	0.519
60	A	11	10	1.16	27	0.370
61	A	7	6	1.12	25	0.240
62	A	10	9	1.03	27	0.333
63	A	8	7	1.03	27	0.259
64	A	10	9	1.05	27	0.333
65	A	20	19	1.19	27	0.704
66	A	16	15	1.19	27	0.556
67	A	12	11	1.17	27	0.407
68	A	8	7	1.14	25	0.280
69	A	12	11	1.06	27	0.407
70	A	14	13	1.07	27	0.481
71	A	10	9	1.05	27	0.333
72	A	21	20	1.20	27	0.741
73	A	17	16	1.19	27	0.593
74	A	13	12	1.18	27	0.444
75	A	9	8	1.16	25	0.320
76	A	14	13	1.08	27	0.481
77	A	16	15	1.08	27	0.556
78	A	16	15	1.02	27	0.556
79	A	18	17	1.19	27	0.630
80	A	14	13	1.17	27	0.481
81	A	10	9	1.15	27	0.333
82	A	6	5	1.08	25	0.200
83	A	6	5	1.04	27	0.185
84	A	8	7	1.03	27	0.259
85	A	10	9	1.05	27	0.333
86	A	15	14	1.16	27	0.519
87	A	12	11	1.10	27	0.407
88	A	8	7	1.10	27	0.259
89	A	5	4	1.00	25	0.160
90	A	8	7	1.03	27	0.259

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	10	9	1.05	27	0.333
92	A	12	11	1.07	27	0.407
93	A	14	13	1.11	27	0.481
94	A	10	9	1.10	27	0.333
95	A	7	6	1.04	27	0.222
96	A	3	3	1.00	25	0.120
97	A	10	9	1.07	27	0.333
98	A	12	11	1.07	27	0.407
99	A	14	13	1.08	27	0.481
100	A	11	10	0.88	27	0.370
101	A	7	6	0.95	25	0.240
102	A	7	6	1.14	27	0.222
103	A	6	5	1.03	27	0.185
104	A	12	11	0.76	27	0.407
105	A	8	7	0.89	25	0.280
106	A	10	9	1.01	27	0.333
107	A	12	11	1.02	27	0.407
108	A	8	7	1.03	27	0.259
109	A	14	13	1.05	27	0.481
110	A	10	9	1.07	27	0.333
111	A	6	5	1.04	25	0.200
112	A	4	3	1.00	27	0.111
113	A	6	5	0.97	27	0.185
114	A	12	11	1.09	27	0.407
115	A	8	7	1.10	27	0.259
116	A	5	4	1.00	25	0.160
117	F	0	0	N/A	0.000	N/A
118	A	10	9	1.05	27	0.333
119	A	7	6	1.05	27	0.222
120	A	3	3	1.00	25	0.120
121	A	6	5	1.00	27	0.185
122	A	4	3	1.00	29	0.103
123	A	4	3	1.00	29	0.103

CHAPTER 3

Listing of integrals

3.1 $\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx$

3.1.1 Optimal result

Integrand size = 29, antiderivative size = 102

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx=\frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(8af-b\left(e+\frac{4df}{e}\right)\right)\operatorname{arctanh}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}$$

output
```
1/8*(8*a*f-b*(e+4*d*f/e))*arctanh(1/2*(2*f*x+e)/f^(1/2)/(f*x^2+e*x+d)^(1/2
))/f^(3/2)+1/4*b*(f*x^2+e*x+d)^(1/2)/f+1/2*b*x*(f*x^2+e*x+d)^(1/2)/e
```

3.1.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx=\frac{b\sqrt{f}(e+2fx)\sqrt{d+x(e+fx)}+(-8aef+b(e^2+4df))\operatorname{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{d}-\sqrt{d+x(e+fx)}}\right)}{4ef^{3/2}}$$

input
```
Integrate[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

output
```
(b*Sqrt[f]*(e + 2*f*x)*Sqrt[d + x*(e + f*x)] + (-8*a*e*f + b*(e^2 + 4*d*f)
)*ArcTanh[(Sqrt[f]*x)/(Sqrt[d] - Sqrt[d + x*(e + f*x)])])/(4*e*f^(3/2))
```

3.1.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{bfx^2}{e}+bx}{\sqrt{d+ex+fx^2}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{f\left(2\left(2a-\frac{bd}{e}\right)+bx\right)}{2\sqrt{fx^2+ex+d}}dx}{2f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{2\left(2a-\frac{bd}{e}\right)+bx}{\sqrt{fx^2+ex+d}}dx+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{4}\left(\frac{b\sqrt{d+ex+fx^2}}{f}-\frac{\left(-8af+\frac{4bdf}{e}+be\right)\int \frac{1}{\sqrt{fx^2+ex+d}}dx}{2f}\right)+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{4}\left(\frac{b\sqrt{d+ex+fx^2}}{f}-\frac{\left(-8af+\frac{4bdf}{e}+be\right)\int \frac{1}{4f-\frac{(e+2fx)^2}{fx^2+ex+d}}d\frac{e+2fx}{\sqrt{fx^2+ex+d}}}{f}\right)+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{b\sqrt{d+ex+fx^2}}{f}-\frac{\left(-8af+\frac{4bdf}{e}+be\right)\text{arctanh}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{2f^{3/2}}\right)+\frac{bx\sqrt{d+ex+fx^2}}{2e}$$

input
```
Int[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

output
```
(b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((b*Sqrt[d + e*x + f*x^2])/f - ((b*e -
 8*a*f + (4*b*d*f)/e)*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2]
)])/(2*f^(3/2)))/4
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.1.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

method	result
risch	$\frac{b(2fx+e)\sqrt{f\,x^2+ex+d}}{4fe}+\frac{(8aef-4bdf-b\,e^2)\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{8f^{\frac{3}{2}}e}$
default	$\frac{\frac{ae\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{\sqrt{f}}+bf\left(\frac{x\sqrt{f\,x^2+ex+d}}{2f}-\frac{3e\left(\frac{\sqrt{f\,x^2+ex+d}}{f}-\frac{e\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{2f^{\frac{3}{2}}}\right)}{4f}-\frac{d\ln\left(\frac{\frac{e}{2}+fx}{\sqrt{f}}+\sqrt{f\,x^2+ex+d}\right)}{2f^{\frac{3}{2}}}\right)+be\left(\right.}{e}$

input
```
int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*(2*f*x+e)*(f*x^2+e*x+d)^(1/2)/f/e+1/8*(8*a*e*f-4*b*d*f-b*e^2)/f^(3/2
)*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))/e
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.01

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx$$

$$=\left[-\frac{(be^2+4\,(bd-2\,ae)f)\sqrt{f}\log\left(-8\,f^2x^2-8\,efx-e^2-4\,\sqrt{fx^2+ex+d}(2\,fx+e)\sqrt{f}-4\,df\right)-4\,(2\,b}{16\,ef^2}\right.$$

input
```
integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqrt(f)*log(-8*f^2*x^2 - 8*e*f*x - e^2
 - 4*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(f) - 4*d*f) - 4*(2*b*f^2*x + b
*e*f)*sqrt(f*x^2 + e*x + d))/(e*f^2), 1/8*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqr
t(-f)*arctan(1/2*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(-f)/(f^2*x^2 + e*f
*x + d*f)) + 2*(2*b*f^2*x + b*e*f)*sqrt(f*x^2 + e*x + d))/(e*f^2)]
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(85) = 170.

Time = 0.38 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.10

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx = \begin{cases} \left(\frac{b}{4f}+\frac{bx}{2e}\right)\sqrt{d+ex+fx^2}+\left(a-\frac{bd}{2e}-\frac{be}{8f}\right)\left(\begin{cases} \frac{\log\left(e+2\sqrt{f}\sqrt{d+ex+fx^2}+2fx\right)}{\sqrt{f}} & \text{for } d-\frac{e^2}{4f}\neq 0 \\ \frac{\left(\frac{e}{2f}+x\right)\log\left(\frac{e}{2f}+x\right)}{\sqrt{f\left(\frac{e}{2f}+x\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for } f\neq 0 \\ \frac{2a\sqrt{d+ex}+\frac{2b\left(-d\sqrt{d+ex}+\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}+\frac{2bf\left(d^2\sqrt{d+ex}-\frac{2d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^3}}{e} & \text{for } e\neq 0 \\ \frac{ax+\tilde{\infty}bfx^3+\frac{bx^2}{2}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

output
```
Piecewise((((b/(4*f) + b*x/(2*e))*sqrt(d + e*x + f*x**2) + (a - b*d/(2*e) -
 b*e/(8*f))*Piecewise((log(e + 2*sqrt(f)*sqrt(d + e*x + f*x**2) + 2*f*x)/s
qrt(f), Ne(d - e**2/(4*f), 0)), ((e/(2*f) + x)*log(e/(2*f) + x)/sqrt(f*(e/
(2*f) + x)**2), True)), Ne(f, 0)), ((2*a*sqrt(d + e*x) + 2*b*(-d*sqrt(d +
e*x) + (d + e*x)**(3/2)/3)/e + 2*b*f*(d**2*sqrt(d + e*x) - 2*d*(d + e*x)**
(3/2)/3 + (d + e*x)**(5/2)/5)/e**3)/e, Ne(e, 0)), ((a*x + zoo*b*f*x**3 + b
*x**2/2)/sqrt(d), True))
```

3.1.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx = \frac{1}{4}\sqrt{fx^2+ex+d}\left(\frac{2\,bx}{e}+\frac{b}{f}\right) + \frac{(be^2+4\,bdf-8\,aef)\log\left(\left|2\left(\sqrt{f}x-\sqrt{fx^2+ex+d}\right)\sqrt{f}+e\right|\right)}{8\,ef^{\frac{3}{2}}}$$

input
```
integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(f*x^2 + e*x + d)*(2*b*x/e + b/f) + 1/8*(b*e^2 + 4*b*d*f - 8*a*e*f
)*log(abs(2*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*sqrt(f) + e))/(e*f^(3/2))
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx = \int \frac{a+b\,x+\frac{b\,f\,x^2}{e}}{\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2),x)
```

output
```
int((a + b*x + (b*f*x^2)/e)/(d + e*x + f*x^2)^(1/2), x)
```

3.2 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$

3.2.1 Optimal result

Integrand size = 31, antiderivative size = 82

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx=-\frac{2\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

output
```
-2*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)/e^(1/2)/(-4*a*f+b*e)^(1/2)/(f*x^2+e*
x+d)^(1/2))*e^(1/2)/(-a*e+b*d)^(1/2)/(-4*a*f+b*e)^(1/2)
```

3.2.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.49

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$$
$$= e\text{RootSum}\Big[aef^2-2b\sqrt{d}ef\#1+be^2\#1^2+4bdf\#1^2-2aef\#1^2-2b\sqrt{d}e\#1^3$$
$$+ae\#1^4\&,\frac{-f\log(x)+f\log\left(-\sqrt{d}+\sqrt{d+ex+fx^2}-x\#1\right)+\log(x)\#1^2-\log\left(-\sqrt{d}+\sqrt{d+ex+f}\right.}{-b\sqrt{d}ef+be^2\#1+4bdf\#1-2aef\#1-3b\sqrt{d}e\#1^2+2ae\#1^3}$$

input
```
Integrate[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

output
```
e*RootSum[a*e*f^2 - 2*b*Sqrt[d]*e*f*#1 + b*e^2*#1^2 + 4*b*d*f*#1^2 - 2*a*e
*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 + a*e*#1^4 & , (-(f*Log[x]) + f*Log[-Sqrt[d]
+ Sqrt[d + e*x + f*x^2] - x*#1] + Log[x]*#1^2 - Log[-Sqrt[d] + Sqrt[d + e*
x + f*x^2] - x*#1]*#1^2)/(-(b*Sqrt[d]*e*f) + b*e^2*#1 + 4*b*d*f*#1 - 2*a*e
*f*#1 - 3*b*Sqrt[d]*e*#1^2 + 2*a*e*#1^3) & ]
```

3.2.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+\frac{bfx^2}{e}+bx\right)}dx$$

$$\Big\downarrow \text{1313}$$

$$-2e\int \frac{1}{e(be-4af)-\frac{(bd-ae)(e+2fx)^2}{fx^2+ex+d}}d\frac{e+2fx}{\sqrt{fx^2+ex+d}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\sqrt{e}\text{arctanh}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

input
```
Int[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

output
```
(-2*Sqrt[e]*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*
f]*Sqrt[d + e*x + f*x^2])])/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

3.2.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

3.2.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(68) = 136$.

Time = 1.00 (sec) , antiderivative size = 491, normalized size of antiderivative = 5.99

method	result
default	$e\left(-\frac{\ln\left(\frac{-\frac{2(ae-bd)}{b}+\frac{\sqrt{-be(4fa-be)}\left(x-\frac{-be+\sqrt{-be(4fa-be)}}{2bf}\right)}{b}+2\sqrt{-\frac{ae-bd}{b}}\sqrt{\left(x-\frac{-be+\sqrt{-be(4fa-be)}}{2bf}\right)^{2}f+\frac{\sqrt{-be(4fa-be)}\left(x-\frac{-be+\sqrt{}}{}\right.}{b}}}{x-\frac{-be+\sqrt{-be(4fa-be)}}{2bf}}\right)}{\sqrt{-be(4fa-be)}\sqrt{-\frac{ae-bd}{b}}}\right.$

input
```
int(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
e*(-1/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b
*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a
*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4
*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b
)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))+1/(-b*e*(4*a*f-b*e))
^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*
(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*
(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*
(4*a*f-b*e))^(1/2))/b/f)))
```

3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(68) = 136.

Time = 0.48 (sec) , antiderivative size = 1079, normalized size of antiderivative = 13.16

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx$$

$$= \left[\frac{1}{2}\sqrt{\frac{e}{b^2de-abe^2-4\,(abd-a^2e)f}}\log\left(\frac{8\,b^2d^2e^4-8\,abde^5+a^2e^6+16\,a^2d^2e^2f^2+(b^2e^4f^2+16\,(b^2d^2-\right.\right.$$

$$-\sqrt{-\frac{e}{b^2de-abe^2-4\,(abd-a^2e)f}}\arctan\left(-\frac{(2\,bde^2-ae^3-4\,adef+(be^2f+4\,(bd-2\,ae)f^2)x^2+(be}{2\,(2\,ef^2x^3+3\,e^2fx}\right.$$

input
```
integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*log((8*b^2*d^2*e^4
- 8*a*b*d*e^5 + a^2*e^6 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2
- 8*a*b*d*e + 8*a^2*e^2)*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b
^2*e^5*f + 16*(b^2*d^2*e - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 -
 4*a*b*e^4)*f^2)*x^3 + (b^2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*
(3*b^2*d^2*e^2 - 13*a*b*d*e^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b
*e^5)*f)*x^2 - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*
e^6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^
4 + 2*a^2*e^5)*f)*x - 4*(2*b^3*d^2*e^4 - 3*a*b^2*d*e^5 + a^2*b*e^6 - 2*(16
*(a*b^2*d^2 - 3*a^2*b*d*e + 2*a^3*e^2)*f^4 - 4*(b^3*d^2*e - 4*a*b^2*d*e^2
+ 3*a^2*b*e^3)*f^3 - (b^3*d*e^3 - a*b^2*e^4)*f^2)*x^3 + 16*(a^2*b*d^2*e^2
- a^3*d*e^3)*f^2 - 3*(16*(a*b^2*d^2*e - 3*a^2*b*d*e^2 + 2*a^3*e^3)*f^3 - 4
*(b^3*d^2*e^2 - 4*a*b^2*d*e^3 + 3*a^2*b*e^4)*f^2 - (b^3*d*e^4 - a*b^2*e^5)
*f)*x^2 - 4*(3*a*b^2*d^2*e^3 - 4*a^2*b*d*e^4 + a^3*e^5)*f + (b^3*d*e^5 - a
*b^2*e^6 + 32*(a^2*b*d^2*e - a^3*d*e^2)*f^3 - 40*(a*b^2*d^2*e^2 - 2*a^2*b*
d*e^3 + a^3*e^4)*f^2 + 2*(4*b^3*d^2*e^3 - 11*a*b^2*d*e^4 + 7*a^2*b*e^5)*f)
*x)*sqrt(f*x^2 + e*x + d)*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f)
))/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b^2*e^2 + 2*a*b
*e*f)*x^2)), -sqrt(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*arctan(-1
/2*(2*b*d*e^2 - a*e^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2...
```

3.2.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = e\int \frac{1}{ae\sqrt{d+ex+fx^2}+bex\sqrt{d+ex+fx^2}+bfx^2\sqrt{d+ex+fx^2}}\,dx$$

input
```
integrate(1/(a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

output
```
e*Integral(1/(a*e*sqrt(d + e*x + f*x**2) + b*e*x*sqrt(d + e*x + f*x**2) +
b*f*x**2*sqrt(d + e*x + f*x**2)), x)
```

3.2.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e*(4*a*f-b*e)>0)', see `assume?`
 for more
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(68) = 136$.

Time = 0.96 (sec) , antiderivative size = 851, normalized size of antiderivative = 10.38

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = -\frac{\sqrt{b^2de^2-abe^3-4\,abdef+4\,a^2e^2f}\log\left(\left|-\left(\sqrt{f}x-\sqrt{fx^2+ex+d}\right)^2be^2f-4\left(\sqrt{f}x-\sqrt{fx^2+ex+d}\right.\right.\right.}{} + \frac{\sqrt{b^2de^2-abe^3-4\,abdef+4\,a^2e^2f}\log\left(\left|-\left(\sqrt{f}x-\sqrt{fx^2+ex+d}\right)^2be^2f-4\left(\sqrt{f}x-\sqrt{fx^2+ex+d}\right.\right.\right.}{}$$

input
```
integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*log(abs(-(sqrt(f)*x
 - sqrt(f*x^2 + e*x + d))^2*b*e^2*f - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d)
)^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*a*e*f^2 - (sqrt(f)*x
 - sqrt(f*x^2 + e*x + d))*b*e^3*sqrt(f) - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x
+ d))*b*d*e*f^(3/2) + 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*a*e^2*f^(3/2)
- 3*b*d*e^2*f + 2*a*e^3*f + 4*b*d^2*f^2 + 4*sqrt(b^2*d*e^2 - a*b*e^3 - 4*a
*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*f^(3/2) + 4*
sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*
x^2 + e*x + d))*e*f + sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f
)*e^2*sqrt(f)))/(b^2*d*e - a*b*e^2 - 4*a*b*d*f + 4*a^2*e*f) + sqrt(b^2*d*e
^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*log(abs(-(sqrt(f)*x - sqrt(f*x^2
 + e*x + d))^2*b*e^2*f - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*b*d*f^2 +
 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*a*e*f^2 - (sqrt(f)*x - sqrt(f*x^2
 + e*x + d))*b*e^3*sqrt(f) - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*b*d*e*f
^(3/2) + 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*a*e^2*f^(3/2) - 3*b*d*e^2*f
 + 2*a*e^3*f + 4*b*d^2*f^2 - 4*sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*
a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*f^(3/2) - 4*sqrt(b^2*d*e^
2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d
))*e*f - sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*e^2*sqrt(f)
))/(b^2*d*e - a*b*e^2 - 4*a*b*d*f + 4*a^2*e*f)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)}\,dx = \int \frac{1}{\sqrt{f\,x^2+e\,x+d}\,\left(a+b\,x+\frac{b\,f\,x^2}{e}\right)}\,dx$$

input
```
int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)),x)
```

output
```
int(1/((d + e*x + f*x^2)^(1/2)*(a + b*x + (b*f*x^2)/e)), x)
```

3.3 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)}\,dx$

3.3.1 Optimal result

Integrand size = 27, antiderivative size = 66

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

output
```
-2*arctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a
-d)^(1/2)/(b^2-4*c*d)^(1/2)
```

3.3.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.86

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx$$
$$=\text{RootSum}\Big[c^2d-2\sqrt{a}bc\#1+b^2\#1^2+4ac\#1^2-2cd\#1^2-2\sqrt{a}b\#1^3$$
$$+d\#1^4\&,\frac{-c\log(x)+c\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\#1\right)+\log(x)\#1^2-\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}\right.}{-\sqrt{a}bc+b^2\#1+4ac\#1-2cd\#1-3\sqrt{a}b\#1^2+2d\#1^3}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

output
```
RootSum[c^2*d - 2*Sqrt[a]*b*c*#1 + b^2*#1^2 + 4*a*c*#1^2 - 2*c*d*#1^2 - 2*
Sqrt[a]*b*#1^3 + d*#1^4 & , (-(c*Log[x]) + c*Log[-Sqrt[a] + Sqrt[a + b*x +
 c*x^2] - x*#1] + Log[x]*#1^2 - Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
1]*#1^2)/(-(Sqrt[a]*b*c) + b^2*#1 + 4*a*c*#1 - 2*c*d*#1 - 3*Sqrt[a]*b*#1^2
 + 2*d*#1^3) & ]
```

3.3.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)}\,dx$$

$$\Big\downarrow \text{1313}$$

$$-2b\int \frac{1}{b\left(b^2-4cd\right)-\frac{b(a-d)(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

output
```
(-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*
x^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

3.3.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

3.3.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(56) = 112$.

Time = 0.93 (sec) , antiderivative size = 307, normalized size of antiderivative = 4.65

method	result
default	$-\frac{\ln\left(\frac{2a-2d+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)^2 c+\sqrt{b^2-4cd}\left(x-\frac{-b+\sqrt{b^2-4cd}}{2c}\right)+a-d}}{x-\frac{-b+\sqrt{b^2-4cd}}{2c}}\right)}{\sqrt{b^2-4cd}\,\sqrt{a-d}}+\ln\left(2a-2d-\sqrt{b^2-}\right.$

input
```
int(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+
(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c))+1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*
c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4
*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)
^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c))
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(56) = 112.

Time = 0.37 (sec) , antiderivative size = 813, normalized size of antiderivative = 12.32

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx$$
$$= \left[\log\left(\frac{8\,a^2b^4+(b^4c^2+24\,ab^2c^3+16\,a^2c^4+128\,c^4d^2-32\,(b^2c^3+4\,ac^4)\,d)\,x^4+2\,(b^5c+24\,ab^3c^2+16\,a^2bc^3+128\,bc^3d^2-32\,(b^3c^2+4\,abc^3)\,d)\,x^3+(b^4+\cdots}{\cdots}\right.\right.$$
$$-\frac{\sqrt{-ab^2-4\,cd^2+(b^2+4\,ac)d}\,\arctan\left(-\frac{(2\,ab^2+(b^2c+4\,ac^2-8\,c^2d)\,x^2-(b^2+4\,ac)\,d+(b^3+4\,abc\cdots}{2\,(a^2b^3+4\,abcd^2+2\,(ab^2c^2+4\,c^3d^2-(b^2c^2+4\,ac^3)\,d)\,x^3+3\,(ab^3c+4\,bc^2d^2-(b^3c+4\,abc^2\cdots}\right.}{ab^2+4\,cd^2-(b^2+4\,ac)d}$$

input
```
integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 -
 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 +
128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a
^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3
)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*
(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2
- (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(
a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^
2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b
^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^
2 + 2*c*d)*x^2 + d^2))/sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d), -sqrt(-a*b
^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 -
8*c^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2
- 4*c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2
+ 2*(a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b
*c^2*d^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2
*a^2*b^2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)
)/(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)]
```

3.3.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)}\,dx$$

input
```
integrate(1/(c*x**2+b*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)), x)
```

3.3.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c*d-b^2>0)', see `assume?` for
 more deta
```

3.3.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)}\,dx=\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+d\right)}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)), x)
```

3.4 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^2}\,dx$

3.4.1 Optimal result

Integrand size = 27, antiderivative size = 129

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\,(b^2-4cd)\,(d+bx+cx^2)} + \frac{(b^2+4c(a-2d))\,\operatorname{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\,(b^2-4cd)^{3/2}}$$

output
```
(b^2+4*c*(a-2*d))*arctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b
*x+a)^(1/2))/(a-d)^(3/2)/(b^2-4*c*d)^(3/2)-(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(
a-d)/(b^2-4*c*d)/(c*x^2+b*x+d)
```

3.4.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.02 (sec) , antiderivative size = 1333, normalized size of antiderivative = 10.33

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = -\frac{\text{RootSum}\left[c^2d-2\sqrt{a}bc\#1+b^2\#1^2+4ac\#1^2-2cd\#1^2-2\sqrt{a}b\#1^3+d\#1^4\&,\frac{-4ab^2\log(x)+b^2d\log(x)+4ac\cdots}{\cdots}\right.}{\cdots} + \frac{\frac{2(b+2cx)\sqrt{a+x(b+cx)}}{d+x(b+cx)}-\frac{\text{RootSum}\left[c^2d-2\sqrt{a}bc\#1+b^2\#1^2+4ac\#1^2-2cd\#1^2-2\sqrt{a}b\#1^3+d\#1^4\&,\frac{-8a^2b^4\log(x)+10ab^4d\log(x)+\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

output
```
-(RootSum[c^2*d - 2*Sqrt[a]*b*c*#1 + b^2*#1^2 + 4*a*c*#1^2 - 2*c*d*#1^2 -
2*Sqrt[a]*b*#1^3 + d*#1^4 & , (-4*a*b^2*Log[x] + b^2*d*Log[x] + 4*a*c*d*Lo
g[x] + c*d^2*Log[x] + 4*a*b^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]
 - b^2*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a*c*d*Log[-Sqrt[
a] + Sqrt[a + b*x + c*x^2] - x*#1] - c*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c
*x^2] - x*#1] - 2*Sqrt[a]*b*d*Log[x]*#1 + 2*Sqrt[a]*b*d*Log[-Sqrt[a] + Sqr
t[a + b*x + c*x^2] - x*#1]*#1 - d^2*Log[x]*#1^2 + d^2*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*b*c) + b^2*#1 + 4*a*c*#1 - 2*c*d
*#1 - 3*Sqrt[a]*b*#1^2 + 2*d*#1^3) & ]/d^3) + ((2*(b + 2*c*x)*Sqrt[a + x*(
b + c*x)])/(d + x*(b + c*x)) - RootSum[c^2*d - 2*Sqrt[a]*b*c*#1 + b^2*#1^2
 + 4*a*c*#1^2 - 2*c*d*#1^2 - 2*Sqrt[a]*b*#1^3 + d*#1^4 & , (-8*a^2*b^4*Log
[x] + 10*a*b^4*d*Log[x] + 40*a^2*b^2*c*d*Log[x] - 2*b^4*d^2*Log[x] - 46*a*
b^2*c*d^2*Log[x] - 32*a^2*c^2*d^2*Log[x] + 7*b^2*c*d^3*Log[x] + 28*a*c^2*d
^3*Log[x] + 8*a^2*b^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 10*a*
b^4*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 40*a^2*b^2*c*d*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2*b^4*d^2*Log[-Sqrt[a] + Sqrt[a +
 b*x + c*x^2] - x*#1] + 46*a*b^2*c*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2
] - x*#1] + 32*a^2*c^2*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] -
7*b^2*c*d^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 28*a*c^2*d^3*Lo
g[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a^(3/2)*b^3*d*Log[x]*#1 ...
```

3.4.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1305, 27, 1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)^2}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{\int -\frac{c^2(b^2+4c(a-2d))(a-d)}{2\sqrt{cx^2+bx+a}(cx^2+bx+d)}dx}{c^2(a-d)^2(b^2-4cd)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)(b^2-4cd)(bx+cx^2+d)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(4c(a-2d)+b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}(cx^2+bx+d)}dx}{2(a-d)\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

$$\Big\downarrow \text{ 1313}$$

$$\frac{b\left(4c(a-2d)+b^2\right)\int \frac{1}{b(b^2-4cd)-\frac{b(a-d)(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{(a-d)\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\left(4c(a-2d)+b^2\right)\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

output
```
-(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^
2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2))
```

3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

3.4.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(117) = 234$.

Time = 1.01 (sec) , antiderivative size = 827, normalized size of antiderivative = 6.41

method	result
default	$-\frac{-\frac{\sqrt{\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)^2 c-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)+a-d}}{(a-d)\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)}-\frac{\sqrt{b^2-4cd}\ln\left(\frac{2a-2d-\sqrt{b^2-4cd}\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)+2\sqrt{a-d}\sqrt{\left(x+\frac{b+\sqrt{b^2-4cd}}{2c}\right)}}{x+\frac{b+\sqrt{b^2-4cd}}{2c}}\right)}{2(a-d)^{\frac{3}{2}}}}{b^2-4cd}$

input
```
int(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b^2-4*c*d)*(-1/(a-d)/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)*((x+1/2*(b+(b^2-4*
c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^
(1/2)-1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1
/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)
^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*
(b+(b^2-4*c*d)^(1/2))/c)))+1/(b^2-4*c*d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^
(1/2))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(
-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((
2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*(
(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))+2/(b^2-4*c*d)
^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^
(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^
(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^
(1/2))/c))-2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)
*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2
))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x
+1/2*(b+(b^2-4*c*d)^(1/2))/c))
```

3.4.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(117) = 234$.

Time = 0.60 (sec) , antiderivative size = 1544, normalized size of antiderivative = 11.97

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2
- 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*log((8*a^2
*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 +
4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 -
 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (
b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b
^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*
c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*
c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2
 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*
(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c
 + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2
+ d^2)) - 4*(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^
2 - (b^2*c + 4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5
 - 8*(b^2*c + 4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^
4 + 4*a^2*b^2*c)*d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3
 + (b^4*c + 16*a*b^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d
)*x^2 + (a^2*b^5 + 16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*
b^3*c + 16*a^2*b*c^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x), -1/2*(sqrt(-a*b
^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*...
```

3.4.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)^2}\,dx$$

input
```
integrate(1/(c*x**2+b*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)**2), x)
```

3.4.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(cx^2+bx+d)^2}\,dx$$

input
```
integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^2), x)
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. 2(117) = 234.

Time = 0.88 (sec) , antiderivative size = 1170, normalized size of antiderivative = 9.07

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*((b^2 + 4*a*c - 8*c*d)*log(abs(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
b^2*c - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^2*c^2*d - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt
(c) - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*b*c^(3/2)*d - 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a
*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2
+ 2*b^2*c*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*b*c + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c
)))/sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) - (b^2 + 4*a*c - 8*c*d)*log(ab
s(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c - 4*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a*c^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^2*d - (sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) - 4*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*a*b*c^(3/2) + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d
 - 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*c^(3/2) + 4*a^2*c^2 + 2*b^2*c*d - 4*sqrt(a*b^2 - b^2*
d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c - sqrt(a*b^
2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^2*d - 4*a*c*d
+ 4*c*d^2))/(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2) + ((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*b^2*sqrt(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(
3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2)*d + (sqrt(c)*x -...
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^2}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+d)^2}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^2), x)
```

3.5 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^3}\,dx$

3.5.1 Optimal result

Integrand size = 27, antiderivative size = 224

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^2}+\frac{3(b^2+4c(a-2d))\,(b+2cx)\sqrt{a+bx+cx^2}}{4(a-d)^2\,(b^2-4cd)^2\,(d+bx+cx^2)} \\ &\quad -\frac{(3b^4+8b^2c(a-4d)+16c^2(3a^2-8ad+8d^2))\,\operatorname{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\,(b^2-4cd)^{5/2}}\end{aligned}$$

output
```
-1/4*(3*b^4+8*b^2*c*(a-4*d)+16*c^2*(3*a^2-8*a*d+8*d^2))*arctanh((2*c*x+b)*
(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-d)^(5/2)/(b^2-4*c*d)
^(5/2)-1/2*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d)/(c*x^2+b*x+d)^2
+3/4*(b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^2/(b^2-4*c*d)^2
/(c*x^2+b*x+d)
```

3.5.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 940 vs. 2(224) = 448.

Time = 14.81 (sec) , antiderivative size = 940, normalized size of antiderivative = 4.20

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx$$

$$\begin{aligned}
&= -\frac{6c^2\sqrt{a+x(b+cx)}}{(a-d)\,(b^2-4cd)^2\,\left(-b+\sqrt{b^2-4cd}-2cx\right)} \\
&+ \frac{2c^2\sqrt{a+x(b+cx)}}{(-a+d)\,(b^2-4cd)^{3/2}\,\left(b-\sqrt{b^2-4cd}+2cx\right)^2} \\
&+ \frac{3c\sqrt{a+x(b+cx)}}{2(a-d)^2\,(b^2-4cd)\,\left(b-\sqrt{b^2-4cd}+2cx\right)} \\
&- \frac{2c^2\sqrt{a+x(b+cx)}}{(-a+d)\,(b^2-4cd)^{3/2}\,\left(b+\sqrt{b^2-4cd}+2cx\right)^2} \\
&+ \frac{6c^2\sqrt{a+x(b+cx)}}{(a-d)\,(b^2-4cd)^2\,\left(b+\sqrt{b^2-4cd}+2cx\right)} \\
&+ \frac{3c\sqrt{a+x(b+cx)}}{2(a-d)^2\,(b^2-4cd)\,\left(b+\sqrt{b^2-4cd}+2cx\right)} \\
&- \frac{(-3b^2+4c(a+2d))\,\text{arctanh}\left(\frac{-b^2+4ac-b\sqrt{b^2-4cd}-2c\sqrt{b^2-4cd}x}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{8(a-d)^{5/2}\,(b^2-4cd)^{3/2}} \\
&+ \frac{6c^2\text{arctanh}\left(\frac{4ac-b\left(b+\sqrt{b^2-4cd}\right)-2c\sqrt{b^2-4cd}x}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a-d}\,(b^2-4cd)^{5/2}} + \frac{3c\text{arctanh}\left(\frac{4ac-b\left(b+\sqrt{b^2-4cd}\right)-2c\sqrt{b^2-4cd}x}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{2(a-d)^{3/2}\,(b^2-4cd)^{3/2}} \\
&+ \frac{6c^2\text{arctanh}\left(\frac{b^2-b\sqrt{b^2-4cd}-2c\left(2a+\sqrt{b^2-4cd}x\right)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a-d}\,(b^2-4cd)^{5/2}} + \frac{3c\text{arctanh}\left(\frac{b^2-b\sqrt{b^2-4cd}-2c\left(2a+\sqrt{b^2-4cd}x\right)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{2(a-d)^{3/2}\,(b^2-4cd)^{3/2}} \\
&- \frac{(-3b^2+4c(a+2d))\,\text{arctanh}\left(\frac{b^2-b\sqrt{b^2-4cd}-2c\left(2a+\sqrt{b^2-4cd}x\right)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{8(a-d)^{5/2}\,(b^2-4cd)^{3/2}}
\end{aligned}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

output
```
(-6*c^2*Sqrt[a + x*(b + c*x)])/((a - d)*(b^2 - 4*c*d)^2*(-b + Sqrt[b^2 - 4
*c*d] - 2*c*x)) + (2*c^2*Sqrt[a + x*(b + c*x)])/((-a + d)*(b^2 - 4*c*d)^(3
/2)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)^2) + (3*c*Sqrt[a + x*(b + c*x)])/(2*(a
 - d)^2*(b^2 - 4*c*d)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)) - (2*c^2*Sqrt[a + x
*(b + c*x)])/((-a + d)*(b^2 - 4*c*d)^(3/2)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)
^2) + (6*c^2*Sqrt[a + x*(b + c*x)])/((a - d)*(b^2 - 4*c*d)^2*(b + Sqrt[b^2
 - 4*c*d] + 2*c*x)) + (3*c*Sqrt[a + x*(b + c*x)])/(2*(a - d)^2*(b^2 - 4*c*
d)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) - ((-3*b^2 + 4*c*(a + 2*d))*ArcTanh[(-
b^2 + 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a -
 d]*Sqrt[a + x*(b + c*x)])])/(8*(a - d)^(5/2)*(b^2 - 4*c*d)^(3/2)) + (6*c^
2*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4
*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/(Sqrt[a - d]*(b^2 - 4*c*d)^(5/2))
+ (3*c*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*
x)/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/(2*(a - d)^(3/2)*(b^2 - 4*c*d
)^(3/2)) + (6*c^2*ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] - 2*c*(2*a + Sqrt[b^2
 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/(Sqrt[a - d]*(b^2
- 4*c*d)^(5/2)) + (3*c*ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] - 2*c*(2*a + Sqr
t[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/(2*(a - d)^(3
/2)*(b^2 - 4*c*d)^(3/2)) - ((-3*b^2 + 4*c*(a + 2*d))*ArcTanh[(b^2 - b*Sqrt
[b^2 - 4*c*d] - 2*c*(2*a + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[...
```

3.5.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1305, 27, 2135, 27, 1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(bx+cx^2+d\right)^3}dx$$

$$\Big\downarrow \; 1305$$

$$\frac{\int -\frac{8(a-d)x^2c^4+8b(a-d)xc^3+(a-d)\left(3b^2+12ac-16cd\right)c^2}{2\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}dx}{2c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{8(a-d)x^2c^4+8b(a-d)xc^3+(a-d)\left(3b^2+12ac-16cd\right)c^2}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}dx}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \text{ 2135}$$

$$-\frac{-\frac{\int \frac{c^4(a-d)^2\left(3b^4+8c(a-4d)b^2+16c^2\left(3a^2-8da+8d^2\right)\right)}{2\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)}dx}{c^2(a-d)^2(b^2-4cd)}-\frac{3c^2(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{(b^2-4cd)(bx+cx^2+d)}}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{c^2\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\int \frac{1}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)}dx}{2(b^2-4cd)}-\frac{3c^2(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{(b^2-4cd)(bx+cx^2+d)}}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \text{ 1313}$$

$$-\frac{\frac{bc^2\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\int \frac{1}{b\left(b^2-4cd\right)-\frac{b(a-d)(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{b^2-4cd}-\frac{3c^2(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{(b^2-4cd)(bx+cx^2+d)}}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{c^2\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}(b^2-4cd)^{3/2}}-\frac{3c^2(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{(b^2-4cd)(bx+cx^2+d)}}{4c^2(a-d)^2\left(b^2-4cd\right)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^2}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

output
```
-1/2*((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x +
 c*x^2)^2) - ((-3*c^2*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x
^2])/((b^2 - 4*c*d)*(d + b*x + c*x^2)) + (c^2*(3*b^4 + 8*b^2*c*(a - 4*d) +
 16*c^2*(3*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b
^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a - d]*(b^2 - 4*c*d)^(3/2)))/(4
*c^2*(a - d)^2*(b^2 - 4*c*d))
```

3.5. $\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^3}\,dx$

3.5.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. 2(204) = 408.

Time = 1.32 (sec) , antiderivative size = 1868, normalized size of antiderivative = 8.34

method	result	size
default	Expression too large to display	1868

input
```
int(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/(b^2-4*c*d)^2*c*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*((x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/
c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1
/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)
^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/
2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))-3/(b^2-4*c*d)^2*c*(-1/(a-d)/(x+1/2*
(b+(b^2-4*c*d)^(1/2))/c)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^
(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-1/2*(b^2-4*c*d)^(1/2)/(a-
d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)+2*(
a-d)^(1/2)*((x+1/2*(b+(b^2-4*c*d)^(1/2))/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*(
b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x+1/2*(b+(b^2-4*c*d)^(1/2))/c)))+1/(b
^2-4*c*d)^(3/2)*(-1/2/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*((x-1/2*(-b
+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))
/c)+a-d)^(1/2)-3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d
)^(1/2))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2
*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln
((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)
*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4
*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)))+1/2*c/(a-d)
^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2...
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(204) = 408$.

Time = 2.89 (sec) , antiderivative size = 3818, normalized size of antiderivative = 17.04

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d^2
- 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c +
8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x
^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^
2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*
c^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 +
 (3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*
a*c)*d)*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^
2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3
 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 1
6*a^2*c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*
c^3)*d^2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 +
 2*(b^2*c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x
^2 - (b^3 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sq
rt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4
*a^2*b^2*c)*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (
3*b^5 + 40*a*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x +
(b^2 + 2*c*d)*x^2 + d^2)) - 4*(2*a^2*b^5 + 128*b*c^2*d^4 - 52*(b^3*c + 4*a
*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + 12*(b^2*c^3 + 4*
a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 + 5*(b^5 + 16...
```

3.5.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**2+b*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Timed out
```

3.5.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(cx^2+bx+d)^3}\,dx$$

input
```
integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^3), x)
```

3.5.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^3}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+d)^3}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^3), x)
```

3.6 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4}\,dx$

3.6.1 Optimal result

Integrand size = 27, antiderivative size = 328

$$\begin{aligned}
&\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\,(b^2-4cd)\,(d+bx+cx^2)^3}+\frac{5(b^2+4c(a-2d))\,(b+2cx)\sqrt{a+bx+cx^2}}{12(a-d)^2\,(b^2-4cd)^2\,(d+bx+cx^2)^2} \\
&\quad -\frac{(15b^4+8b^2c(7a-22d)+16c^2(15a^2-44ad+44d^2))\,(b+2cx)\sqrt{a+bx+cx^2}}{24(a-d)^3\,(b^2-4cd)^3\,(d+bx+cx^2)} \\
&\quad +\frac{(b^2+4c(a-2d))\,(5b^4-8b^2c(a+4d)+16c^2(5a^2-8ad+8d^2))\,\mathrm{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{8(a-d)^{7/2}\,(b^2-4cd)^{7/2}}
\end{aligned}$$

output
```
1/8*(b^2+4*c*(a-2*d))*(5*b^4-8*b^2*c*(a+4*d)+16*c^2*(5*a^2-8*a*d+8*d^2))*a
rctanh((2*c*x+b)*(a-d)^(1/2)/(b^2-4*c*d)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a-d)^
(7/2)/(b^2-4*c*d)^(7/2)-1/3*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)/(b^2-4*c*d
)/(c*x^2+b*x+d)^3+5/12*(b^2+4*c*(a-2*d))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-
d)^2/(b^2-4*c*d)^2/(c*x^2+b*x+d)^2-1/24*(15*b^4+8*b^2*c*(7*a-22*d)+16*c^2*
(15*a^2-44*a*d+44*d^2))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/(a-d)^3/(b^2-4*c*d)^
3/(c*x^2+b*x+d)
```

3.6.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3382 vs. 2(328) = 656.

Time = 16.44 (sec) , antiderivative size = 3382, normalized size of antiderivative = 10.31

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

output
```
(-8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*
d] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) + (8*c^3*(a + b*x + c*x^2))/((a - d)*
(b^2 - 4*c*d)^(5/2)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)
]) - (20*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^3*(b - Sqrt[b^2 - 4
*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (8*c^3*(a + b*x + c*x^2))/(3*(a -
d)*(b^2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]
) - (8*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(5/2)*(b + Sqrt[b^2 -
 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) - (20*c^3*(a + b*x + c*x^2))/((a
 - d)*(b^2 - 4*c*d)^3*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)
]) - (20*c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c
*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/(
Sqrt[a - d]*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*c^2*Sqrt[a + b
*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*
c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/((a - d)^(3/2)*(b^2 - 4*
c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) - (20*c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[
(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a
- d]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[a - d]*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*
(b + c*x)]) - (5*c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^
2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x
^2])]])/((a - d)^(3/2)*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) - (16*...
```

3.6.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1305, 27, 2135, 27, 2135, 27, 1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(bx+cx^2+d)^4}\,dx$$

$\big\downarrow$ 1305

$$\frac{\int -\frac{16(a-d)x^2c^4+16b(a-d)xc^3+(a-d)(5b^2+20ac-24cd)c^2}{2\sqrt{cx^2+bx+a}(cx^2+bx+d)^3}dx}{3c^2(a-d)^2(b^2-4cd)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(bx+cx^2+d)^3}$$

$\big\downarrow$ 27

$$-\frac{\int \frac{16(a-d)x^2c^4+16b(a-d)xc^3+(a-d)(5b^2+20ac-24cd)c^2}{\sqrt{cx^2+bx+a}(cx^2+bx+d)^3}dx}{6c^2(a-d)^2(b^2-4cd)}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(bx+cx^2+d)^3}$$

$\big\downarrow$ 2135

$$-\frac{-\frac{\int \frac{40\left(b^2+4c(a-2d)\right)(a-d)^2x^2c^6+40b\left(b^2+4c(a-2d)\right)(a-d)^2xc^5+(a-d)^2\left(15b^4+8c(7a-17d)b^2+16c^2\left(15a^2-34da+24d^2\right)\right)c^4}{2\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}dx}{2c^2(a-d)^2(b^2-4cd)}-\frac{5c^2(b+2cx)(4c(a-2d}{2(b^2-4cd)(bx}}{6c^2(a-d)^2(b^2-4cd)}$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(bx+cx^2+d)^3}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\int \frac{40\left(b^2+4c(a-2d)\right)(a-d)^2x^2c^6+40b\left(b^2+4c(a-2d)\right)(a-d)^2xc^5+(a-d)^2\left(15b^4+8c(7a-17d)b^2+16c^2\left(15a^2-34da+24d^2\right)\right)c^4}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^2}dx}{4c^2(a-d)^2(b^2-4cd)}-\frac{5c^2(b+2cx)(4c(a-2d}{2(b^2-4cd)(bx}}{6c^2(a-d)^2(b^2-4cd)}$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(bx+cx^2+d)^3}$$

$\big\downarrow$ 2135

$$-\frac{-\frac{-\frac{\int \frac{3c^6\left(b^2+4c(a-2d)\right)(a-d)^3\left(5b^4-8c(a+4d)b^2+16c^2\left(5a^2-8da+8d^2\right)\right)}{2\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)}dx}{c^2(a-d)^2\left(b^2-4cd\right)}-\frac{c^4(a-d)(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-22d)+15b^4\right)\sqrt{a+bx+cx^2}}{\left(b^2-4cd\right)\left(bx+cx^2+d\right)}}{4c^2(a-d)^2(b^2-4cd)}}{6c^2(a-d)^2(b^2-4cd)}$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)(b^2-4cd)(bx+cx^2+d)^3}$$

$\big\downarrow$ 27

3.6. $\int \frac{1}{\sqrt{a+bx+cx^2}(d+bx+cx^2)^4}\,dx$

$$-\frac{-\frac{-\frac{3c^4(a-d)\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad+8d^2\right)-8b^2c(a+4d)+5b^4\right)\int \frac{1}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)}dx}{2\left(b^2-4cd\right)}-\frac{c^4(a-d)(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-\cdots\right.}{\left(b^2-4cd\right)\left(bx+cx^2+d\right)}}{4c^2(a-d)^2(b^2-4cd)}}{6c^2(a-d)^2\left(b^2-4cd\right)}$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$\downarrow$ 1313

$$-\frac{-\frac{\frac{3bc^4(a-d)\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad+8d^2\right)-8b^2c(a+4d)+5b^4\right)\int \frac{1}{b\left(b^2-4cd\right)-\frac{b(a-d)(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{b^2-4cd}-\frac{c^4(a-d)(b+2cx)\left(16c^2\left(15a^2-44ad+4\cdots\right.\right.}{\left(b^2-4cd\right)\cdots}}{4c^2(a-d)^2(b^2-4cd)}}{6c^2(a-d)^2\left(b^2-4cd\right)}$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

$\downarrow$ 221

$$-\frac{-\frac{\frac{3c^4\sqrt{a-d}\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad+8d^2\right)-8b^2c(a+4d)+5b^4\right)\text{arctanh}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\left(b^2-4cd\right)^{3/2}}-\frac{c^4(a-d)(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c\cdots\right.}{\left(b^2-4cd\right)\left(bx+cx^2+\cdots\right.}}{4c^2(a-d)^2(b^2-4cd)}}{6c^2(a-d)^2\left(b^2-4cd\right)}$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)^3}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

output
```
-1/3*((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x +
 c*x^2)^3) - ((-5*c^2*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x
^2])/(2*(b^2 - 4*c*d)*(d + b*x + c*x^2)^2) - (-((c^4*(a - d)*(15*b^4 + 8*b
^2*c*(7*a - 22*d) + 16*c^2*(15*a^2 - 44*a*d + 44*d^2))*(b + 2*c*x)*Sqrt[a
+ b*x + c*x^2])/((b^2 - 4*c*d)*(d + b*x + c*x^2))) + (3*c^4*(b^2 + 4*c*(a
- 2*d))*Sqrt[a - d]*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2*(5*a^2 - 8*a*d + 8
*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x +
 c*x^2])])/(b^2 - 4*c*d)^(3/2))/(4*c^2*(a - d)^2*(b^2 - 4*c*d)))/(6*c^2*(a
 - d)^2*(b^2 - 4*c*d))
```

3.6. $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4}\,dx$

3.6.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3648 vs. 2(304) = 608.

Time = 1.95 (sec) , antiderivative size = 3649, normalized size of antiderivative = 11.12

method	result	size
default	Expression too large to display	3649

input
```
int(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b^2-4*c*d)^2*(-1/3/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^3*((x-1/2*(-b
+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))
/c)+a-d)^(1/2)-5/6*(b^2-4*c*d)^(1/2)/(a-d)*(-1/2/(a-d)/(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)^2*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-3/4*(b^2-4*c*d)^(1/2)/(a-d)*(-1/
(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2
*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-
4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c
*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c
*d)^(1/2))/c)))+1/2*c/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b
+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c
+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b
+(b^2-4*c*d)^(1/2))/c)))-2/3*c/(a-d)*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(
b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2*(b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-
2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1
/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^
(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))))+10*c^2/(b^2-4*c*
d)^3*(-1/(a-d)/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)*((x-1/2*(-b+(b^2-4*c*d)...
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3962 vs. $2(304) = 608$.

Time = 11.83 (sec) , antiderivative size = 8134, normalized size of antiderivative = 24.80

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.6.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(c*x**2+b*x+d)**4/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Timed out
```

3.6.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}(cx^2+bx+d)^4}\,dx$$

input
```
integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^4), x)
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30280 vs. $2(304) = 608$.

Time $= 4.51$ (sec) , antiderivative size $= 30280$, normalized size of antiderivative $= 92.32$

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx=\text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/16*((5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a^3*c^3 - 72*b^4*c*d - 1
92*a*b^2*c^2*d - 1152*a^2*c^3*d + 384*b^2*c^2*d^2 + 1536*a*c^3*d^2 - 1024*
c^3*d^3)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c + 4*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqrt(c) + 4*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b*c^(3/2)*d + 3*a*b^2*c + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2 - 2*b^2*c*d + 4*sqr
t(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b
*c + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c)))/sqrt(a*b^2 - b^
2*d - 4*a*c*d + 4*c*d^2) - (5*b^6 + 12*a*b^4*c + 48*a^2*b^2*c^2 + 320*a^3*
c^3 - 72*b^4*c*d - 192*a*b^2*c^2*d - 1152*a^2*c^3*d + 384*b^2*c^2*d^2 + 15
36*a*c^3*d^2 - 1024*c^3*d^3)*log(abs((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*b^2*c + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^2 - 8*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2*c^2*d + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*sqr
t(c) + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(3/2) - 8*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*b*c^(3/2)*d + 3*a*b^2*c - 4*sqrt(a*b^2 - b^2*d - 4*
a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(3/2) - 4*a^2*c^2
 - 2*b^2*c*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*b*c - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sq...
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+bx+cx^2)^4}\,dx=\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+d)^4}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + b*x + c*x^2)^4), x)
```

3.7 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$

3.7.1 Optimal result

Integrand size = 31, antiderivative size = 162

$$\begin{aligned}\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx &= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)} \\ &\quad - \frac{(8aef-b(e^2+4df))\,\operatorname{arctanh}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}\end{aligned}$$

output
```
-(8*a*e*f-b*(4*d*f+e^2))*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)/e^(1/2)/(-4*a*
f+b*e)^(1/2)/(f*x^2+e*x+d)^(1/2))/e^(3/2)/(-a*e+b*d)^(3/2)/(-4*a*f+b*e)^(3
/2)-b*(2*f*x+e)*(f*x^2+e*x+d)^(1/2)/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e
*x+a*e)
```

3.7.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.12 (sec) , antiderivative size = 1420, normalized size of antiderivative = 8.77

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

output
```
((-2*RootSum[a*e*f^2 - 2*b*Sqrt[d]*e*f*#1 + b*e^2*#1^2 + 4*b*d*f*#1^2 - 2*
a*e*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 + a*e*#1^4 & , (-4*b^2*d*e*Log[x] + a*b*e^
2*Log[x] + 4*a*b*d*f*Log[x] + a^2*e*f*Log[x] + 4*b^2*d*e*Log[-Sqrt[d] + Sq
rt[d + e*x + f*x^2] - x*#1] - a*b*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2]
 - x*#1] - 4*a*b*d*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - a^2*e*
f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 2*a*b*Sqrt[d]*e*Log[x]*#1
 + 2*a*b*Sqrt[d]*e*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 - a^2*e
*Log[x]*#1^2 + a^2*e*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1^2)/(-
(b*Sqrt[d]*e*f) + b*e^2*#1 + 4*b*d*f*#1 - 2*a*e*f*#1 - 3*b*Sqrt[d]*e*#1^2
+ 2*a*e*#1^3) & ])/a^3 + (b*((-2*e*(e + 2*f*x)*Sqrt[d + x*(e + f*x)])/(a*e
 + b*x*(e + f*x)) + RootSum[a*e*f^2 - 2*b*Sqrt[d]*e*f*#1 + b*e^2*#1^2 + 4*
b*d*f*#1^2 - 2*a*e*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 + a*e*#1^4 & , (-8*b^3*d^2*
e^2*Log[x] + 10*a*b^2*d*e^3*Log[x] - 2*a^2*b*e^4*Log[x] + 40*a*b^2*d^2*e*f
*Log[x] - 46*a^2*b*d*e^2*f*Log[x] + 7*a^3*e^3*f*Log[x] - 32*a^2*b*d^2*f^2*
Log[x] + 28*a^3*d*e*f^2*Log[x] + 8*b^3*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x
 + f*x^2] - x*#1] - 10*a*b^2*d*e^3*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] -
x*#1] + 2*a^2*b*e^4*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 40*a*b^
2*d^2*e*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] + 46*a^2*b*d*e^2*f*
Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 7*a^3*e^3*f*Log[-Sqrt[d] +
Sqrt[d + e*x + f*x^2] - x*#1] + 32*a^2*b*d^2*f^2*Log[-Sqrt[d] + Sqrt[d ...
```

3.7.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1305, 27, 1313, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}dx$$

$$\Big\downarrow \ 1305$$

$$\frac{\int \frac{b(bd-ae)f^2\left(8aef-b\left(e^2+4df\right)\right)}{2\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{bef^2(bd-ae)^2(be-4af)}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(8aef-b\left(4df+e^2\right)\right)\int \frac{1}{\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{2e(bd-ae)(be-4af)}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \text{1313}$$

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\int \frac{1}{e^2(be-4af)-\frac{e(bd-ae)(e+2fx)^2}{fx^2+ex+d}}d\frac{e+2fx}{\sqrt{fx^2+ex+d}}}{(bd-ae)(be-4af)}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\text{arctanh}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

input
```
Int[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

output
```
-((b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e
+ b*e*x + b*f*x^2))) - ((8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*
e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(e^(3/
2)*(b*d - a*e)^(3/2)*(b*e - 4*a*f)^(3/2))
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. $2(146) = 292$.

Time = 1.18 (sec) , antiderivative size = 1352, normalized size of antiderivative = 8.35

method	result	size
default	Expression too large to display	1352

input
```
int(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/e/(4*a*f-b*e)/b*(1/(a*e-b*d)*b/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b
/f)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1
/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)-1/2*(
-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+
(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(
-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e
*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d
)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)))-1/e/(4*a*f-b*e)/
b*(1/(a*e-b*d)*b/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)*((x+1/2*(b*e+(
-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(
-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)+1/2*(-b*e*(4*a*f-b*e))^(1
/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(
1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*(
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e
+(-b*e*(4*a*f-b*e))^(1/2))/b/f)))-2/e/(4*a*f-b*e)*f/(-b*e*(4*a*f-b*e))^(1/
2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1
/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b
*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-
b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b...
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(146) = 292$.

Time = 0.99 (sec) , antiderivative size = 2005, normalized size of antiderivative = 12.38

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/4*(sqrt(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2
*e^2*f + 4*(b^2*d - 2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e
^3 + 4*(b^2*d*e - 2*a*b*e^2)*f)*x)*log((8*b^2*d^2*e^4 - 8*a*b*d*e^5 + a^2*
e^6 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 - 8*a*b*d*e + 8*a^2*
e^2)*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*e^5*f + 16*(b^2*d
^2*e - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a*b*e^4)*f^2)*x^3
 + (b^2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^2*d^2*e^2 - 13*
a*b*d*e^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*f)*x^2 - 4*sqr
t(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(2*b*d*e^3 - a*e^4 - 4*a*
d*e^2*f + 2*(b*e^2*f^2 + 4*(b*d - 2*a*e)*f^3)*x^3 + 3*(b*e^3*f + 4*(b*d*e
- 2*a*e^2)*f^2)*x^2 + (b*e^4 - 8*a*d*e*f^2 + 2*(4*b*d*e^2 - 5*a*e^3)*f)*x)
*sqrt(f*x^2 + e*x + d) - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^
5 - 3*a*b*e^6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 -
5*a*b*d*e^4 + 2*a^2*e^5)*f)*x)/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x
+ a^2*e^2 + (b^2*e^2 + 2*a*b*e*f)*x^2)) + 4*(b^3*d*e^3 - a*b^2*e^4 - 4*(a*
b^2*d*e^2 - a^2*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d*e^2 -
 a*b^2*e^3)*f)*x)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d*e^6
+ a^3*b^2*e^7 + 16*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (16*(
a^2*b^3*d^2*e^2 - 2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 - 2*
a^2*b^3*d*e^4 + a^3*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b...
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.7.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \int \frac{1}{(bfx^2+bex+ae)^2\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(146) = 292$.

Time = 1.06 (sec) , antiderivative size = 1305, normalized size of antiderivative = 8.06

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

output
```
1/2*((b*e^2 + 4*b*d*f - 8*a*e*f)*log(abs(-(sqrt(f)*x - sqrt(f*x^2 + e*x +
d))^2*b*e^2*f - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*b*d*f^2 + 8*(sqrt(
f)*x - sqrt(f*x^2 + e*x + d))^2*a*e*f^2 - (sqrt(f)*x - sqrt(f*x^2 + e*x +
d))*b*e^3*sqrt(f) - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*b*d*e*f^(3/2) +
8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*a*e^2*f^(3/2) - 3*b*d*e^2*f + 2*a*e^
3*f + 4*b*d^2*f^2 + 4*sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f
)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*f^(3/2) + 4*sqrt(b^2*d*e^2 - a*b*e
^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*e*f +
sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*e^2*sqrt(f)))/sqrt(b
^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f) - (b*e^2 + 4*b*d*f - 8*a*e
*f)*log(abs(-(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*b*e^2*f - 4*(sqrt(f)*x
- sqrt(f*x^2 + e*x + d))^2*b*d*f^2 + 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))
^2*a*e*f^2 - (sqrt(f)*x - sqrt(f*x^2 + e*x + d))*b*e^3*sqrt(f) - 4*(sqrt(f
)*x - sqrt(f*x^2 + e*x + d))*b*d*e*f^(3/2) + 8*(sqrt(f)*x - sqrt(f*x^2 + e
*x + d))*a*e^2*f^(3/2) - 3*b*d*e^2*f + 2*a*e^3*f + 4*b*d^2*f^2 - 4*sqrt(b^
2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f)*(sqrt(f)*x - sqrt(f*x^2 + e
*x + d))^2*f^(3/2) - 4*sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*
f)*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*e*f - sqrt(b^2*d*e^2 - a*b*e^3 - 4*
a*b*d*e*f + 4*a^2*e^2*f)*e^2*sqrt(f)))/sqrt(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*
e*f + 4*a^2*e^2*f))/(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f) +...
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \int \frac{1}{\left(b\,f\,x^2+b\,e\,x+a\,e\right)^2\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
```

output
```
int(1/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

3.8 $\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

3.8.1 Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx=\frac{\arctan\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}$$

output
```
1/3*arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(1/2))*3^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx=-\frac{\arctan\left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

output
```
-(ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]]/Sqrt[3])
```

3.8.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1313, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{1313}$$

$$-4\int \frac{1}{-\frac{8(x+1)^2}{x^2+2x+5}-24}d\frac{2(x+1)}{\sqrt{x^2+2x+5}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

input
```
Int[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

output
```
ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]
```

3.8.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

3.8.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^2+2x+5}}\right)}{3}$	27
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+3\right)x^2+3\sqrt{x^2+2x+5}\,x-2\,\mathrm{RootOf}\left(\_Z^2+3\right)x+3\sqrt{x^2+2x+5}-7\,\mathrm{RootOf}\left(\_Z^2+3\right)}{x^2+2x+4}\right)}{6}$	75

input
```
int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^2+2\,x+5}(x+1) - \frac{1}{3}\sqrt{3}(x^2+2\,x+4)\right)$$

input
```
integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)*(x + 1) - 1/3*sqrt(3)*(
x^2 + 2*x + 4))
```

3.8.6 Sympy [F]

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}\,dx$$

input
```
integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

output
```
Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

3.8.7 Maxima [F]

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2x+5}(x^2+2x+4)}\,dx$$

input
```
integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(24) = 48.

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)$$

input
```
integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + 1/3*sqrt
(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5)))
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}\,dx$$

input
```
int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

output
```
int(1/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

3.9 $\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx$

3.9.1 Optimal result

Integrand size = 31, antiderivative size = 136

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = -\frac{2^{1+q}\left(-\frac{e-\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-1-p-q}\left(2a+ex+2cx^2\right)^{1+p+q}\text{Hypergeometric2F1}\left(-p-q,1+p+q,2+p+\right.}{\sqrt{-16ac+e^2}(1+p+q)}$$

output
```
-2^(1+q)*(2*c*x^2+e*x+2*a)^(1+p+q)*hypergeom([-p-q, 1+p+q],[2+p+q],1/2*(e+
4*c*x+(-16*a*c+e^2)^(1/2))/(-16*a*c+e^2)^(1/2))*((-e-4*c*x+(-16*a*c+e^2)^(
1/2))/(-16*a*c+e^2)^(1/2))^(-1-p-q)/(1+p+q)/(-16*a*c+e^2)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \, dx = \frac{2^{-2+q}\left(e-\sqrt{-16ac+e^2}+4cx\right)\left(\frac{e+\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-p-q}(2a+x(e+2cx))^{p+q}\text{Hypergeometric2F1}\left(-p-\right.}{c(1+p+q)}$$

input
```
Integrate[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

output
```
(2^(-2 + q)*(e - Sqrt[-16*a*c + e^2] + 4*c*x)*((e + Sqrt[-16*a*c + e^2] +
4*c*x)/Sqrt[-16*a*c + e^2])^(-p - q)*(2*a + x*(e + 2*c*x))^(p + q)*Hyperge
ometric2F1[-p - q, 1 + p + q, 2 + p + q, (-e + Sqrt[-16*a*c + e^2] - 4*c*x
)/(2*Sqrt[-16*a*c + e^2])])/(c*(1 + p + q))
```

3.9.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1295, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^2+\frac{ex}{2}\right)^p \left(2a+2cx^2+ex\right)^q dx$$

$$\Big\downarrow \text{ 1295}$$

$$2^{-p}\int \left(2cx^2+ex+2a\right)^{p+q}dx$$

$$\Big\downarrow \text{ 1096}$$

$$-\frac{2^{q+1}\left(-\frac{-\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q-1}\left(2a+2cx^2+ex\right)^{p+q+1}\text{Hypergeometric2F1}\left(-p-q,p+q+1,p+q+2,\frac{e+4c}{2}\right.}{(p+q+1)\sqrt{e^2-16ac}}$$

input
```
Int[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

output
```
-((2^(1 + q)*(-((e - Sqrt[-16*a*c + e^2] + 4*c*x)/Sqrt[-16*a*c + e^2]))^(-
1 - p - q)*(2*a + e*x + 2*c*x^2)^(1 + p + q)*Hypergeometric2F1[-p - q, 1 +
 p + q, 2 + p + q, (e + Sqrt[-16*a*c + e^2] + 4*c*x)/(2*Sqrt[-16*a*c + e^2
])])/(Sqrt[-16*a*c + e^2]*(1 + p + q)))
```

3.9.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1295
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_) + (e_.)*(x_) + (f_.)*(x_
)^2)^(q_.), x_Symbol] :> Simp[(c/f)^p   Int[(d + e*x + f*x^2)^(p + q), x],
x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d -
a*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && ( !IntegerQ[q] || LeafCount[d +
e*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

3.9.4 Maple [F]

$$\int \left(a+\frac{1}{2}ex+c\,x^{2}\right)^{p} \left(2c\,x^{2}+ex+2a\right)^{q} dx$$

input
```
int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

output
```
int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

3.9.5 Fricas [F]

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q \,dx = \int \left(2\,cx^2+ex+2\,a\right)^q \left(cx^2+\frac{1}{2}\,ex+a\right)^p \,dx$$

input
```
integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="fricas")
```

output
```
integral((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q\,dx=\text{Timed out}$$

input
```
integrate((a+1/2*e*x+c*x**2)**p*(2*c*x**2+e*x+2*a)**q,x)
```

output
```
Timed out
```

3.9.7 Maxima [F]

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q\,dx=\int \left(2\,cx^2+ex+2\,a\right)^q \left(cx^2+\frac{1}{2}\,ex+a\right)^p\,dx$$

input
```
integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="maxima")
```

output
```
integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

3.9.8 Giac [F]

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q\,dx=\int \left(2\,cx^2+ex+2\,a\right)^q \left(cx^2+\frac{1}{2}\,ex+a\right)^p\,dx$$

input
```
integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="giac")
```

output
```
integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p (2a+ex+2cx^2)^q\,dx=\int \left(c\,x^2+\frac{e\,x}{2}+a\right)^p (2\,c\,x^2+e\,x+2\,a)^q\,dx$$

input
```
int((a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x)
```

output
```
int((a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q, x)
```

3.10 $\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx$

3.10.1 Optimal result

Integrand size = 34, antiderivative size = 200

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx = -\frac{2^{1+p+q}\sqrt{c}\left(-\frac{\sqrt{c}\left(e-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-1-p-q} \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^{1+q} \text{Hypergeometric2F1}\left(\right.}{\sqrt{ce^2-4af^2}(1+p+q)}$$

output
```
-2^(1+p+q)*(a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^(1+q)*hypergeom([-p-q, 1+
p+q],[2+p+q],1/2*c^(1/2)*(e+2*f*x+(-4*a*f^2+c*e^2)^(1/2)/c^(1/2))/(-4*a*f^
2+c*e^2)^(1/2))*c^(1/2)*(-c^(1/2)*(e+2*f*x-(-4*a*f^2+c*e^2)^(1/2)/c^(1/2))
/(-4*a*f^2+c*e^2)^(1/2))^(-1-p-q)/(1+p+q)/(-4*a*f^2+c*e^2)^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.86

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx = \frac{2^{-1+p+q}\left(\frac{af}{c}+x(e+fx)\right)^q \left(a+\frac{cx(e+fx)}{f}\right)^p \left(-\sqrt{ce^2-4af^2}+\sqrt{c}(e+2fx)\right)\left(1+\frac{\sqrt{c}(e+2fx)}{\sqrt{ce^2-4af^2}}\right)^{-p-q} \text{Hyperge}}{\sqrt{c}f(1+p+q)}$$

input
```
Integrate[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

output
```
(2^(-1 + p + q)*((a*f)/c + x*(e + f*x))^q*(a + (c*x*(e + f*x))/f)^p*(-Sqrt
[c*e^2 - 4*a*f^2] + Sqrt[c]*(e + 2*f*x))*(1 + (Sqrt[c]*(e + 2*f*x))/Sqrt[c
*e^2 - 4*a*f^2])^(-p - q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q,
1/2 - (Sqrt[c]*(e + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f^2])])/(Sqrt[c]*f*(1 + p
+ q))
```

3.10.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1296, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx$$

$$\Big\downarrow \; 1296$$

$$\left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^{-p} \int \left(fx^2+ex+\frac{af}{c}\right)^{p+q} dx$$

$$\Big\downarrow \; 1096$$

$$-\frac{\sqrt{c}2^{p+q+1}\left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^{q+1} \left(-\frac{\sqrt{c}\left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+e+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-p-q-1} \text{Hypergeometric2F1}\left(-p \cdots\right.}{(p+q+1)\sqrt{ce^2-4af^2}}$$

input
```
Int[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

output
```
-((2^(1 + p + q)*Sqrt[c]*(-((Sqrt[c]*(e - Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] +
2*f*x))/Sqrt[c*e^2 - 4*a*f^2]))^(-1 - p - q)*(a + (c*e*x)/f + c*x^2)^p*((a
*f)/c + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p +
q, (Sqrt[c]*(e + Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2*f*x))/(2*Sqrt[c*e^2 - 4
*a*f^2])])/(Sqrt[c*e^2 - 4*a*f^2]*(1 + p + q)))
```

3.10.3.1 Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1296
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x + c*x^2)^FracPart[p]/(d
^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]))   Int[(d + e*x + f*x^2)^(p + q)
, x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[
b*d - a*e, 0] &&  !IntegerQ[p] &&  !IntegerQ[q] &&  !GtQ[c/f, 0]
```

3.10.4 Maple [F]

$$\int \left(a+\frac{cex}{f}+c\,x^2\right)^p \left(\frac{af}{c}+ex+f\,x^2\right)^q dx$$

input
```
int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

output
```
int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

3.10.5 Fricas [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx = \int \left(cx^2+\frac{cex}{f}+a\right)^p \left(fx^2+ex+\frac{af}{c}\right)^q dx$$

input
```
integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="fricas")
```

output
```
integral((((c*f*x^2 + c*e*x + a*f)/c)^q*((c*f*x^2 + c*e*x + a*f)/f)^p, x)
```

3.10.6 Sympy [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx=\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx$$

input
```
integrate((a+c*e*x/f+c*x**2)**p*(a*f/c+e*x+f*x**2)**q,x)
```

output
```
Integral((a + c*e*x/f + c*x**2)**p*(a*f/c + e*x + f*x**2)**q, x)
```

3.10.7 Maxima [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx=\int \left(cx^2+\frac{cex}{f}+a\right)^p \left(fx^2+ex+\frac{af}{c}\right)^q dx$$

input
```
integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)
```

3.10.8 Giac [F]

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q dx=\int \left(cx^2+\frac{cex}{f}+a\right)^p \left(fx^2+ex+\frac{af}{c}\right)^q dx$$

input
```
integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="giac")
```

output
```
integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q\,dx=\int \left(e\,x+f\,x^2+\frac{a\,f}{c}\right)^q \left(a+c\,x^2+\frac{c\,e\,x}{f}\right)^p\,dx$$

input
```
int((e*x + f*x^2 + (a*f)/c)^q*(a + c*x^2 + (c*e*x)/f)^p,x)
```

output
```
int((e*x + f*x^2 + (a*f)/c)^q*(a + c*x^2 + (c*e*x)/f)^p, x)
```

3.11 $\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx$

3.11.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx=\frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x}+\frac{\sqrt{1+2x+x^2}\operatorname{arcsinh}(x)}{1+x}$$

output
```
arcsinh(x)*((1+x)^2)^(1/2)/(1+x)+(x^2+1)^(1/2)*((1+x)^2)^(1/2)/(1+x)
```

3.11.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx=\frac{\sqrt{(1+x)^2}\left(\sqrt{1+x^2}-\log\left(-x+\sqrt{1+x^2}\right)\right)}{1+x}$$

input
```
Integrate[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[(1 + x)^2]*(Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]))/(1 + x)
```

3.11.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1298, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2x+1}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \ 1298$$

$$\frac{\sqrt{x^2+2x+1}\int \frac{2(x+1)}{\sqrt{x^2+1}}dx}{2(x+1)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{x^2+2x+1}\int \frac{x+1}{\sqrt{x^2+1}}dx}{x+1}$$

$$\Big\downarrow \ 455$$

$$\frac{\sqrt{x^2+2x+1}\left(\int \frac{1}{\sqrt{x^2+1}}dx+\sqrt{x^2+1}\right)}{x+1}$$

$$\Big\downarrow \ 222$$

$$\frac{\sqrt{x^2+2x+1}\left(\operatorname{arcsinh}(x)+\sqrt{x^2+1}\right)}{x+1}$$

input
```
Int[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[1 + 2*x + x^2]*(Sqrt[1 + x^2] + ArcSinh[x]))/(1 + x)
```

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1298
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a
, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.11.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

method	result	size
default	$\operatorname{csgn}\left(1+x\right)\left(\operatorname{arcsinh}\left(x\right)+\sqrt{x^{2}+1}\right)$	16
risch	$\frac{\operatorname{arcsinh}(x)\sqrt{(1+x)^{2}}}{1+x}+\frac{\sqrt{x^{2}+1}\,\sqrt{(1+x)^{2}}}{1+x}$	37
meijerg	$\frac{\operatorname{arcsinh}(x)\sqrt{(1+x)^{2}}}{1+x}+\frac{\sqrt{(1+x)^{2}}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^{2}+1}\right)}{2(1+x)\sqrt{\pi}}$	52

input
```
int(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(1+x)*(arcsinh(x)+(x^2+1)^(1/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx = \sqrt{x^2+1} - \log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

3.11.6 Sympy [F]

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate(((1+x)**2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt((x + 1)**2)/sqrt(x**2 + 1), x)
```

3.11.7 Maxima [F]

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((x + 1)^2)/sqrt(x^2 + 1), x)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx = -\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}(x+1) - \log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}(x+1)+\sqrt{x^2+1}\mathrm{sgn}(x+1)$$

input
```
integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
 1) + sqrt(x^2 + 1)*sgn(x + 1)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

input
```
int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int(((x + 1)^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.12 $\int \frac{1}{(-1+x^2)^2\sqrt{-1+x+x^2}}\,dx$

3.12.1 Optimal result

Integrand size = 18, antiderivative size = 70

$$\int \frac{1}{(-1+x^2)^2\sqrt{-1+x+x^2}}\,dx = \frac{\sqrt{-1+x+x^2}}{2(1-x^2)} - \frac{1}{8}\arctan\left(\frac{3+x}{2\sqrt{-1+x+x^2}}\right) - \frac{5}{8}\text{arctanh}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)$$

output
```
-1/8*arctan(1/2*(3+x)/(x^2+x-1)^(1/2))-5/8*arctanh(1/2*(1-3*x)/(x^2+x-1)^(
1/2))+1/2*(x^2+x-1)^(1/2)/(-x^2+1)
```

3.12.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{1}{(-1+x^2)^2\sqrt{-1+x+x^2}}\,dx = -\frac{\sqrt{-1+x+x^2}}{2(-1+x^2)} - \frac{1}{4}\arctan\left(1+x-\sqrt{-1+x+x^2}\right) + \frac{5}{4}\text{arctanh}\left(1-x+\sqrt{-1+x+x^2}\right)$$

input
```
Integrate[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

output
```
-1/2*Sqrt[-1 + x + x^2]/(-1 + x^2) - ArcTan[1 + x - Sqrt[-1 + x + x^2]]/4
+ (5*ArcTanh[1 - x + Sqrt[-1 + x + x^2]])/4
```

3.12.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1307, 25, 1366, 25, 1154, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-1)^2\sqrt{x^2+x-1}}dx$$

$$\Big\downarrow \text{ 1307}$$

$$\frac{\sqrt{x^2+x-1}}{2(1-x^2)}-\frac{1}{4}\int -\frac{2x+3}{(1-x^2)\sqrt{x^2+x-1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\int \frac{2x+3}{(1-x^2)\sqrt{x^2+x-1}}dx+\frac{\sqrt{x^2+x-1}}{2(1-x^2)}$$

$$\Big\downarrow \text{ 1366}$$

$$\frac{1}{4}\left(\frac{5}{2}\int \frac{1}{(1-x)\sqrt{x^2+x-1}}dx-\frac{1}{2}\int -\frac{1}{(x+1)\sqrt{x^2+x-1}}dx\right)+\frac{\sqrt{x^2+x-1}}{2(1-x^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{5}{2}\int \frac{1}{(1-x)\sqrt{x^2+x-1}}dx+\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+x-1}}dx\right)+\frac{\sqrt{x^2+x-1}}{2(1-x^2)}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{4}\left(-5\int \frac{1}{4-\frac{(1-3x)^2}{x^2+x-1}}d\frac{1-3x}{\sqrt{x^2+x-1}}-\int \frac{1}{-\frac{(x+3)^2}{x^2+x-1}-4}d\left(-\frac{x+3}{\sqrt{x^2+x-1}}\right)\right)+\frac{\sqrt{x^2+x-1}}{2(1-x^2)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{4}\left(-5\int \frac{1}{4-\frac{(1-3x)^2}{x^2+x-1}}d\frac{1-3x}{\sqrt{x^2+x-1}}-\frac{1}{2}\arctan\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)\right)+\frac{\sqrt{x^2+x-1}}{2(1-x^2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}\left(-\frac{1}{2}\arctan\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-\frac{5}{2}\text{arctanh}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)\right)+\frac{\sqrt{x^2+x-1}}{2(1-x^2)}$$

input
```
Int[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

output
```
Sqrt[-1 + x + x^2]/(2*(1 - x^2)) + (-1/2*ArcTan[(3 + x)/(2*Sqrt[-1 + x + x
^2])] - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/2)/4
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1307
```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p + 1
)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))),
 x] - Simp[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))   Int[(a + c*x^2)
^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p +
 1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
 + q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*((-c)*e*(2
*p + q + 4)))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
 /; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && Ne
Q[a*c*e^2 + (c*d - a*f)^2, 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ
[q, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.12.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

method	result	s
risch	$-\frac{\sqrt{x^2+x-1}}{2(x^2-1)}+\frac{\arctan\left(\frac{-3-x}{2\sqrt{(1+x)^2-2-x}}\right)}{8}+\frac{5\,\mathrm{arctanh}\left(\frac{-1+3x}{2\sqrt{(-1+x)^2-2+3x}}\right)}{8}$	6
default	$-\frac{\sqrt{(-1+x)^2-2+3x}}{4(-1+x)}+\frac{5\,\mathrm{arctanh}\left(\frac{-1+3x}{2\sqrt{(-1+x)^2-2+3x}}\right)}{8}+\frac{\sqrt{(1+x)^2-2-x}}{4+4x}+\frac{\arctan\left(\frac{-3-x}{2\sqrt{(1+x)^2-2-x}}\right)}{8}$	8
trager	$-\frac{\sqrt{x^2+x-1}}{2(x^2-1)}+\frac{5\ln\left(-\frac{2\sqrt{x^2+x-1}-1+3x}{-1+x}\right)}{8}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{x^2+x-1}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)}{1+x}\right)}{8}$	8

input
```
int(1/(x^2-1)^2/(x^2+x-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^2-1)*(x^2+x-1)^(1/2)+1/8*arctan(1/2*(-3-x)/((1+x)^2-2-x)^(1/2))+5/
8*arctanh(1/2*(-1+3*x)/((-1+x)^2-2+3*x)^(1/2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{1}{(-1+x^2)^2\sqrt{-1+x+x^2}}\,dx$$
$$= \frac{2\left(x^2-1\right)\arctan\left(-x+\sqrt{x^2+x-1}-1\right)+5\left(x^2-1\right)\log\left(-x+\sqrt{x^2+x-1}+2\right)-5\left(x^2-1\right)\log\left(-x\right.}{8\left(x^2-1\right)}$$

input
```
integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*(x^2 - 1)*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5*(x^2 - 1)*log(-x +
 sqrt(x^2 + x - 1) + 2) - 5*(x^2 - 1)*log(-x + sqrt(x^2 + x - 1)) - 4*sqrt
(x^2 + x - 1))/(x^2 - 1)
```

3.12.6 Sympy [F]

$$\int \frac{1}{(-1+x^2)^2\sqrt{-1+x+x^2}}\,dx = \int \frac{1}{(x-1)^2\,(x+1)^2\,\sqrt{x^2+x-1}}\,dx$$

input
```
integrate(1/(x**2-1)**2/(x**2+x-1)**(1/2),x)
```

output
```
Integral(1/((x - 1)**2*(x + 1)**2*sqrt(x**2 + x - 1)), x)
```

3.12.7 Maxima [F]

$$\int \frac{1}{(-1+x^2)^2\sqrt{-1+x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+x-1}(x^2-1)^2}\,dx$$

input
```
integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + x - 1)*(x^2 - 1)^2), x)
```

3.12.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(52) = 104$.

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

$$\begin{aligned}\int \frac{1}{(-1+x^2)^2\sqrt{-1+x+x^2}}\,dx &= \frac{2\left(x-\sqrt{x^2+x-1}\right)^3+3\left(x-\sqrt{x^2+x-1}\right)^2-x+\sqrt{x^2+x-1}-1}{2\left(\left(x-\sqrt{x^2+x-1}\right)^4-2\left(x-\sqrt{x^2+x-1}\right)^2-4\,x+4\sqrt{x^2+x-1}\right)}\\ &\quad+\frac{1}{4}\arctan\left(-x+\sqrt{x^2+x-1}-1\right)\\ &\quad+\frac{5}{8}\log\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right)-\frac{5}{8}\log\left(\left|-x+\sqrt{x^2+x-1}\right|\right)\end{aligned}$$

input
```
integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*(x - sqrt(x^2 + x - 1))^3 + 3*(x - sqrt(x^2 + x - 1))^2 - x + sqrt(
x^2 + x - 1) - 1)/((x - sqrt(x^2 + x - 1))^4 - 2*(x - sqrt(x^2 + x - 1))^2
 - 4*x + 4*sqrt(x^2 + x - 1)) + 1/4*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5
/8*log(abs(-x + sqrt(x^2 + x - 1) + 2)) - 5/8*log(abs(-x + sqrt(x^2 + x -
1)))
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+x^2)^2 \sqrt{-1+x+x^2}}\,dx = \int \frac{1}{(x^2-1)^2 \sqrt{x^2+x-1}}\,dx$$

input
```
int(1/((x^2 - 1)^2*(x + x^2 - 1)^(1/2)),x)
```

output
```
int(1/((x^2 - 1)^2*(x + x^2 - 1)^(1/2)), x)
```

3.13 $\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx$

3.13.1 Optimal result

Integrand size = 26, antiderivative size = 1077

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx =$$

$$-\frac{\sqrt[4]{b^2d+b\sqrt{b^2-4ac}\,d-2a(cd-af)}\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(\cdots\right.\right.}{\left(\left(b+\sqrt{b^2-4ac}\right)\cdots\right.}}\cdots}{\cdots}$$

output
```
-(cos(2*arctan((2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/2))
^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2)))^2)^(1/2)/cos(2*arctan((2*c^2*d
-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^
(1/2)/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b^2)^(1/2))^(1/4)/(b+2*c*x+(-4*a*c
+b^2)^(1/2))^(1/2)))*EllipticF(sin(2*arctan((2*c^2*d-2*a*c*f+b*f*(b+(-4*a*
c+b^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d-2*a*(-a*f
+c*d)+b*d*(-4*a*c+b^2)^(1/2))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2))),1
/2*(2+2*(a*f+c*d)*(b+(-4*a*c+b^2)^(1/2))/(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c
+b^2)^(1/2))^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/2))^(1/
2))*(b+2*c*x+(-4*a*c+b^2)^(1/2))^(3/2)*(b^2*d-2*a*(-a*f+c*d)+b*d*(-4*a*c+b
^2)^(1/2))^(1/4)*(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d-2*a*c*f+b*f*(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d-2*a*(-a*f+
c*d)+b*d*(-4*a*c+b^2)^(1/2))^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(
(f*x^2+d)*(4*a*c-(b+(-4*a*c+b^2)^(1/2))^2)^2/(b+2*c*x+(-4*a*c+b^2)^(1/2))^
2/(4*a^2*f+d*(b+(-4*a*c+b^2)^(1/2))^2))^(1/2)*((1-4*(a*f+c*d)*(b+(-4*a*c+b
^2)^(1/2))*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(4*
a^2*f+d*(b+(-4*a*c+b^2)^(1/2))^2)+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^2*(4*c^2*
d+f*(b+(-4*a*c+b^2)^(1/2))^2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f+d*(b
+(-4*a*c+b^2)^(1/2))^2))/(1+(2*a+x*(b+(-4*a*c+b^2)^(1/2)))*(2*c^2*d-2*a...
```

3.13.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 600, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx =$$

$$-\frac{2\sqrt{2}\left(-b+\sqrt{b^2-4ac}-2cx\right)\left(-i\sqrt{d}+\sqrt{f}x\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(i\sqrt{d}+\sqrt{f}x\right)}{\left(-2ic\sqrt{d}+\left(b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\sqrt{\frac{c\left(-i\sqrt{d}\left(\sqrt{b^2 \cdots}\right.\right.}{\left(2ic \cdots\right.}}}{\left(-2ic\sqrt{d}+\left(-b+\sqrt{b^2-4ac}\right)\sqrt{f}\right)\sqrt{\cdots}}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]
```

output
```
(-2*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*((-I)*Sqrt[d] + Sqrt[f]*x)*Sq
rt[-((c*Sqrt[b^2 - 4*a*c]*(I*Sqrt[d] + Sqrt[f]*x))/(((-2*I)*c*Sqrt[d] + (b
 + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*Sqrt[(c
*((-I)*Sqrt[d]*(Sqrt[b^2 - 4*a*c] + 2*c*x) + Sqrt[f]*(-2*a + Sqrt[b^2 - 4*
a*c]*x) + b*((-I)*Sqrt[d] - Sqrt[f]*x)))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2
 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*EllipticF[ArcSin[Sq
rt[(((-2*I)*c*Sqrt[d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(b + Sqrt[b^2 -
4*a*c] + 2*c*x))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x))]], (c*d - I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f
] + a*f)/(c*d + I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)])/(((-2*I)*c*Sq
rt[d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*Sqrt[(I*c*Sqrt[b^2 - 4*a*c]*(Sqr
t[d] + I*Sqrt[f]*x))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*
(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*Sqrt[d + f*x^2]*Sqrt[a + x*(b + c*x)])
```

3.13.3 Rubi [A] (warning: unable to verify)

Time = 1.22 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1324, 732, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+fx^2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \; 1324$$

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{x\left(\sqrt{b^2-4ac}+b\right)+2a}\int \frac{1}{\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{fx^2+d}}dx}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 732$$

$$-\frac{2\left(\sqrt{b^2-4ac}+b+2cx\right)^{3/2}\sqrt{x\left(\sqrt{b^2-4ac}+b\right)+2a}\sqrt{\frac{\left(4ac-\left(\sqrt{b^2-4ac}+b\right)^2\right)^2(d+fx^2)}{\left(\sqrt{b^2-4ac}+b+2cx\right)^2\left(4a^2f+d\left(\sqrt{b^2-4ac}+b\right)^2\right)}}\int \frac{}{\sqrt{\frac{\left(4dc^2+\left(b+\sqrt{b^2 \ldots\right.\right.}{\left(4fa^2+\left(b+\sqrt{\ldots\right.\right.}}}}{\left(4ac-\left(\sqrt{b^2-4ac}+b\right)^2\right)\sqrt{d+fx^2}\sqrt{a \ldots}}$$

$$\Big\downarrow \; 1416$$

$$-\frac{\sqrt[4]{db^2+\sqrt{b^2-4ac}db-2a(cd-af)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2-4\right.\right.}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2\right.}}}{}$$

input

```
Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]
```

output

```
-(((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c
- (b + Sqrt[b^2 - 4*a*c])^2)^2*(d + f*x^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d
+ 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - 2*a*c*
f + b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[
b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] +
2*c*x)))*Sqrt[(1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt
[b^2 - 4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2
- 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b +
Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2
 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d + b*Sqrt[b^2
- 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*Ellipti
cF[2*ArcTan[((2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[
2*a + (b + Sqrt[b^2 - 4*a*c])*x])/((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c
*d - a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (1 + ((b + Sqrt[b^
2 - 4*a*c])*(c*d + a*f))/(Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c
])*f]*Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]))/2])/((4*a*c
- (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c]
)*f)^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]*Sqrt[1 - (4*(b + Sqrt[...
```

3.13. $\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx$

3.13.3.1 Defintions of rubi rules used

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 1324
```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (f_.)*(x_)^2]), x
_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b + r + 2*c*x]*(Sqrt[2
*a + (b + r)*x]/Sqrt[a + b*x + c*x^2])   Int[1/(Sqrt[b + r + 2*c*x]*Sqrt[2*
a + (b + r)*x]*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && Ne
Q[b^2 - 4*a*c, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.13.4 Maple [A] (warning: unable to verify)

Time = 5.29 (sec) , antiderivative size = 721, normalized size of antiderivative = 0.67

method	result
default	$\frac{16\left(bcf\,x^{2}-2c^{2}x^{2}\sqrt{-df}-cf\,x^{2}\sqrt{-4ac+b^{2}}+4acfx-2bcx\sqrt{-df}-2cx\sqrt{-df}\,\sqrt{-4ac+b^{2}}+baf+2ac\sqrt{-df}+af\sqrt{-4ac+b^{2}}-b^{2}\sqrt{-df}-b\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{2\sqrt{(c\,x^{2}+bx+a)(f\,x^{2}+d)}\left(\frac{\sqrt{-df}}{f}+\frac{-b+\sqrt{-4ac+b^{2}}}{2c}\right)\sqrt{\frac{\left(-\frac{\sqrt{-df}}{f}+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)\left(x-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}\right)}{\left(-\frac{\sqrt{-df}}{f}-\frac{-b+\sqrt{-4ac+b^{2}}}{2c}\right)\left(x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)}}\left(x+\frac{b+\sqrt{-4ac+b^{2}}}{2c}\right)^{2}\sqrt{\frac{\left(-\frac{b+\sqrt{\cdots}}{\cdots}\right.}{\left(\frac{\sqrt{-d\cdots}}{f}\right.}}\cdots}{\sqrt{c\,x^{2}+bx+a}\,\sqrt{f\,x^{2}+d}\left(-\frac{\sqrt{-df}}{f}+\frac{b+\sqrt{-4ac-\cdots}}{2c}\cdots\right.}$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16*(b*c*f*x^2-2*c^2*x^2*(-d*f)^(1/2)-c*f*x^2*(-4*a*c+b^2)^(1/2)+4*a*c*f*x-
2*b*c*x*(-d*f)^(1/2)-2*c*x*(-d*f)^(1/2)*(-4*a*c+b^2)^(1/2)+b*a*f+2*a*c*(-d
*f)^(1/2)+a*f*(-4*a*c+b^2)^(1/2)-b^2*(-d*f)^(1/2)-b*(-d*f)^(1/2)*(-4*a*c+b
^2)^(1/2))*EllipticF((-(2*(-d*f)^(1/2)*c-f*(-4*a*c+b^2)^(1/2)-b*f)*(-b-2*c
*x+(-4*a*c+b^2)^(1/2))/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)/(b+2*c*
x+(-4*a*c+b^2)^(1/2)))^(1/2),((f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c+b*f)*
(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)/(2*(-d*f)^(1/2)*c-f*(-4*a*c+b^
2)^(1/2)+b*f)/(2*(-d*f)^(1/2)*c-f*(-4*a*c+b^2)^(1/2)-b*f))^(1/2))*((-4*a*c
+b^2)^(1/2)*(f*x+(-d*f)^(1/2))*c/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*
f)/(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)*((-4*a*c+b^2)^(1/2)*(-f*x+(-d*f)^(1
/2))*c/(2*(-d*f)^(1/2)*c-f*(-4*a*c+b^2)^(1/2)+b*f)/(b+2*c*x+(-4*a*c+b^2)^(
1/2)))^(1/2)*(-(2*(-d*f)^(1/2)*c-f*(-4*a*c+b^2)^(1/2)-b*f)*(-b-2*c*x+(-4*a
*c+b^2)^(1/2))/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)/(b+2*c*x+(-4*a*
c+b^2)^(1/2)))^(1/2)*(f*x^2+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/(1/c/f*(-b-2*c*x+
(-4*a*c+b^2)^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2))*(-f*x+(-d*f)^(1/2))*(f*x+
(-d*f)^(1/2)))^(1/2)/(-4*a*c+b^2)^(1/2)/(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/
2)*c+b*f)/((c*x^2+b*x+a)*(f*x^2+d))^(1/2)
```

3.13.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)/(c*f*x^4 + b*f*x^3 + b*d*x
+ (c*d + a*f)*x^2 + a*d), x)
```

3.13.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx = \int \frac{1}{\sqrt{d+fx^2}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+d)**(1/2),x)
```

output
```
Integral(1/(sqrt(d + f*x**2)*sqrt(a + b*x + c*x**2)), x)
```

3.13.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

3.13.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx = \int \frac{1}{\sqrt{f\,x^2+d}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.14 $\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx$

3.14.1 Optimal result

Integrand size = 27, antiderivative size = 98

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx = -\frac{1}{2}\arcsin(2+x) - \frac{\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
-1/2*arcsin(2+x)-1/2*arctanh(x/(-x^2-4*x-3)^(1/2))-1/2*arctan(1/2*(1+(-3-x
)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(1/2*(1+(3+x)/(-x^2-4*x-3
)^(1/2))*2^(1/2))*2^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx = \frac{\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{\sqrt{2}} + \arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) - \frac{1}{2}\operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

input
```
Integrate[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

output
```
ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]/Sqrt[2] + ArcTan[Sqrt[-3
- 4*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

3.14.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {1320, 1090, 223, 1361, 27, 1317, 27, 1359, 27, 1360, 219, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-x^2-4x-3}}{2x^2+4x+3}dx$$

$$\Big\downarrow \text{1320}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{-x^2-4x-3}}dx-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{1-\frac{1}{4}(-2x-4)^2}}d(-2x-4)-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{1361}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx+\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)+\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-2\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)+\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{1317}$$

$$\frac{1}{2}\left(3\left(\frac{1}{6}\int -\frac{4x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{1}{6}\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)-2\int \frac{2x}{\sqrt{-x^2-4x-}}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{2}{3}\int \frac{x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)-2\int \frac{2x+}{\sqrt{-x^2-4x-3}\,(2}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{1359}$$

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{16}{3}\int -\frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{4\left(\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1\right)}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)-2\int\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{1360}$$

$$\frac{1}{2}\left(3\left(\int \frac{1}{3-\frac{3x^2}{-x^2-4x-3}}d\frac{x}{\sqrt{-x^2-4x-3}}+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)-6\int \frac{}{3-}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(3\left(\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)-2\text{arctanh}\left(\frac{}{\sqrt{-}}\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

3.14. $\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\left(3\left(\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{4}{3}\left(-\frac{1}{6}\int \frac{1}{\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{2(x+3)}{9\sqrt{-x^2-4x-3}}+\frac{1}{3}}d\frac{x+3}{3\sqrt{-x^2-4x-3}}-\frac{1}{6}\int \frac{(x+}{9(-x^2-}\right.\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\left(3\left(\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{4}{3}\left(\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{8}{9}}d\left(\frac{2(x+3)}{3\sqrt{-x^2-4x-3}}-\frac{2}{3}\right)+\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}}\right.\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)+$$
$$\frac{1}{2}\left(3\left(\frac{2}{3}\sqrt{2}\arctan\left(\frac{x+3}{2\sqrt{2}\sqrt{-x^2-4x-3}}\right)+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)-2\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)$$

input
```
Int[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

output
```
ArcSin[(-4 - 2*x)/2]/2 + (3*((2*Sqrt[2]*ArcTan[(3 + x)/(2*Sqrt[2]*Sqrt[-3
- 4*x - x^2])])/3 + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/3) - 2*ArcTanh[x/Sqrt[
-3 - 4*x - x^2]])/2
```

3.14.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

rule 1320
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f
  Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0]
```

rule 1359
```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2
*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
 4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

3.14. $\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx$

rule 1360
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[g   Subst[Int[1/(a + (c*d - a*f
)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &
& EqQ[2*h*d - g*e, 0]
```

rule 1361
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(2*h*d - g*e)/e   Int[1/((a +
b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/e   Int[(2*d + e*x)/((
a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e,
0] && NeQ[2*h*d - g*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.14.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.14 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.29

method	result
trager	$\operatorname{RootOf}\left(16\_Z^2+8\_Z+3\right)\ln\left(\frac{16\operatorname{RootOf}\left(16\_Z^2+8\_Z+3\right)^2x+24\operatorname{RootOf}\left(16\_Z^2+8\_Z+3\right)x+24\operatorname{RootOf}\left(16\_\right.}{4\operatorname{RootOf}\left(16\_Z^2+8\_Z+3\right)x-x-3}\right.$
default	$-\frac{\arcsin(2+x)}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}-\frac{\sqrt{3}\,\sqrt{4}}{}$

input
```
int((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x,method=_RETURNVERBOSE)
```

output
```
RootOf(16*_Z^2+8*_Z+3)*ln((16*RootOf(16*_Z^2+8*_Z+3)^2*x+24*RootOf(16*_Z^2
+8*_Z+3)*x+24*RootOf(16*_Z^2+8*_Z+3)-6*(-x^2-4*x-3)^(1/2)+5*x+6)/(4*RootOf
(16*_Z^2+8*_Z+3)*x-x-3))-1/2*ln((16*RootOf(16*_Z^2+8*_Z+3)^2*x-8*RootOf(16
*_Z^2+8*_Z+3)*x-24*RootOf(16*_Z^2+8*_Z+3)-6*(-x^2-4*x-3)^(1/2)-3*x-6)/(4*R
ootOf(16*_Z^2+8*_Z+3)*x+3*x+3))-ln((16*RootOf(16*_Z^2+8*_Z+3)^2*x-8*RootOf
(16*_Z^2+8*_Z+3)*x-24*RootOf(16*_Z^2+8*_Z+3)-6*(-x^2-4*x-3)^(1/2)-3*x-6)/(
4*RootOf(16*_Z^2+8*_Z+3)*x+3*x+3))*RootOf(16*_Z^2+8*_Z+3)+1/2*RootOf(_Z^2+
1)*ln(RootOf(_Z^2+1)*x+2*RootOf(_Z^2+1)+(-x^2-4*x-3)^(1/2))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.64

$$\begin{aligned}\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = &-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) \\ &-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) \\ &+\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) \\ &+\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) \\ &-\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)\end{aligned}$$

input
```
integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x
+ 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3
))/(2*x + 3)) + 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) +
 1/8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2
 - 4*x - 3)*x - 4*x - 3)/x^2)
```

3.14.6 Sympy [F]

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = \int \frac{\sqrt{-\left(x+1\right)\left(x+3\right)}}{2x^2+4x+3}\,dx$$

input
```
integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3),x)
```

output
```
Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)
```

3.14.7 Maxima [F]

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = \int \frac{\sqrt{-x^2-4\,x-3}}{2\,x^2+4\,x+3}\,dx$$

input
```
integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 - 4*x - 3)/(2*x^2 + 4*x + 3), x)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(82) = 164$.

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\begin{aligned}\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = &-\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)\\ &-\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)\\ &-\frac{1}{2}\,\arcsin\left(x+2\right)\\ &-\frac{1}{4}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{\left(x+2\right)^2}+1\right)\\ &+\frac{1}{4}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{\left(x+2\right)^2}+3\right)\end{aligned}$$

input
```
integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1
)) - 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*
(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = \int \frac{\sqrt{-x^2-4\,x-3}}{2\,x^2+4\,x+3}\,dx$$

input
```
int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3),x)
```

output
```
int((- 4*x - x^2 - 3)^(1/2)/(4*x + 2*x^2 + 3), x)
```

3.15 $\int (3-x+2x^2)(2+3x+5x^2)^4\,dx$

3.15.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx = 48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x^8}{4}+\frac{5075x^9}{9}+\frac{475x^{10}}{2}+\frac{1250x^{11}}{11}$$

output
```
48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+
5075/9*x^9+475/2*x^10+1250/11*x^11
```

3.15.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)(2+3x+5x^2)^4\,dx = 48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x^8}{4}+\frac{5075x^9}{9}+\frac{475x^{10}}{2}+\frac{1250x^{11}}{11}$$

input
```
Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 11
76*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

3.15.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)\left(5x^2+3x+2\right)^4dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(1250x^{10}+2375x^9+5075x^8+6830x^7+8232x^6+7131x^5+5099x^4+2624x^3+1064x^2+272x+48\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

input
```
Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 11
76*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.15.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result
gosper	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{125}{11}$
default	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{125}{11}$
norman	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{125}{11}$
risch	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{125}{11}$
parallelrisch	$48x+136x^2+\frac{1064}{3}x^3+656x^4+\frac{5099}{5}x^5+\frac{2377}{2}x^6+1176x^7+\frac{3415}{4}x^8+\frac{5075}{9}x^9+\frac{475}{2}x^{10}+\frac{125}{11}$

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+
5075/9*x^9+475/2*x^10+1250/11*x^11
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^4\,dx = \frac{1250}{11}x^{11}+\frac{475}{2}x^{10}+\frac{5075}{9}x^9+\frac{3415}{4}x^8+1176\,x^7 + \frac{2377}{2}x^6+\frac{5099}{5}x^5+656\,x^4+\frac{1064}{3}x^3+136\,x^2+48\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^
6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^4\,dx = \frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

input
```
integrate((2*x**2-x+3)*(5*x**2+3*x+2)**4,x)
```

output
```
1250*x**11/11 + 475*x**10/2 + 5075*x**9/9 + 3415*x**8/4 + 1176*x**7 + 2377
*x**6/2 + 5099*x**5/5 + 656*x**4 + 1064*x**3/3 + 136*x**2 + 48*x
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^4\,dx = \frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^9+\frac{3415}{4}\,x^8+1176\,x^7+\frac{2377}{2}\,x^6+\frac{5099}{5}\,x^5+656\,x^4+\frac{1064}{3}\,x^3+136\,x^2+48\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^
6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^4\,dx = \frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^9+\frac{3415}{4}\,x^8+1176\,x^7+\frac{2377}{2}\,x^6+\frac{5099}{5}\,x^5+656\,x^4+\frac{1064}{3}\,x^3+136\,x^2+48\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^
6 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^4\,dx = \frac{1250\,x^{11}}{11}+\frac{475\,x^{10}}{2}+\frac{5075\,x^9}{9}+\frac{3415\,x^8}{4}+1176\,x^7+\frac{2377\,x^6}{2}+\frac{5099\,x^5}{5}+656\,x^4+\frac{1064\,x^3}{3}+136\,x^2+48\,x$$

input
```
int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^4,x)
```

output
```
48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 11
76*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

3.16 $\int (3-x+2x^2) (2+3x+5x^2)^3 \, dx$

3.16.1 Optimal result

Integrand size = 23, antiderivative size = 56

$$\int (3-x+2x^2) (2+3x+5x^2)^3 \, dx = 24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{250x^9}{9}$$

output
```
24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/
9*x^9
```

3.16.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2) (2+3x+5x^2)^3 \, dx = 24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{250x^9}{9}$$

input
```
Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x
^7)/7 + (325*x^8)/8 + (250*x^9)/9
```

3.16.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2 x^2-x+3\right) \left(5 x^2+3 x+2\right)^3 \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(250 x^8+325 x^7+720 x^6+804 x^5+876 x^4+579 x^3+322 x^2+100 x+24\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{250 x^9}{9}+\frac{325 x^8}{8}+\frac{720 x^7}{7}+134 x^6+\frac{876 x^5}{5}+\frac{579 x^4}{4}+\frac{322 x^3}{3}+50 x^2+24 x$$

input
```
Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x
^7)/7 + (325*x^8)/8 + (250*x^9)/9
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.16.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

method	result	size
gosper	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
default	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
norman	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
risch	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45
parallelrisch	$24x+50x^2+\frac{322}{3}x^3+\frac{579}{4}x^4+\frac{876}{5}x^5+134x^6+\frac{720}{7}x^7+\frac{325}{8}x^8+\frac{250}{9}x^9$	45

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/
9*x^9
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^3\,dx=\frac{250}{9}\,x^9+\frac{325}{8}\,x^8+\frac{720}{7}\,x^7+134\,x^6+\frac{876}{5}\,x^5+\frac{579}{4}\,x^4+\frac{322}{3}\,x^3+50\,x^2+24\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/
3*x^3 + 50*x^2 + 24*x
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int (3-x+2x^2)(2+3x+5x^2)^3\,dx = \frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

input
```
integrate((2*x**2-x+3)*(5*x**2+3*x+2)**3,x)
```

output
```
250*x**9/9 + 325*x**8/8 + 720*x**7/7 + 134*x**6 + 876*x**5/5 + 579*x**4/4
+ 322*x**3/3 + 50*x**2 + 24*x
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)(2+3x+5x^2)^3\,dx = \frac{250}{9}x^9+\frac{325}{8}x^8+\frac{720}{7}x^7+134x^6+\frac{876}{5}x^5+\frac{579}{4}x^4+\frac{322}{3}x^3+50x^2+24x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/
3*x^3 + 50*x^2 + 24*x
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)(2+3x+5x^2)^3\,dx = \frac{250}{9}x^9+\frac{325}{8}x^8+\frac{720}{7}x^7+134x^6+\frac{876}{5}x^5+\frac{579}{4}x^4+\frac{322}{3}x^3+50x^2+24x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/
3*x^3 + 50*x^2 + 24*x
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2) (2+3x+5x^2)^3 \, dx = \frac{250\,x^9}{9} + \frac{325\,x^8}{8} + \frac{720\,x^7}{7} + 134\,x^6 + \frac{876\,x^5}{5} + \frac{579\,x^4}{4} + \frac{322\,x^3}{3} + 50\,x^2 + 24\,x$$

input
```
int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3,x)
```

output
```
24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x
^7)/7 + (325*x^8)/8 + (250*x^9)/9
```

3.17 $\int (3 - x + 2x^2) (2 + 3x + 5x^2)^2 \, dx$

3.17.1 Optimal result

Integrand size = 23, antiderivative size = 44

$$\int (3 - x + 2x^2) \left(2 + 3x + 5x^2\right)^2 \, dx = 12x + 16x^2 + \frac{83x^3}{3} + \frac{85x^4}{4} + \frac{103x^5}{5} + \frac{35x^6}{6} + \frac{50x^7}{7}$$

output
```
12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

3.17.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (3 - x + 2x^2) \left(2 + 3x + 5x^2\right)^2 \, dx = 12x + 16x^2 + \frac{83x^3}{3} + \frac{85x^4}{4} + \frac{103x^5}{5} + \frac{35x^6}{6} + \frac{50x^7}{7}$$

input
```
Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x
^7)/7
```

3.17.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2 \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

input
```
Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x
^7)/7
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.17.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
gosper	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
default	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
norman	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
risch	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35
parallelrisch	$12x+16x^2+\frac{83}{3}x^3+\frac{85}{4}x^4+\frac{103}{5}x^5+\frac{35}{6}x^6+\frac{50}{7}x^7$	35

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^2\,dx=\frac{50}{7}\,x^7+\frac{35}{6}\,x^6+\frac{103}{5}\,x^5+\frac{85}{4}\,x^4+\frac{83}{3}\,x^3+16\,x^2+12\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (3-x+2x^2)\left(2+3x+5x^2\right)^2\,dx=\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

input
```
integrate((2*x**2-x+3)*(5*x**2+3*x+2)**2,x)
```

output
```
50*x**7/7 + 35*x**6/6 + 103*x**5/5 + 85*x**4/4 + 83*x**3/3 + 16*x**2 + 12*
x
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)(2+3x+5x^2)^2\,dx = \frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)(2+3x+5x^2)^2\,dx = \frac{50}{7}x^7 + \frac{35}{6}x^6 + \frac{103}{5}x^5 + \frac{85}{4}x^4 + \frac{83}{3}x^3 + 16x^2 + 12x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)(2+3x+5x^2)^2\,dx = \frac{50x^7}{7} + \frac{35x^6}{6} + \frac{103x^5}{5} + \frac{85x^4}{4} + \frac{83x^3}{3} + 16x^2 + 12x$$

input
```
int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2,x)
```

output
```
12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x
^7)/7
```

3.18 $\int (3-x+2x^2)(2+3x+5x^2)\,dx$

3.18.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int (3-x+2x^2)(2+3x+5x^2)\,dx = 6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5$$

output
```
6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

3.18.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)(2+3x+5x^2)\,dx = 6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5$$

input
```
Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

output
```
6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

3.18.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)\left(5x^2+3x+2\right)dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(10x^4+x^3+16x^2+7x+6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

input
```
Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

output
```
6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.18.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$6x + \frac{7}{2}x^2 + \frac{16}{3}x^3 + \frac{1}{4}x^4 + 2x^5$	25
default	$6x + \frac{7}{2}x^2 + \frac{16}{3}x^3 + \frac{1}{4}x^4 + 2x^5$	25
norman	$6x + \frac{7}{2}x^2 + \frac{16}{3}x^3 + \frac{1}{4}x^4 + 2x^5$	25
risch	$6x + \frac{7}{2}x^2 + \frac{16}{3}x^3 + \frac{1}{4}x^4 + 2x^5$	25
parallelrisch	$6x + \frac{7}{2}x^2 + \frac{16}{3}x^3 + \frac{1}{4}x^4 + 2x^5$	25

input
```
int((2*x^2-x+3)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (3 - x + 2x^2)(2 + 3x + 5x^2)\, dx = 2\,x^5 + \frac{1}{4}\,x^4 + \frac{16}{3}\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (3 - x + 2x^2)(2 + 3x + 5x^2)\, dx = 2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

input
```
integrate((2*x**2-x+3)*(5*x**2+3*x+2),x)
```

output
```
2*x**5 + x**4/4 + 16*x**3/3 + 7*x**2/2 + 6*x
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)\,(2+3x+5x^2)\;dx = 2\,x^5+\frac{1}{4}\,x^4+\frac{16}{3}\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)\,(2+3x+5x^2)\;dx = 2\,x^5+\frac{1}{4}\,x^4+\frac{16}{3}\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

3.18.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)\,(2+3x+5x^2)\;dx = 2\,x^5+\frac{x^4}{4}+\frac{16\,x^3}{3}+\frac{7\,x^2}{2}+6\,x$$

input
```
int((2*x^2 - x + 3)*(3*x + 5*x^2 + 2),x)
```

output
```
6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

3.19 $\int \frac{3-x+2x^2}{2+3x+5x^2} dx$

3.19.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{3-x+2x^2}{2+3x+5x^2} dx = \frac{2x}{5} + \frac{143 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}} - \frac{11}{50} \log\left(2+3x+5x^2\right)$$

output
```
2/5*x-11/50*ln(5*x^2+3*x+2)+143/775*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2
)
```

3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{3-x+2x^2}{2+3x+5x^2} dx = \frac{2x}{5} + \frac{143 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}} - \frac{11}{50} \log\left(2+3x+5x^2\right)$$

input
```
Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

output
```
(2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*
x + 5*x^2])/50
```

3.19.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-x+3}{5x^2+3x+2} \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{11(1-x)}{5\left(5x^2+3x+2\right)}+\frac{2}{5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{143 \arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}$$

input
```
Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

output
```
(2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*
x + 5*x^2])/50
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.19.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2x}{5}-\frac{11\ln\left(5x^2+3x+2\right)}{50}+\frac{143\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{775}$	34
risch	$\frac{2x}{5}-\frac{11\ln\left(100x^2+60x+40\right)}{50}+\frac{143\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{775}$	34

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x-11/50*ln(5*x^2+3*x+2)+143/775*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2
)
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx=\frac{143}{775}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^
2 + 3*x + 2)
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx=\frac{2x}{5}-\frac{11\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{50}+\frac{143\sqrt{31}\,\text{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{775}$$

input
```
integrate((2*x**2-x+3)/(5*x**2+3*x+2),x)
```

output
```
2*x/5 - 11*log(x**2 + 3*x/5 + 2/5)/50 + 143*sqrt(31)*atan(10*sqrt(31)*x/31
 + 3*sqrt(31)/31)/775
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx = \frac{143}{775}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^
2 + 3*x + 2)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx = \frac{143}{775}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^
2 + 3*x + 2)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx = \frac{2\,x}{5}-\frac{11\,\ln\left(5\,x^2+3\,x+2\right)}{50}+\frac{143\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{775}$$

input
```
int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2),x)
```

output
```
(2*x)/5 - (11*log(3*x + 5*x^2 + 2))/50 + (143*31^(1/2)*atan((10*31^(1/2)*x
)/31 + (3*31^(1/2))/31))/775
```

3.20 $\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx$

3.20.1 Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx=\frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{82\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}$$

output
```
11/155*(7+13*x)/(5*x^2+3*x+2)+82/961*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/
2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx=\frac{11(7+13x)}{155\left(2+3x+5x^2\right)}+\frac{82\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}$$

input
```
Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])
/(31*Sqrt[31])
```

3.20.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-x+3}{\left(5x^2+3x+2\right)^2}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{31}\int \frac{41}{5x^2+3x+2}dx+\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{41}{31}\int \frac{1}{5x^2+3x+2}dx+\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}-\frac{82}{31}\int \frac{1}{-(10x+3)^2-31}d(10x+3)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{82\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}$$

input
```
Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])
/(31*Sqrt[31])
```

3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.20.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\frac{143x}{775}+\frac{77}{775}}{x^2+\frac{3}{5}x+\frac{2}{5}}+\frac{82\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{961}$	34
risch	$\frac{\frac{143x}{775}+\frac{77}{775}}{x^2+\frac{3}{5}x+\frac{2}{5}}+\frac{82\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{961}$	34

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(143/775*x+77/775)/(x^2+3/5*x+2/5)+82/961*arctan(1/31*(10*x+3)*31^(1/2))*3
1^(1/2)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{410\,\sqrt{31}(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+4433\,x+2387}{4805\,(5\,x^2+3\,x+2)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/4805*(410*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) +
4433*x + 2387)/(5*x^2 + 3*x + 2)
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{143x+77}{775x^2+465x+310}+\frac{82\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

input
```
integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

output
```
(143*x + 77)/(775*x**2 + 465*x + 310) + 82*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/961
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{82}{961}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{11\,(13\,x+7)}{155\,(5\,x^2+3\,x+2)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^
2 + 3*x + 2)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{82}{961}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{11\,(13\,x+7)}{155\,(5\,x^2+3\,x+2)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^
2 + 3*x + 2)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^2}\,dx = \frac{\frac{143\,x}{775}+\frac{77}{775}}{x^2+\frac{3\,x}{5}+\frac{2}{5}} + \frac{82\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

input
```
int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^2,x)
```

output
```
((143*x)/775 + 77/775)/((3*x)/5 + x^2 + 2/5) + (82*31^(1/2)*atan((10*31^(1
/2)*x)/31 + (3*31^(1/2))/31))/961
```

3.21 $\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx$

3.21.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx = \frac{11(7+13x)}{310(2+3x+5x^2)^2} + \frac{553(3+10x)}{9610(2+3x+5x^2)} + \frac{1106\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

output
```
11/310*(7+13*x)/(5*x^2+3*x+2)^2+553/9610*(3+10*x)/(5*x^2+3*x+2)+1106/29791
*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} dx = \frac{\frac{31(1141+4094x+4977x^2+5530x^3)}{(2+3x+5x^2)^2} + 2212\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{59582}$$

input
```
Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((31*(1141 + 4094*x + 4977*x^2 + 5530*x^3))/(2 + 3*x + 5*x^2)^2 + 2212*Sqr
t[31]*ArcTan[(3 + 10*x)/Sqrt[31]])/59582
```

3.21.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2191, 27, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2-x+3}{(5x^2+3x+2)^3}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{62}\int \frac{553}{5\left(5x^2+3x+2\right)^2}dx+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{553}{310}\int \frac{1}{\left(5x^2+3x+2\right)^2}dx+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{553}{310}\left(\frac{10}{31}\int \frac{1}{5x^2+3x+2}dx+\frac{10x+3}{31\left(5x^2+3x+2\right)}\right)+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{553}{310}\left(\frac{10x+3}{31\left(5x^2+3x+2\right)}-\frac{20}{31}\int \frac{1}{-(10x+3)^2-31}d(10x+3)\right)+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{553}{310}\left(\frac{20\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{10x+3}{31\left(5x^2+3x+2\right)}\right)+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}$$

input
```
Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*((3 + 10*x)/(31*(2 + 3*x
+ 5*x^2)) + (20*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31])))/310
```

3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.21.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{2765}{961}x^3+\frac{4977}{1922}x^2+\frac{2047}{961}x+\frac{1141}{1922}}{(5x^2+3x+2)^2}+\frac{1106\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47
risch	$\frac{\frac{2765}{961}x^3+\frac{4977}{1922}x^2+\frac{2047}{961}x+\frac{1141}{1922}}{(5x^2+3x+2)^2}+\frac{1106\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47

input
```
int((2*x^2-x+3)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
25*(553/4805*x^3+4977/48050*x^2+2047/24025*x+1141/48050)/(5*x^2+3*x+2)^2+1
106/29791*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{171430\,x^3+2212\,\sqrt{31}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+154287\,x^2+126914}{59582\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/59582*(171430*x^3 + 2212*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*
arctan(1/31*sqrt(31)*(10*x + 3)) + 154287*x^2 + 126914*x + 35371)/(25*x^4
+ 30*x^3 + 29*x^2 + 12*x + 4)
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{5530x^3+4977x^2+4094x+1141}{48050x^4+57660x^3+55738x^2+23064x+7688} + \frac{1106\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

input
```
integrate((2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

output
```
(5530*x**3 + 4977*x**2 + 4094*x + 1141)/(48050*x**4 + 57660*x**3 + 55738*x
**2 + 23064*x + 7688) + 1106*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/3
1)/29791
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{5530\,x^3+4977\,x^2+4094\,x+1141}{1922\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 +
4977*x^2 + 4094*x + 1141)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{5530\,x^3+4977\,x^2+4094\,x+1141}{1922\,(5\,x^2+3\,x+2)^2}$$

input
```
integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 +
4977*x^2 + 4094*x + 1141)/(5*x^2 + 3*x + 2)^2
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{3-x+2x^2}{(2+3x+5x^2)^3}\,dx = \frac{1106\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791} + \frac{\frac{553\,x^3}{4805}+\frac{4977\,x^2}{48050}+\frac{2047\,x}{24025}+\frac{1141}{48050}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

input
```
int((2*x^2 - x + 3)/(3*x + 5*x^2 + 2)^3,x)
```

output
```
(1106*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((2047*
x)/24025 + (4977*x^2)/48050 + (553*x^3)/4805 + 1141/48050)/((12*x)/25 + (2
9*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

3.21. $\int \frac{3-x+2x^2}{(2+3x+5x^2)^3} \, dx$

3.22 $\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx$

3.22.1 Optimal result

Integrand size = 25, antiderivative size = 80

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = 144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3315x^8+\frac{24859x^9}{9}+1571x^{10}+\frac{11525x^{11}}{11}+\frac{875x^{12}}{3}+\frac{2500x^{13}}{13}$$

output
```
144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315
*x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

3.22.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = 144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3315x^8+\frac{24859x^9}{9}+1571x^{10}+\frac{11525x^{11}}{11}+\frac{875x^{12}}{3}+\frac{2500x^{13}}{13}$$

input
```
Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

output
```
144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3
+ (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 +
 (875*x^12)/3 + (2500*x^13)/13
```

3.22.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^2 \left(5x^2+3x+2\right)^4 \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(2500x^{12}+3500x^{11}+11525x^{10}+15710x^9+24859x^8+26520x^7+27763x^6+21542x^5+14801x^4+7352x^3+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+ 1838x^4+\frac{3016x^3}{3}+384x^2+144x$$

input
```
Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

output
```
144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3
+ (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 +
 (875*x^12)/3 + (2500*x^13)/13
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.22.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

method	result
gosper	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x$
default	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x$
norman	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x$
risch	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x$
parallelrisch	$144x+384x^2+\frac{3016}{3}x^3+1838x^4+\frac{14801}{5}x^5+\frac{10771}{3}x^6+\frac{27763}{7}x^7+3315x^8+\frac{24859}{9}x^9+1571x$

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315
*x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \frac{2500}{13}x^{13}+\frac{875}{3}x^{12}+\frac{11525}{11}x^{11}+1571x^{10} + \frac{24859}{9}x^9+3315x^8+\frac{27763}{7}x^7+\frac{10771}{3}x^6 + \frac{14801}{5}x^5+1838x^4+\frac{3016}{3}x^3+384x^2+144x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315
*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 3
84*x^2 + 144*x
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \frac{2500x^{13}}{13} + \frac{875x^{12}}{3} + \frac{11525x^{11}}{11} + 1571x^{10} + \frac{24859x^9}{9} + 3315x^8 + \frac{27763x^7}{7} + \frac{10771x^6}{3} + \frac{14801x^5}{5} + 1838x^4 + \frac{3016x^3}{3} + 384x^2 + 144x$$

input
```
integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**4,x)
```

output
```
2500*x**13/13 + 875*x**12/3 + 11525*x**11/11 + 1571*x**10 + 24859*x**9/9 +
 3315*x**8 + 27763*x**7/7 + 10771*x**6/3 + 14801*x**5/5 + 1838*x**4 + 3016
*x**3/3 + 384*x**2 + 144*x
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \frac{2500}{13} x^{13} + \frac{875}{3} x^{12} + \frac{11525}{11} x^{11} + 1571 x^{10} + \frac{24859}{9} x^9 + 3315 x^8 + \frac{27763}{7} x^7 + \frac{10771}{3} x^6 + \frac{14801}{5} x^5 + 1838 x^4 + \frac{3016}{3} x^3 + 384 x^2 + 144 x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315
*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 3
84*x^2 + 144*x
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \frac{2500}{13}\,x^{13} + \frac{875}{3}\,x^{12} + \frac{11525}{11}\,x^{11} + 1571\,x^{10} + \frac{24859}{9}\,x^9 + 3315\,x^8 + \frac{27763}{7}\,x^7 + \frac{10771}{3}\,x^6 + \frac{14801}{5}\,x^5 + 1838\,x^4 + \frac{3016}{3}\,x^3 + 384\,x^2 + 144\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315
*x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 3
84*x^2 + 144*x
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \frac{2500\,x^{13}}{13} + \frac{875\,x^{12}}{3} + \frac{11525\,x^{11}}{11} + 1571\,x^{10} + \frac{24859\,x^9}{9} + 3315\,x^8 + \frac{27763\,x^7}{7} + \frac{10771\,x^6}{3} + \frac{14801\,x^5}{5} + 1838\,x^4 + \frac{3016\,x^3}{3} + 384\,x^2 + 144\,x$$

input
```
int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^4,x)
```

output
```
144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3
+ (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 +
 (875*x^12)/3 + (2500*x^13)/13
```

3.23 $\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx$

3.23.1 Optimal result

Integrand size = 25, antiderivative size = 66

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx = 72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6 +444x^7+\frac{1863x^8}{8}+\frac{1865x^9}{9}+40x^{10}+\frac{500x^{11}}{11}$$

output
```
72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+18
65/9*x^9+40*x^10+500/11*x^11
```

3.23.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx = 72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6 +444x^7+\frac{1863x^8}{8}+\frac{1865x^9}{9}+40x^{10}+\frac{500x^{11}}{11}$$

input
```
Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

output
```
72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444
*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

3.23.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^2 \left(5x^2+3x+2\right)^3 \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(500x^{10}+400x^9+1865x^8+1863x^7+3108x^6+2694x^5+2693x^4+1615x^3+914x^2+276x+72\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

input
```
Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

output
```
72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444
*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.23.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result
gosper	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^1$
default	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^1$
norman	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^1$
risch	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^1$
parallelrisch	$72x+138x^2+\frac{914}{3}x^3+\frac{1615}{4}x^4+\frac{2693}{5}x^5+449x^6+444x^7+\frac{1863}{8}x^8+\frac{1865}{9}x^9+40x^{10}+\frac{500}{11}x^1$

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+18
65/9*x^9+40*x^10+500/11*x^11
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx = \frac{500}{11}x^{11}+40x^{10}+\frac{1865}{9}x^9+\frac{1863}{8}x^8+444x^7+449x^6 + \frac{2693}{5}x^5+\frac{1615}{4}x^4+\frac{914}{3}x^3+138x^2+72x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693
/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^9}{9} + \frac{1863x^8}{8} + 444x^7 + 449x^6 + \frac{2693x^5}{5} + \frac{1615x^4}{4} + \frac{914x^3}{3} + 138x^2 + 72x$$

input
```
integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**3,x)
```

output
```
500*x**11/11 + 40*x**10 + 1865*x**9/9 + 1863*x**8/8 + 444*x**7 + 449*x**6
+ 2693*x**5/5 + 1615*x**4/4 + 914*x**3/3 + 138*x**2 + 72*x
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \frac{500}{11}\,x^{11} + 40\,x^{10} + \frac{1865}{9}\,x^9 + \frac{1863}{8}\,x^8 + 444\,x^7 + 449\,x^6 + \frac{2693}{5}\,x^5 + \frac{1615}{4}\,x^4 + \frac{914}{3}\,x^3 + 138\,x^2 + 72\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693
/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \frac{500}{11}\,x^{11} + 40\,x^{10} + \frac{1865}{9}\,x^9 + \frac{1863}{8}\,x^8 + 444\,x^7 + 449\,x^6 + \frac{2693}{5}\,x^5 + \frac{1615}{4}\,x^4 + \frac{914}{3}\,x^3 + 138\,x^2 + 72\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693
/5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^3 \, dx = \frac{500\,x^{11}}{11} + 40\,x^{10} + \frac{1865\,x^9}{9} + \frac{1863\,x^8}{8} + 444\,x^7 + 449\,x^6 + \frac{2693\,x^5}{5} + \frac{1615\,x^4}{4} + \frac{914\,x^3}{3} + 138\,x^2 + 72\,x$$

input
```
int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3,x)
```

output
```
72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444
*x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

3.24 $\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx$

3.24.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx = 36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5+\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}$$

output
```
36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

3.24.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^2 \, dx = 36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5+\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}$$

input
```
Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

output
```
36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
 (5*x^8)/2 + (100*x^9)/9
```

3.24.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^2 \left(5x^2+3x+2\right)^2 dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(100x^8+20x^7+321x^6+172x^5+390x^4+236x^3+241x^2+84x+36\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

input
```
Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

output
```
36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
 (5*x^8)/2 + (100*x^9)/9
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.24.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
gosper	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
default	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
norman	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
risch	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45
parallelrisch	$36x+42x^2+\frac{241}{3}x^3+59x^4+78x^5+\frac{86}{3}x^6+\frac{321}{7}x^7+\frac{5}{2}x^8+\frac{100}{9}x^9$	45

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2\,dx=\frac{100}{9}\,x^9+\frac{5}{2}\,x^8+\frac{321}{7}\,x^7+\frac{86}{3}\,x^6+78\,x^5+59\,x^4+\frac{241}{3}\,x^3+42\,x^2+36\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
 42*x^2 + 36*x
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2\,dx=\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

input
```
integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**2,x)
```

output
```
100*x**9/9 + 5*x**8/2 + 321*x**7/7 + 86*x**6/3 + 78*x**5 + 59*x**4 + 241*x
**3/3 + 42*x**2 + 36*x
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx = \frac{100}{9}\,x^9 + \frac{5}{2}\,x^8 + \frac{321}{7}\,x^7 + \frac{86}{3}\,x^6 + 78\,x^5 + 59\,x^4 + \frac{241}{3}\,x^3 + 42\,x^2 + 36\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
 42*x^2 + 36*x
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx = \frac{100}{9}\,x^9 + \frac{5}{2}\,x^8 + \frac{321}{7}\,x^7 + \frac{86}{3}\,x^6 + 78\,x^5 + 59\,x^4 + \frac{241}{3}\,x^3 + 42\,x^2 + 36\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
 42*x^2 + 36*x
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx = \frac{100\,x^9}{9}+\frac{5\,x^8}{2}+\frac{321\,x^7}{7}+\frac{86\,x^6}{3}+78\,x^5 +59\,x^4+\frac{241\,x^3}{3}+42\,x^2+36\,x$$

input
```
int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2,x)
```

output
```
36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
 (5*x^8)/2 + (100*x^9)/9
```

3.25 $\int (3-x+2x^2)^2 (2+3x+5x^2) \, dx$

3.25.1 Optimal result

Integrand size = 23, antiderivative size = 46

$$\int (3-x+2x^2)^2 (2+3x+5x^2) \, dx = 18x+\frac{15x^2}{2}+\frac{53x^3}{3}+\frac{x^4}{4}+\frac{61x^5}{5}-\frac{4x^6}{3}+\frac{20x^7}{7}$$

output
```
18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7
```

3.25.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^2 (2+3x+5x^2) \, dx = 18x+\frac{15x^2}{2}+\frac{53x^3}{3}+\frac{x^4}{4}+\frac{61x^5}{5}-\frac{4x^6}{3}+\frac{20x^7}{7}$$

input
```
Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

output
```
18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)
/7
```

3.25.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2-x+3)^2 (5x^2+3x+2)\, dx$$

$$\Big\downarrow \text{2188}$$

$$\int (20x^6-8x^5+61x^4+x^3+53x^2+15x+18)\, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

input
```
Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

output
```
18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)
/7
```

3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.25.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
gosper	$18x+\frac{15}{2}x^{2}+\frac{53}{3}x^{3}+\frac{1}{4}x^{4}+\frac{61}{5}x^{5}-\frac{4}{3}x^{6}+\frac{20}{7}x^{7}$	35
default	$18x+\frac{15}{2}x^{2}+\frac{53}{3}x^{3}+\frac{1}{4}x^{4}+\frac{61}{5}x^{5}-\frac{4}{3}x^{6}+\frac{20}{7}x^{7}$	35
norman	$18x+\frac{15}{2}x^{2}+\frac{53}{3}x^{3}+\frac{1}{4}x^{4}+\frac{61}{5}x^{5}-\frac{4}{3}x^{6}+\frac{20}{7}x^{7}$	35
risch	$18x+\frac{15}{2}x^{2}+\frac{53}{3}x^{3}+\frac{1}{4}x^{4}+\frac{61}{5}x^{5}-\frac{4}{3}x^{6}+\frac{20}{7}x^{7}$	35
parallelrisch	$18x+\frac{15}{2}x^{2}+\frac{53}{3}x^{3}+\frac{1}{4}x^{4}+\frac{61}{5}x^{5}-\frac{4}{3}x^{6}+\frac{20}{7}x^{7}$	35

input
```
int((2*x^2-x+3)^2*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \left(3-x+2x^{2}\right)^{2}\left(2+3x+5x^{2}\right)\,dx=\frac{20}{7}\,x^{7}-\frac{4}{3}\,x^{6}+\frac{61}{5}\,x^{5}+\frac{1}{4}\,x^{4}+\frac{53}{3}\,x^{3}+\frac{15}{2}\,x^{2}+18\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \left(3-x+2x^{2}\right)^{2}\left(2+3x+5x^{2}\right)\,dx=\frac{20x^{7}}{7}-\frac{4x^{6}}{3}+\frac{61x^{5}}{5}+\frac{x^{4}}{4}+\frac{53x^{3}}{3}+\frac{15x^{2}}{2}+18x$$

input
```
integrate((2*x**2-x+3)**2*(5*x**2+3*x+2),x)
```

output
```
20*x**7/7 - 4*x**6/3 + 61*x**5/5 + x**4/4 + 53*x**3/3 + 15*x**2/2 + 18*x
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int (3-x+2x^2)^2 (2+3x+5x^2)\,dx = \frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int (3-x+2x^2)^2 (2+3x+5x^2)\,dx = \frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18\,x$$

input
```
integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int (3-x+2x^2)^2 (2+3x+5x^2)\,dx = \frac{20\,x^7}{7} - \frac{4\,x^6}{3} + \frac{61\,x^5}{5} + \frac{x^4}{4} + \frac{53\,x^3}{3} + \frac{15\,x^2}{2} + 18\,x$$

input
```
int((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2),x)
```

output
```
18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)
/7
```

3.26 $\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx$

3.26.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx=\frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{8349\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}}-\frac{1573\log(2+3x+5x^2)}{1250}$$

output
```
381/125*x-16/25*x^2+4/15*x^3-1573/1250*ln(5*x^2+3*x+2)+8349/19375*arctan(1
/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx=\frac{8349\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}}+\frac{10x(1143-240x+100x^2)-4719\log(2+3x+5x^2)}{3750}$$

input
```
Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

output
```
(8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) + (10*x*(1143 - 240*x +
100*x^2) - 4719*Log[2 + 3*x + 5*x^2])/3750
```

3.26.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^2}{5x^2+3x+2}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{4x^2}{5}+\frac{121(3-13x)}{125(5x^2+3x+2)}-\frac{32x}{25}+\frac{381}{125}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8349\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}+\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log(5x^2+3x+2)}{1250}+\frac{381x}{125}$$

input
```
Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

output
```
(381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]]
)/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.26.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\ln(5x^2+3x+2)}{1250}+\frac{8349\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{19375}$	44
risch	$\frac{4x^3}{15}-\frac{16x^2}{25}+\frac{381x}{125}-\frac{1573\ln(100x^2+60x+40)}{1250}+\frac{8349\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{19375}$	44

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
381/125*x-16/25*x^2+4/15*x^3-1573/1250*ln(5*x^2+3*x+2)+8349/19375*arctan(1
/31*(10*x+3)*31^(1/2))*31^(1/2)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{4}{15}\,x^3-\frac{16}{25}\,x^2+\frac{8349}{19375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{381}{125}\,x-\frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)
) + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{4x^3}{15}-\frac{16x^2}{25}+\frac{381x}{125}-\frac{1573\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{1250}+\frac{8349\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{19375}$$

input
```
integrate((2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

output
```
4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*log(x**2 + 3*x/5 + 2/5)/1250 + 8
349*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19375
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{\left(3-x+2x^2\right)^2}{2+3x+5x^2}\,dx = \frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)
) + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{\left(3-x+2x^2\right)^2}{2+3x+5x^2}\,dx = \frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)
) + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 12.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \frac{381\,x}{125} - \frac{1573\,\ln\left(5\,x^2+3\,x+2\right)}{1250} + \frac{8349\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{19375} - \frac{16\,x^2}{25} + \frac{4\,x^3}{15}$$

input
```
int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2),x)
```

output
```
(381*x)/125 - (1573*log(3*x + 5*x^2 + 2))/1250 + (8349*31^(1/2)*atan((10*3
1^(1/2)*x)/31 + (3*31^(1/2))/31))/19375 - (16*x^2)/25 + (4*x^3)/15
```

3.27 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx$

3.27.1 Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{4x}{25}+\frac{121(61+69x)}{3875\,(2+3x+5x^2)} + \frac{41932\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}}-\frac{22}{125}\log\left(2+3x+5x^2\right)$$

output
```
4/25*x+121/3875*(61+69*x)/(5*x^2+3*x+2)-22/125*ln(5*x^2+3*x+2)+41932/12012
5*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{19220x+\frac{3751(61+69x)}{2+3x+5x^2}+41932\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)-21142\log\left(2+3x+5x^2\right)}{120125}$$

input
```
Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(19220*x + (3751*(61 + 69*x))/(2 + 3*x + 5*x^2) + 41932*Sqrt[31]*ArcTan[(3
 + 10*x)/Sqrt[31]] - 21142*Log[2 + 3*x + 5*x^2])/120125
```

3.27.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^2}{(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{31}\int \frac{4(155x^2-248x+1008)}{25\,(5x^2+3x+2)}dx+\frac{121(69x+61)}{3875\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{775}\int \frac{155x^2-248x+1008}{5x^2+3x+2}dx+\frac{121(69x+61)}{3875\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{4}{775}\int \left(\frac{11(86-31x)}{5x^2+3x+2}+31\right)dx+\frac{121(69x+61)}{3875\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{775}\left(\frac{10483\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{5\sqrt{31}}-\frac{341}{10}\log\left(5x^2+3x+2\right)+31x\right)+\frac{121(69x+61)}{3875\,(5x^2+3x+2)}$$

input
```
Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (4*(31*x + (10483*ArcTan[(3 +
 10*x)/Sqrt[31]])/(5*Sqrt[31]) - (341*Log[2 + 3*x + 5*x^2])/10))/775
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.27.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{4x}{25}+\frac{\frac{8349x}{19375}+\frac{7381}{19375}}{x^2+\frac{3}{5}x+\frac{2}{5}}-\frac{22\ln\left(100x^2+60x+40\right)}{125}+\frac{41932\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{120125}$	50
default	$\frac{4x}{25}-\frac{11\left(-\frac{759x}{775}-\frac{671}{775}\right)}{25\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{22\ln\left(5x^2+3x+2\right)}{125}+\frac{41932\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{120125}$	51

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
4/25*x+(8349/19375*x+7381/19375)/(x^2+3/5*x+2/5)-22/125*ln(100*x^2+60*x+40
)+41932/120125*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{96100\,x^3 + 41932\,\sqrt{31}(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + 57660\,x^2 - 21142\,(5\,x^2+3\,x+2)\log(}{120125\,(5\,x^2+3\,x+2)}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/120125*(96100*x^3 + 41932*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31
)*(10*x + 3)) + 57660*x^2 - 21142*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) +
 297259*x + 228811)/(5*x^2 + 3*x + 2)
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{4x}{25} + \frac{8349x+7381}{19375x^2+11625x+7750} - \frac{22\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{125} + \frac{41932\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{120125}$$

input
```
integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

output
```
4*x/25 + (8349*x + 7381)/(19375*x**2 + 11625*x + 7750) - 22*log(x**2 + 3*x
/5 + 2/5)/125 + 41932*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/1201
25
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{4}{25}\,x + \frac{121\,(69\,x+61)}{3875\,(5\,x^2+3\,x+2)} - \frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875
*(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{4}{25}\,x + \frac{121\,(69\,x+61)}{3875\,(5\,x^2+3\,x+2)} - \frac{22}{125}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875
*(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx = \frac{4\,x}{25} - \frac{22\,\ln\left(5\,x^2+3\,x+2\right)}{125} + \frac{\frac{8349\,x}{19375}+\frac{7381}{19375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}} + \frac{41932\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{120125}$$

input
```
int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^2,x)
```

output
```
(4*x)/25 - (22*log(3*x + 5*x^2 + 2))/125 + ((8349*x)/19375 + 7381/19375)/(
(3*x)/5 + x^2 + 2/5) + (41932*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/
2))/31))/120125
```

3.27. $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx$

3.28 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx$

3.28.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx = \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{4330\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

output
```
121/7750*(61+69*x)/(5*x^2+3*x+2)^2+11/240250*(17557+45710*x)/(5*x^2+3*x+2)
+4330/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} dx = \frac{11(11183+33524x+44983x^2+45710x^3)}{48050(2+3x+5x^2)^2} + \frac{4330\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

input
```
Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(11*(11183 + 33524*x + 44983*x^2 + 45710*x^3))/(48050*(2 + 3*x + 5*x^2)^2)
 + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

3.28.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2191, 27, 2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^2}{(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{62}\int \frac{6200x^2-9920x+48669}{125(5x^2+3x+2)^2}dx+\frac{121(69x+61)}{7750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{6200x^2-9920x+48669}{(5x^2+3x+2)^2}dx}{7750}+\frac{121(69x+61)}{7750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{\frac{1}{31}\int \frac{541250}{5x^2+3x+2}dx+\frac{11(45710x+17557)}{31(5x^2+3x+2)}}{7750}+\frac{121(69x+61)}{7750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{541250}{31}\int \frac{1}{5x^2+3x+2}dx+\frac{11(45710x+17557)}{31(5x^2+3x+2)}}{7750}+\frac{121(69x+61)}{7750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{11(45710x+17557)}{31(5x^2+3x+2)}-\frac{1082500}{31}\int \frac{1}{-(10x+3)^2-31}d(10x+3)}{7750}+\frac{121(69x+61)}{7750(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{1082500\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{11(45710x+17557)}{31(5x^2+3x+2)}}{7750}+\frac{121(69x+61)}{7750(5x^2+3x+2)^2}$$

input
```
Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + ((11*(17557 + 45710*x))/(31
*(2 + 3*x + 5*x^2)) + (1082500*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31]))
/7750
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.28.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{50281}{4805}x^3+\frac{494813}{48050}x^2+\frac{184382}{24025}x+\frac{123013}{48050}}{(5x^2+3x+2)^2}+\frac{4330\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47
risch	$\frac{\frac{50281}{4805}x^3+\frac{494813}{48050}x^2+\frac{184382}{24025}x+\frac{123013}{48050}}{(5x^2+3x+2)^2}+\frac{4330\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{29791}$	47

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
25*(50281/120125*x^3+494813/1201250*x^2+184382/600625*x+123013/1201250)/(5
*x^2+3*x+2)^2+4330/29791*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{15587110\,x^3+216500\,\sqrt{31}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+15339203\,x^2+}{1489550\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/1489550*(15587110*x^3 + 216500*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
 + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 15339203*x^2 + 11431684*x + 38134
03)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{502810x^3+494813x^2+368764x+123013}{1201250x^4+1441500x^3+1393450x^2+576600x+192200} + \frac{4330\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

input
```
integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

output
```
(502810*x**3 + 494813*x**2 + 368764*x + 123013)/(1201250*x**4 + 1441500*x*
*3 + 1393450*x**2 + 576600*x + 192200) + 4330*sqrt(31)*atan(10*sqrt(31)*x/
31 + 3*sqrt(31)/31)/29791
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{11\,(45710\,x^3+44983\,x^2+33524\,x+11183)}{48050\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
 + 44983*x^2 + 33524*x + 11183)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{11\,(45710\,x^3+44983\,x^2+33524\,x+11183)}{48050\,(5\,x^2+3\,x+2)^2}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
 + 44983*x^2 + 33524*x + 11183)/(5*x^2 + 3*x + 2)^2
```

3.28.9 Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{4330\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791} + \frac{\frac{50281\,x^3}{120125}+\frac{494813\,x^2}{1201250}+\frac{184382\,x}{600625}+\frac{123013}{1201250}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

input
```
int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^3,x)
```

output
```
(4330*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/29791 + ((18438
2*x)/600625 + (494813*x^2)/1201250 + (50281*x^3)/120125 + 123013/1201250)/
((12*x)/25 + (29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

3.28. $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx$

3.29 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx$

3.29.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx = \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}+\frac{66752\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

output
```
121/11625*(61+69*x)/(5*x^2+3*x+2)^3+11/120125*(4579+12060*x)/(5*x^2+3*x+2)
^2+16688/148955*(3+10*x)/(5*x^2+3*x+2)+66752/923521*arctan(1/31*(3+10*x)*3
1^(1/2))*31^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4} dx = \frac{1259239+5674908x+12780597x^2+21491796x^3+18774000x^4+12516000x^5}{446865(2+3x+5x^2)^3}+\frac{66752\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

input
```
Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

output
```
(1259239 + 5674908*x + 12780597*x^2 + 21491796*x^3 + 18774000*x^4 + 125160
00*x^5)/(446865*(2 + 3*x + 5*x^2)^3) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])
/(29791*Sqrt[31])
```

3.29.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2191, 27, 2191, 27, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2x^2-x+3\right)^2}{\left(5x^2+3x+2\right)^4}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{93}\int \frac{6\left(1550x^2-2480x+12863\right)}{125\left(5x^2+3x+2\right)^3}dx+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{1550x^2-2480x+12863}{\left(5x^2+3x+2\right)^3}dx}{3875}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(\frac{1}{62}\int \frac{417200}{\left(5x^2+3x+2\right)^2}dx+\frac{11(12060x+4579)}{62\left(5x^2+3x+2\right)^2}\right)}{3875}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{208600}{31}\int \frac{1}{\left(5x^2+3x+2\right)^2}dx+\frac{11(12060x+4579)}{62\left(5x^2+3x+2\right)^2}\right)}{3875}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}$$

$$\Big\downarrow \text{1086}$$

$$\frac{2\left(\frac{208600}{31}\left(\frac{10}{31}\int \frac{1}{5x^2+3x+2}dx+\frac{10x+3}{31(5x^2+3x+2)}\right)+\frac{11(12060x+4579)}{62\left(5x^2+3x+2\right)^2}\right)}{3875}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2\left(\frac{208600}{31}\left(\frac{10x+3}{31(5x^2+3x+2)}-\frac{20}{31}\int \frac{1}{-(10x+3)^2-31}d(10x+3)\right)+\frac{11(12060x+4579)}{62(5x^2+3x+2)^2}\right)}{3875}+\frac{121(69x+61)}{11625\,(5x^2+3x+2)^3}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{2\left(\frac{208600}{31}\left(\frac{20\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}+\frac{10x+3}{31(5x^2+3x+2)}\right)+\frac{11(12060x+4579)}{62(5x^2+3x+2)^2}\right)}{3875}+\frac{121(69x+61)}{11625\,(5x^2+3x+2)^3}$$

input
```
Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

output
```
(121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (2*((11*(4579 + 12060*x))/
(62*(2 + 3*x + 5*x^2)^2) + (208600*((3 + 10*x)/(31*(2 + 3*x + 5*x^2)) + (2
0*ArcTan[(3 + 10*x)/Sqrt[31]])/(31*Sqrt[31])))/31))/3875
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.29.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\frac{834400}{29791}x^5+\frac{1251600}{29791}x^4+\frac{7163932}{148955}x^3+\frac{4260199}{148955}x^2+\frac{1891636}{148955}x+\frac{1259239}{446865}}{(5x^2+3x+2)^3}+\frac{66752\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{923521}$	57
risch	$\frac{\frac{834400}{29791}x^5+\frac{1251600}{29791}x^4+\frac{7163932}{148955}x^3+\frac{4260199}{148955}x^2+\frac{1891636}{148955}x+\frac{1259239}{446865}}{(5x^2+3x+2)^3}+\frac{66752\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{923521}$	57

input
```
int((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
125*(33376/148955*x^5+50064/148955*x^4+7163932/18619375*x^3+4260199/186193
75*x^2+1891636/18619375*x+1259239/55858125)/(5*x^2+3*x+2)^3+66752/923521*a
rctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx$$
$$= \frac{387996000\,x^5+581994000\,x^4+666245676\,x^3+1001280\,\sqrt{31}(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2}{13852815\,(125\,x^6+225\,x^5+285\,x^4+207\,x^3}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1/13852815*(387996000*x^5 + 581994000*x^4 + 666245676*x^3 + 1001280*sqrt(3
1)*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*arctan(1/3
1*sqrt(31)*(10*x + 3)) + 396198507*x^2 + 175922148*x + 39036409)/(125*x^6
+ 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+1259239}{55858125x^6+100544625x^5+127356525x^4+92501055x^3+50942610x^2+16087140x+3574920} + \frac{66752\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{923521}$$

input
```
integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**4,x)
```

output
```
(12516000*x**5 + 18774000*x**4 + 21491796*x**3 + 12780597*x**2 + 5674908*x
 + 1259239)/(55858125*x**6 + 100544625*x**5 + 127356525*x**4 + 92501055*x*
*3 + 50942610*x**2 + 16087140*x + 3574920) + 66752*sqrt(31)*atan(10*sqrt(3
1)*x/31 + 3*sqrt(31)/31)/923521
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+5674908\,x+1259239}{446865\,(125\,x^6+225\,x^5+285\,x^4+207\,x^3+114\,x^2+36\,x+8)}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(1251600
0*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/
(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx$$
$$= \frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+5674908\,x+1259239}{446865\,(5\,x^2+3\,x+2)^3}$$

input
```
integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(1251600
0*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/
(5*x^2 + 3*x + 2)^3
```

3.29.9 Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{66752\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}\,x}{31}+\frac{3\sqrt{31}}{31}\right)}{923521} + \frac{\frac{33376\,x^5}{148955}+\frac{50064\,x^4}{148955}+\frac{7163932\,x^3}{18619375}+\frac{4260199\,x^2}{18619375}+\frac{1891636\,x}{18619375}+\frac{1259239}{55858125}}{x^6+\frac{9\,x^5}{5}+\frac{57\,x^4}{25}+\frac{207\,x^3}{125}+\frac{114\,x^2}{125}+\frac{36\,x}{125}+\frac{8}{125}}$$

input
```
int((2*x^2 - x + 3)^2/(3*x + 5*x^2 + 2)^4,x)
```

output
```
(66752*31^(1/2)*atan((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/923521 + ((189
1636*x)/18619375 + (4260199*x^2)/18619375 + (7163932*x^3)/18619375 + (5006
4*x^4)/148955 + (33376*x^5)/148955 + 1259239/55858125)/((36*x)/125 + (114*
x^2)/125 + (207*x^3)/125 + (57*x^4)/25 + (9*x^5)/5 + x^6 + 8/125)
```

3.30 $\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx$

3.30.1 Optimal result

Integrand size = 25, antiderivative size = 96

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = 432x+1080x^2+2856x^3+5144x^4 + \frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}+\frac{94881x^8}{8} + \frac{103583x^9}{9}+\frac{75311x^{10}}{10}+\frac{68583x^{11}}{11} + \frac{30395x^{12}}{12}+\frac{27050x^{13}}{13}+\frac{2250x^{14}}{7}+\frac{1000x^{15}}{3}$$

output
```
432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881
/8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^1
3+2250/7*x^14+1000/3*x^15
```

3.30.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = 432x+1080x^2+2856x^3+5144x^4 + \frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}+\frac{94881x^8}{8} + \frac{103583x^9}{9}+\frac{75311x^{10}}{10}+\frac{68583x^{11}}{11} + \frac{30395x^{12}}{12}+\frac{27050x^{13}}{13}+\frac{2250x^{14}}{7}+\frac{1000x^{15}}{3}$$

input
```
Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

output
```
432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (
91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x
^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/
3
```

3.30.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^3 \left(5x^2+3x+2\right)^4 dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(5000x^{14}+4500x^{13}+27050x^{12}+30395x^{11}+68583x^{10}+75311x^9+103583x^8+94881x^7+91349x^6+64529$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+\frac{43083x^5}{5}+5144x^4+2856x^3+1080x^2+432x$$

input
```
Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

output
```
432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (
91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x
^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/
3
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.30.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

method	result
gosper	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{753}{10}$
default	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{753}{10}$
norman	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{753}{10}$
risch	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{753}{10}$
parallelrisch	$432x + 1080x^2 + 2856x^3 + 5144x^4 + \frac{43083}{5}x^5 + \frac{64529}{6}x^6 + \frac{91349}{7}x^7 + \frac{94881}{8}x^8 + \frac{103583}{9}x^9 + \frac{753}{10}$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881
/8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^1
3+2250/7*x^14+1000/3*x^15
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^4\,dx = {} & \frac{1000}{3}\,x^{15} + \frac{2250}{7}\,x^{14} + \frac{27050}{13}\,x^{13} + \frac{30395}{12}\,x^{12} \\ & + \frac{68583}{11}\,x^{11} + \frac{75311}{10}\,x^{10} + \frac{103583}{9}\,x^{9} \\ & + \frac{94881}{8}\,x^{8} + \frac{91349}{7}\,x^{7} + \frac{64529}{6}\,x^{6} + \frac{43083}{5}\,x^{5} \\ & + 5144\,x^{4} + 2856\,x^{3} + 1080\,x^{2} + 432\,x\end{aligned}$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11
+ 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 +
 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = \frac{1000x^{15}}{3} + \frac{2250x^{14}}{7} + \frac{27050x^{13}}{13} + \frac{30395x^{12}}{12} + \frac{68583x^{11}}{11} + \frac{75311x^{10}}{10} + \frac{103583x^9}{9} + \frac{94881x^8}{8} + \frac{91349x^7}{7} + \frac{64529x^6}{6} + \frac{43083x^5}{5} + 5144x^4 + 2856x^3 + 1080x^2 + 432x$$

input
```
integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**4,x)
```

output
```
1000*x**15/3 + 2250*x**14/7 + 27050*x**13/13 + 30395*x**12/12 + 68583*x**1
1/11 + 75311*x**10/10 + 103583*x**9/9 + 94881*x**8/8 + 91349*x**7/7 + 6452
9*x**6/6 + 43083*x**5/5 + 5144*x**4 + 2856*x**3 + 1080*x**2 + 432*x
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^4 \, dx = \frac{1000}{3}\,x^{15} + \frac{2250}{7}\,x^{14} + \frac{27050}{13}\,x^{13} + \frac{30395}{12}\,x^{12} + \frac{68583}{11}\,x^{11} + \frac{75311}{10}\,x^{10} + \frac{103583}{9}\,x^9 + \frac{94881}{8}\,x^8 + \frac{91349}{7}\,x^7 + \frac{64529}{6}\,x^6 + \frac{43083}{5}\,x^5 + 5144\,x^4 + 2856\,x^3 + 1080\,x^2 + 432\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11
+ 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 +
 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx = \frac{1000}{3}\,x^{15} + \frac{2250}{7}\,x^{14} + \frac{27050}{13}\,x^{13} + \frac{30395}{12}\,x^{12} + \frac{68583}{11}\,x^{11} + \frac{75311}{10}\,x^{10} + \frac{103583}{9}\,x^{9} + \frac{94881}{8}\,x^{8} + \frac{91349}{7}\,x^{7} + \frac{64529}{6}\,x^{6} + \frac{43083}{5}\,x^{5} + 5144\,x^{4} + 2856\,x^{3} + 1080\,x^{2} + 432\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11
+ 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 +
 43083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx = \frac{1000\,x^{15}}{3} + \frac{2250\,x^{14}}{7} + \frac{27050\,x^{13}}{13} + \frac{30395\,x^{12}}{12} + \frac{68583\,x^{11}}{11} + \frac{75311\,x^{10}}{10} + \frac{103583\,x^{9}}{9} + \frac{94881\,x^{8}}{8} + \frac{91349\,x^{7}}{7} + \frac{64529\,x^{6}}{6} + \frac{43083\,x^{5}}{5} + 5144\,x^{4} + 2856\,x^{3} + 1080\,x^{2} + 432\,x$$

input
```
int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^4,x)
```

output
```
432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (
91349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x
^11)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/
3
```

3.31 $\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx$

3.31.1 Optimal result

Integrand size = 25, antiderivative size = 82

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = 216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{7869x^8}{8}+\frac{3316x^9}{3}+\frac{3061x^{10}}{10}+\frac{4830x^{11}}{11}+25x^{12}+\frac{1000x^{13}}{13}$$

output
```
216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*
x^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

3.31.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = 216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{7869x^8}{8}+\frac{3316x^9}{3}+\frac{3061x^{10}}{10}+\frac{4830x^{11}}{11}+25x^{12}+\frac{1000x^{13}}{13}$$

input
```
Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

output
```
216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (
12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/
11 + 25*x^12 + (1000*x^13)/13
```

3.31.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^3 \left(5x^2+3x+2\right)^3 \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(1000x^{12}+300x^{11}+4830x^{10}+3061x^9+9948x^8+7869x^7+12016x^6+8619x^5+8292x^4+4483x^3+2610x^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+378x^2+216x$$

input
```
Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

output
```
216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (
12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/
11 + 25*x^12 + (1000*x^13)/13
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.31.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

method	result
gosper	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+$
default	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+$
norman	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+$
risch	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+$
parallelrisch	$216x+378x^2+870x^3+\frac{4483}{4}x^4+\frac{8292}{5}x^5+\frac{2873}{2}x^6+\frac{12016}{7}x^7+\frac{7869}{8}x^8+\frac{3316}{3}x^9+\frac{3061}{10}x^{10}+$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*
x^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = \frac{1000}{13}\,x^{13}+25\,x^{12}+\frac{4830}{11}\,x^{11}+\frac{3061}{10}\,x^{10} + \frac{3316}{3}\,x^9+\frac{7869}{8}\,x^8+\frac{12016}{7}\,x^7+\frac{2873}{2}\,x^6 + \frac{8292}{5}\,x^5+\frac{4483}{4}\,x^4+870\,x^3+378\,x^2+216\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8
*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*
x^2 + 216*x
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = \frac{1000x^{13}}{13} + 25x^{12} + \frac{4830x^{11}}{11} + \frac{3061x^{10}}{10} + \frac{3316x^9}{3} + \frac{7869x^8}{8} + \frac{12016x^7}{7} + \frac{2873x^6}{2} + \frac{8292x^5}{5} + \frac{4483x^4}{4} + 870x^3 + 378x^2 + 216x$$

input
```
integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**3,x)
```

output
```
1000*x**13/13 + 25*x**12 + 4830*x**11/11 + 3061*x**10/10 + 3316*x**9/3 + 7
869*x**8/8 + 12016*x**7/7 + 2873*x**6/2 + 8292*x**5/5 + 4483*x**4/4 + 870*
x**3 + 378*x**2 + 216*x
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = \frac{1000}{13}\,x^{13} + 25\,x^{12} + \frac{4830}{11}\,x^{11} + \frac{3061}{10}\,x^{10} + \frac{3316}{3}\,x^9 + \frac{7869}{8}\,x^8 + \frac{12016}{7}\,x^7 + \frac{2873}{2}\,x^6 + \frac{8292}{5}\,x^5 + \frac{4483}{4}\,x^4 + 870\,x^3 + 378\,x^2 + 216\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8
*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*
x^2 + 216*x
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = \frac{1000}{13}\,x^{13}+25\,x^{12}+\frac{4830}{11}\,x^{11}+\frac{3061}{10}\,x^{10}+\frac{3316}{3}\,x^{9}+\frac{7869}{8}\,x^{8}+\frac{12016}{7}\,x^{7}+\frac{2873}{2}\,x^{6}+\frac{8292}{5}\,x^{5}+\frac{4483}{4}\,x^{4}+870\,x^{3}+378\,x^{2}+216\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8
*x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*
x^2 + 216*x
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^3 \, dx = \frac{1000\,x^{13}}{13}+25\,x^{12}+\frac{4830\,x^{11}}{11}+\frac{3061\,x^{10}}{10}+\frac{3316\,x^{9}}{3}+\frac{7869\,x^{8}}{8}+\frac{12016\,x^{7}}{7}+\frac{2873\,x^{6}}{2}+\frac{8292\,x^{5}}{5}+\frac{4483\,x^{4}}{4}+870\,x^{3}+378\,x^{2}+216\,x$$

input
```
int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3,x)
```

output
```
216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (
12016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/
11 + 25*x^12 + (1000*x^13)/13
```

3.32 $\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx$

3.32.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx = 108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+\frac{922x^9}{9}-6x^{10}+\frac{200x^{11}}{11}$$

output
```
108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+9
22/9*x^9-6*x^10+200/11*x^11
```

3.32.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx = 108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+\frac{922x^9}{9}-6x^{10}+\frac{200x^{11}}{11}$$

input
```
Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

output
```
108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (15
71*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

3.32.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^3 \left(5x^2+3x+2\right)^2 dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(200x^{10}-60x^9+922x^8+83x^7+1571x^6+598x^5+1416x^4+635x^3+711x^2+216x+108\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

input
```
Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

output
```
108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (15
71*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.32.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result
gosper	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$
default	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$
norman	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$
risch	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$
parallelrisch	$108x+108x^2+237x^3+\frac{635}{4}x^4+\frac{1416}{5}x^5+\frac{299}{3}x^6+\frac{1571}{7}x^7+\frac{83}{8}x^8+\frac{922}{9}x^9-6x^{10}+\frac{200}{11}x^{11}$

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+9
22/9*x^9-6*x^10+200/11*x^11
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx = \frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^9 + \frac{83}{8}x^8 + \frac{1571}{7}x^7 + \frac{299}{3}x^6 + \frac{1416}{5}x^5 + \frac{635}{4}x^4 + 237x^3 + 108x^2 + 108x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 141
6/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx = \frac{200x^{11}}{11} - 6x^{10} + \frac{922x^9}{9} + \frac{83x^8}{8} + \frac{1571x^7}{7} + \frac{299x^6}{3} + \frac{1416x^5}{5} + \frac{635x^4}{4} + 237x^3 + 108x^2 + 108x$$

input
```
integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**2,x)
```

output
```
200*x**11/11 - 6*x**10 + 922*x**9/9 + 83*x**8/8 + 1571*x**7/7 + 299*x**6/3
 + 1416*x**5/5 + 635*x**4/4 + 237*x**3 + 108*x**2 + 108*x
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx = \frac{200}{11}\,x^{11} - 6\,x^{10} + \frac{922}{9}\,x^9 + \frac{83}{8}\,x^8 + \frac{1571}{7}\,x^7 + \frac{299}{3}\,x^6 + \frac{1416}{5}\,x^5 + \frac{635}{4}\,x^4 + 237\,x^3 + 108\,x^2 + 108\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 141
6/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx = \frac{200}{11}\,x^{11} - 6\,x^{10} + \frac{922}{9}\,x^9 + \frac{83}{8}\,x^8 + \frac{1571}{7}\,x^7 + \frac{299}{3}\,x^6 + \frac{1416}{5}\,x^5 + \frac{635}{4}\,x^4 + 237\,x^3 + 108\,x^2 + 108\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 141
6/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)^2 \, dx = \frac{200\,x^{11}}{11} - 6\,x^{10} + \frac{922\,x^9}{9} + \frac{83\,x^8}{8} + \frac{1571\,x^7}{7} + \frac{299\,x^6}{3} + \frac{1416\,x^5}{5} + \frac{635\,x^4}{4} + 237\,x^3 + 108\,x^2 + 108\,x$$

input
```
int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2,x)
```

output
```
108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (15
71*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

3.33 $\int (3-x+2x^2)^3 (2+3x+5x^2) \, dx$

3.33.1 Optimal result

Integrand size = 23, antiderivative size = 56

$$\int (3-x+2x^2)^3 (2+3x+5x^2) \, dx = 54x + \frac{27x^2}{2} + 60x^3 - 5x^4 + \frac{288x^5}{5} - \frac{83x^6}{6} + \frac{190x^7}{7} - \frac{9x^8}{2} + \frac{40x^9}{9}$$

output
```
54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9
```

3.33.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (3-x+2x^2)^3 (2+3x+5x^2) \, dx = 54x + \frac{27x^2}{2} + 60x^3 - 5x^4 + \frac{288x^5}{5} - \frac{83x^6}{6} + \frac{190x^7}{7} - \frac{9x^8}{2} + \frac{40x^9}{9}$$

input
```
Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

output
```
54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/
7 - (9*x^8)/2 + (40*x^9)/9
```

3.33.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^3 \left(5x^2+3x+2\right) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(40x^8-36x^7+190x^6-83x^5+288x^4-20x^3+180x^2+27x+54\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

input
```
Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

output
```
54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/
7 - (9*x^8)/2 + (40*x^9)/9
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.33.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

method	result	size
gosper	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
default	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
norman	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
risch	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45
parallelrisch	$54x+\frac{27}{2}x^2+60x^3-5x^4+\frac{288}{5}x^5-\frac{83}{6}x^6+\frac{190}{7}x^7-\frac{9}{2}x^8+\frac{40}{9}x^9$	45

input
```
int((2*x^2-x+3)^3*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\,dx = \frac{40}{9}\,x^9-\frac{9}{2}\,x^8+\frac{190}{7}\,x^7-\frac{83}{6}\,x^6+\frac{288}{5}\,x^5-5\,x^4+60\,x^3+\frac{27}{2}\,x^2+54\,x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 2
7/2*x^2 + 54*x
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\,dx = \frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

input
```
integrate((2*x**2-x+3)**3*(5*x**2+3*x+2),x)
```

output
```
40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x
**3 + 27*x**2/2 + 54*x
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\, dx = \frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 2
7/2*x^2 + 54*x
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (3-x+2x^2)^3 (2+3x+5x^2)\, dx = \frac{40}{9}x^9 - \frac{9}{2}x^8 + \frac{190}{7}x^7 - \frac{83}{6}x^6 + \frac{288}{5}x^5 - 5x^4 + 60x^3 + \frac{27}{2}x^2 + 54x$$

input
```
integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 2
7/2*x^2 + 54*x
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)\,dx = \frac{40\,x^9}{9}-\frac{9\,x^8}{2}+\frac{190\,x^7}{7}-\frac{83\,x^6}{6}+\frac{288\,x^5}{5}-5\,x^4+60\,x^3+\frac{27\,x^2}{2}+54\,x$$

input

```
int((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2),x)
```

output

```
54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/
7 - (9*x^8)/2 + (40*x^9)/9
```

3.34 $\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx$

3.34.1 Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx=\frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}+\frac{328757\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}}-\frac{158389\log(2+3x+5x^2)}{31250}$$

output
```
49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln
(5*x^2+3*x+2)+328757/484375*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx =\frac{1972542\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)+31(5x(297048-111765x+61100x^2-15750x^3+6000x^4)-475167\log(2}{2906250}$$

input
```
Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

output
```
(1972542*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 31*(5*x*(297048 - 111765*x
 + 61100*x^2 - 15750*x^3 + 6000*x^4) - 475167*Log[2 + 3*x + 5*x^2]))/29062
50
```

3.34.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^3}{5x^2+3x+2}\,dx$$

$$\big\downarrow \text{2188}$$

$$\int \left(\frac{8x^4}{5}-\frac{84x^3}{25}+\frac{1222x^2}{125}-\frac{1331(119x+11)}{3125\,(5x^2+3x+2)}-\frac{7451x}{625}+\frac{49508}{3125}\right)dx$$

$$\big\downarrow \text{2009}$$

$$\frac{328757\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}+\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}$$

input
```
Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

output
```
(49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/
25 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2
 + 3*x + 5*x^2])/31250
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.34.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\ln(5x^2+3x+2)}{31250}+\frac{328757\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{484375}$	54
risch	$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}+\frac{49508x}{3125}-\frac{158389\ln(100x^2+60x+40)}{31250}+\frac{328757\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{484375}$	54

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln
(5*x^2+3*x+2)+328757/484375*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{8}{25}\,x^5-\frac{21}{25}\,x^4+\frac{1222}{375}\,x^3-\frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{49508}{3125}\,x-\frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(3
1)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^
2 + 3*x + 2)
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} - \frac{158389\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{31250} + \frac{328757\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{484375}$$

input
```
integrate((2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

output
```
8*x**5/25 - 21*x**4/25 + 1222*x**3/375 - 7451*x**2/1250 + 49508*x/3125 - 1
58389*log(x**2 + 3*x/5 + 2/5)/31250 + 328757*sqrt(31)*atan(10*sqrt(31)*x/3
1 + 3*sqrt(31)/31)/484375
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{8}{25}\,x^5 - \frac{21}{25}\,x^4 + \frac{1222}{375}\,x^3 - \frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{49508}{3125}\,x - \frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(3
1)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^
2 + 3*x + 2)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{8}{25}\,x^5 - \frac{21}{25}\,x^4 + \frac{1222}{375}\,x^3 - \frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{49508}{3125}\,x - \frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(3
1)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^
2 + 3*x + 2)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \frac{49508\,x}{3125} - \frac{158389\,\ln\left(5\,x^2+3\,x+2\right)}{31250} + \frac{328757\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{484375} - \frac{7451\,x^2}{1250} + \frac{1222\,x^3}{375} - \frac{21\,x^4}{25} + \frac{8\,x^5}{25}$$

input
```
int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2),x)
```

output
```
(49508*x)/3125 - (158389*log(3*x + 5*x^2 + 2))/31250 + (328757*31^(1/2)*at
an((10*31^(1/2)*x)/31 + (3*31^(1/2))/31))/484375 - (7451*x^2)/1250 + (1222
*x^3)/375 - (21*x^4)/25 + (8*x^5)/25
```

3.35 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx$

3.35.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\,(2+3x+5x^2)}+\frac{3819607\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}}-\frac{10769\log\left(2+3x+5x^2\right)}{6250}$$

output
```
1466/625*x-54/125*x^2+8/75*x^3+1331/96875*(443+247*x)/(5*x^2+3*x+2)-10769/
6250*ln(5*x^2+3*x+2)+3819607/3003125*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/
2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875\,(2+3x+5x^2)}+\frac{3819607\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}}-\frac{10769\log\left(2+3x+5x^2\right)}{6250}$$

input
```
Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2
+ 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) -
 (10769*Log[2 + 3*x + 5*x^2])/6250
```

3.35.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^3}{(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{31}\int \frac{31000x^4-65100x^3+189410x^2-230981x+372701}{625(5x^2+3x+2)}dx+\frac{1331(247x+443)}{96875(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{31000x^4-65100x^3+189410x^2-230981x+372701}{5x^2+3x+2}dx}{19375}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\int \left(6200x^2-16740x+\frac{121(2329-2759x)}{5x^2+3x+2}+45446\right)dx}{19375}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3819607\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{5\sqrt{31}}+\frac{6200x^3}{3}-8370x^2-\frac{333839}{10}\log(5x^2+3x+2)+45446x}{19375}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}$$

input
```
Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(1331*(443 + 247*x))/(96875*(2 + 3*x + 5*x^2)) + (45446*x - 8370*x^2 + (62
00*x^3)/3 + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(5*Sqrt[31]) - (333839*L
og[2 + 3*x + 5*x^2])/10)/19375
```

3.35. $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}dx$

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.35.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}+\frac{\frac{328757x}{484375}+\frac{589633}{484375}}{x^2+\frac{3}{5}x+\frac{2}{5}}-\frac{10769\ln\left(100x^2+60x+40\right)}{6250}+\frac{3819607\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{3003125}$	60
default	$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1466x}{625}-\frac{121\left(-\frac{2717x}{775}-\frac{4873}{775}\right)}{625\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{10769\ln\left(5x^2+3x+2\right)}{6250}+\frac{3819607\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{3003125}$	61

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
8/75*x^3-54/125*x^2+1466/625*x+(328757/484375*x+589633/484375)/(x^2+3/5*x+
2/5)-10769/6250*ln(100*x^2+60*x+40)+3819607/3003125*arctan(1/31*(10*x+3)*3
1^(1/2))*31^(1/2)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx$$
$$= \frac{9610000\,x^5 - 33154500\,x^4 + 191815600\,x^3 + 22917642\,\sqrt{31}(5\,x^2+3\,x+2)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + 1}{18018750\,(5\,x^2+3\,x}$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/18018750*(9610000*x^5 - 33154500*x^4 + 191815600*x^3 + 22917642*sqrt(31)
*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 111226140*x^2 - 3104
7027*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 145678362*x + 109671738)/(5*
x^2 + 3*x + 2)
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{328757x+589633}{484375x^2+290625x+193750} - \frac{10769\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{6250} + \frac{3819607\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{3003125}$$

input
```
integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

output
```
8*x**3/75 - 54*x**2/125 + 1466*x/625 + (328757*x + 589633)/(484375*x**2 +
290625*x + 193750) - 10769*log(x**2 + 3*x/5 + 2/5)/6250 + 3819607*sqrt(31)
*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3003125
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{8}{75}\,x^3-\frac{54}{125}\,x^2+\frac{3819607}{3003125}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{1466}{625}\,x+\frac{1331\,(247\,x+443)}{96875\,(5\,x^2+3\,x+2)}-\frac{10769}{6250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*
x + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/
6250*log(5*x^2 + 3*x + 2)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{8}{75}\,x^3-\frac{54}{125}\,x^2+\frac{3819607}{3003125}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{1466}{625}\,x+\frac{1331\,(247\,x+443)}{96875\,(5\,x^2+3\,x+2)}-\frac{10769}{6250}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*
x + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/
6250*log(5*x^2 + 3*x + 2)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx = \frac{1466\,x}{625} - \frac{10769\,\ln\left(5\,x^2+3\,x+2\right)}{6250} + \frac{\frac{328757\,x}{484375}+\frac{589633}{484375}}{x^2+\frac{3\,x}{5}+\frac{2}{5}} + \frac{3819607\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{3003125} - \frac{54\,x^2}{125} + \frac{8\,x^3}{75}$$

input
```
int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^2,x)
```

output
```
(1466*x)/625 - (10769*log(3*x + 5*x^2 + 2))/6250 + ((328757*x)/484375 + 58
9633/484375)/((3*x)/5 + x^2 + 2/5) + (3819607*31^(1/2)*atan((10*31^(1/2)*x
)/31 + (3*31^(1/2))/31))/3003125 - (54*x^2)/125 + (8*x^3)/75
```

3.36 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} \, dx$

3.36.1 Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} \, dx = \frac{8x}{125} + \frac{1331(443+247x)}{193750\,(2+3x+5x^2)^2} + \frac{121(188381+342840x)}{6006250\,(2+3x+5x^2)} + \frac{11341176 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}} - \frac{66}{625} \log\left(2+3x+5x^2\right)$$

output
```
8/125*x+1331/193750*(443+247*x)/(5*x^2+3*x+2)^2+121/6006250*(188381+342840
*x)/(5*x^2+3*x+2)-66/625*ln(5*x^2+3*x+2)+11341176/18619375*arctan(1/31*(3+
10*x)*31^(1/2))*31^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3} \, dx = \frac{11916400x + \frac{1279091(443+247x)}{(2+3x+5x^2)^2} + \frac{3751(188381+342840x)}{2+3x+5x^2} + 113411760\sqrt{31} \arctan\left(\frac{3+10x}{\sqrt{31}}\right) - 19662060 \log(2+3}{186193750}$$

input
```
Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(11916400*x + (1279091*(443 + 247*x))/(2 + 3*x + 5*x^2)^2 + (3751*(188381
+ 342840*x))/(2 + 3*x + 5*x^2) + 113411760*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt
[31]] - 19662060*Log[2 + 3*x + 5*x^2])/186193750
```

3.36.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2191, 27, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2x^2-x+3\right)^3}{\left(5x^2+3x+2\right)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{62}\int \frac{310000x^4-651000x^3+1894100x^2-2309810x+4055767}{3125\left(5x^2+3x+2\right)^2}dx+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{310000x^4-651000x^3+1894100x^2-2309810x+4055767}{\left(5x^2+3x+2\right)^2}dx}{193750}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{1}{31}\int \frac{100\left(19220x^2-51894x+555719\right)}{5x^2+3x+2}dx+\frac{121(342840x+188381)}{31\left(5x^2+3x+2\right)}}{193750}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{100}{31}\int \frac{19220x^2-51894x+555719}{5x^2+3x+2}dx+\frac{121(342840x+188381)}{31\left(5x^2+3x+2\right)}}{193750}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\frac{100}{31}\int \left(\frac{33(16607-1922x)}{5x^2+3x+2}+3844\right)dx+\frac{121(342840x+188381)}{31\left(5x^2+3x+2\right)}}{193750}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{100}{31}\left(\frac{5670588\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{5\sqrt{31}}-\frac{31713}{5}\log\left(5x^2+3x+2\right)+3844x\right)+\frac{121(342840x+188381)}{31(5x^2+3x+2)}}{193750}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}$$

input
```
Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

output
```
(1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + ((121*(188381 + 342840
*x))/(31*(2 + 3*x + 5*x^2)) + (100*(3844*x + (5670588*ArcTan[(3 + 10*x)/Sq
rt[31]])/(5*Sqrt[31]) - (31713*Log[2 + 3*x + 5*x^2])/5))/31)/193750
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.36.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{8x}{125}-\frac{11\left(-\frac{377124}{24025}x^3-\frac{866987}{48050}x^2-\frac{293711}{24025}x-\frac{232243}{48050}\right)}{5\left(5x^2+3x+2\right)^2}-\frac{66\ln\left(5x^2+3x+2\right)}{625}+\frac{11341176\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{18619375}$	63
risch	$\frac{8x}{125}+\frac{\frac{4148364}{120125}x^3+\frac{9536857}{240250}x^2+\frac{3230821}{120125}x+\frac{2554673}{240250}}{\left(5x^2+3x+2\right)^2}-\frac{66\ln\left(100x^2+60x+40\right)}{625}+\frac{11341176\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{18619375}$	63

input
```
int((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
8/125*x-11/5*(-377124/24025*x^3-866987/48050*x^2-293711/24025*x-232243/480
50)/(5*x^2+3*x+2)^2-66/625*ln(5*x^2+3*x+2)+11341176/18619375*arctan(1/31*(
10*x+3)*31^(1/2))*31^(1/2)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$$
$$=\frac{59582000\,x^5+71498400\,x^4+1355107960\,x^3+22682352\,\sqrt{31}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\arctan\left(\frac{1}{3}\right.}{37238750}$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/37238750*(59582000*x^5 + 71498400*x^4 + 1355107960*x^3 + 22682352*sqrt(3
1)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3))
+ 1506812195*x^2 - 3932412*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2
 + 3*x + 2) + 1011087630*x + 395974315)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x +
 4)
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{8x}{125} + \frac{8296728x^3+9536857x^2+6461642x+2554673}{6006250x^4+7207500x^3+6967250x^2+2883000x+961000} - \frac{66\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{625} + \frac{11341176\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{18619375}$$

input
```
integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

output
```
8*x/125 + (8296728*x**3 + 9536857*x**2 + 6461642*x + 2554673)/(6006250*x**
4 + 7207500*x**3 + 6967250*x**2 + 2883000*x + 961000) - 66*log(x**2 + 3*x/
5 + 2/5)/625 + 11341176*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/18
619375
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{8}{125}\,x + \frac{121\,(68568\,x^3+78817\,x^2+53402\,x+21113)}{240250\,(25\,x^4+30\,x^3+29\,x^2+12\,x+4)} - \frac{66}{625}\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 12
1/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(25*x^4 + 30*x^3 + 29*x
^2 + 12*x + 4) - 66/625*log(5*x^2 + 3*x + 2)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{8}{125}\,x + \frac{121\,(68568\,x^3+78817\,x^2+53402\,x+21113)}{240250\,(5\,x^2+3\,x+2)^2} - \frac{66}{625}\,\log\left(5\,x^2+3\,x+2\right)$$

input
```
integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 12
1/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(5*x^2 + 3*x + 2)^2 - 6
6/625*log(5*x^2 + 3*x + 2)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx = \frac{8\,x}{125}-\frac{66\,\ln\left(5\,x^2+3\,x+2\right)}{625}+\frac{11341176\,\sqrt{31}\,\mathrm{atan}\left(\frac{10\,\sqrt{31}\,x}{31}+\frac{3\,\sqrt{31}}{31}\right)}{18619375} + \frac{\frac{4148364\,x^3}{3003125}+\frac{9536857\,x^2}{6006250}+\frac{3230821\,x}{3003125}+\frac{2554673}{6006250}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$

input
```
int((2*x^2 - x + 3)^3/(3*x + 5*x^2 + 2)^3,x)
```

output
```
(8*x)/125 - (66*log(3*x + 5*x^2 + 2))/625 + (11341176*31^(1/2)*atan((10*31
^(1/2)*x)/31 + (3*31^(1/2))/31))/18619375 + ((3230821*x)/3003125 + (953685
7*x^2)/6006250 + (4148364*x^3)/3003125 + 2554673/6006250)/((12*x)/25 + (29
*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

3.37 $\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx$

3.37.1 Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{122691x}{128}-\frac{28747x^2}{128}-\frac{21229x^3}{96}+\frac{6245x^4}{64}+\frac{1855x^5}{8}+\frac{3625x^6}{24}+\frac{625x^7}{14}+\frac{1156639\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}}+\frac{307461}{512}\log\left(3-x+2x^2\right)$$

output
```
122691/128*x-28747/128*x^2-21229/96*x^3+6245/64*x^4+1855/8*x^5+3625/24*x^6
+625/14*x^7+307461/512*ln(2*x^2-x+3)+1156639/5888*arctan(1/23*(1-4*x)*23^(
1/2))*23^(1/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{x(2576511-603687x-594412x^2+262290x^3+623280x^4+406000x^5+120000x^6)}{2688} - \frac{1156639\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{256\sqrt{23}}+\frac{307461}{512}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

output
```
(x*(2576511 - 603687*x - 594412*x^2 + 262290*x^3 + 623280*x^4 + 406000*x^5
 + 120000*x^6))/2688 - (1156639*ArcTan[(-1 + 4*x)/Sqrt[23]])/(256*Sqrt[23]
) + (307461*Log[3 - x + 2*x^2])/512
```

3.37.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(5x^2+3x+2\right)^4}{2x^2-x+3}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{625x^6}{2}+\frac{3625x^5}{4}+\frac{9275x^4}{8}+\frac{6245x^3}{16}-\frac{21229x^2}{32}-\frac{14641(25-21x)}{128\left(2x^2-x+3\right)}-\frac{28747x}{64}+\frac{122691}{128}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1156639\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{23}}+\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

output
```
(122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*
x^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]
])/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.37.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

method	result
default	$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\ln(2x^2-x+3)}{512}-\frac{1156639\sqrt{23}\arctan}{5888}$
risch	$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\ln(16x^2-8x+24)}{512}-\frac{1156639\sqrt{23}\text{arc}}{58}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

output
```
625/14*x^7+3625/24*x^6+1855/8*x^5+6245/64*x^4-21229/96*x^3-28747/128*x^2+1
22691/128*x+307461/512*ln(2*x^2-x+3)-1156639/5888*23^(1/2)*arctan(1/23*(-1
+4*x)*23^(1/2))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{625}{14}\,x^7+\frac{3625}{24}\,x^6+\frac{1855}{8}\,x^5+\frac{6245}{64}\,x^4-\frac{21229}{96}\,x^3 -\frac{28747}{128}\,x^2-\frac{1156639}{5888}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) +\frac{122691}{128}\,x+\frac{307461}{512}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="fricas")
```

output
```
625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747
/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/
128*x + 307461/512*log(2*x^2 - x + 3)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{122691x}{128}+\frac{307461\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{512}-\frac{1156639\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{5888}$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3),x)
```

output
```
625*x**7/14 + 3625*x**6/24 + 1855*x**5/8 + 6245*x**4/64 - 21229*x**3/96 -
28747*x**2/128 + 122691*x/128 + 307461*log(x**2 - x/2 + 3/2)/512 - 1156639
*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/5888
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{625}{14}x^7+\frac{3625}{24}x^6+\frac{1855}{8}x^5+\frac{6245}{64}x^4-\frac{21229}{96}x^3-\frac{28747}{128}x^2-\frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{122691}{128}x+\frac{307461}{512}\log\left(2x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="maxima")
```

output
```
625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747
/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/
128*x + 307461/512*log(2*x^2 - x + 3)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{625}{14}\,x^7 + \frac{3625}{24}\,x^6 + \frac{1855}{8}\,x^5 + \frac{6245}{64}\,x^4 - \frac{21229}{96}\,x^3 - \frac{28747}{128}\,x^2 - \frac{1156639}{5888}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{122691}{128}\,x + \frac{307461}{512}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="giac")
```

output
```
625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747
/128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/
128*x + 307461/512*log(2*x^2 - x + 3)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \frac{122691\,x}{128} + \frac{307461\,\ln\left(2\,x^2-x+3\right)}{512} - \frac{1156639\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{5888} - \frac{28747\,x^2}{128} - \frac{21229\,x^3}{96} + \frac{6245\,x^4}{64} + \frac{1855\,x^5}{8} + \frac{3625\,x^6}{24} + \frac{625\,x^7}{14}$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3),x)
```

output
```
(122691*x)/128 + (307461*log(2*x^2 - x + 3))/512 - (1156639*23^(1/2)*atan(
(4*23^(1/2)*x)/23 - 23^(1/2)/23))/5888 - (28747*x^2)/128 - (21229*x^3)/96
+ (6245*x^4)/64 + (1855*x^5)/8 + (3625*x^6)/24 + (625*x^7)/14
```

3.38 $\int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx$

3.38.1 Optimal result

Integrand size = 25, antiderivative size = 70

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx = -\frac{4795x}{32} - \frac{829x^2}{32} + \frac{965x^3}{24} + \frac{575x^4}{16} + \frac{25x^5}{2} - \frac{59895 \arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}} + \frac{1331}{128} \log\left(3-x+2x^2\right)$$

output
```
-4795/32*x-829/32*x^2+965/24*x^3+575/16*x^4+25/2*x^5+1331/128*ln(2*x^2-x+3
)-59895/1472*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx = \frac{59895 \arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{64\sqrt{23}} + \frac{1}{384}\left(4x\left(-14385-2487x+3860x^2+3450x^3+1200x^4\right) + 3993\log\left(3-x+2x^2\right)\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

output
```
(59895*ArcTan[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (4*x*(-14385 - 2487*x
+ 3860*x^2 + 3450*x^3 + 1200*x^4) + 3993*Log[3 - x + 2*x^2])/384
```

3.38.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{2x^2-x+3}dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{125x^4}{2}+\frac{575x^3}{4}+\frac{965x^2}{8}+\frac{1331(x+11)}{32(2x^2-x+3)}-\frac{829x}{16}-\frac{4795}{32}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{59895\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

output
```
(-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (
59895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2]
)/128
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.38.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\ln(2x^2-x+3)}{128}+\frac{59895\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{1472}$	54
risch	$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\ln(16x^2-8x+24)}{128}+\frac{59895\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{1472}$	54

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

output
```
25/2*x^5+575/16*x^4+965/24*x^3-829/32*x^2-4795/32*x+1331/128*ln(2*x^2-x+3)
+59895/1472*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{25}{2}\,x^5+\frac{575}{16}\,x^4+\frac{965}{24}\,x^3-\frac{829}{32}\,x^2 + \frac{59895}{1472}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{4795}{32}\,x+\frac{1331}{128}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="fricas")
```

output
```
25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arct
an(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{59895\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3),x)
```

output
```
25*x**5/2 + 575*x**4/16 + 965*x**3/24 - 829*x**2/32 - 4795*x/32 + 1331*log
(x**2 - x/2 + 3/2)/128 + 59895*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/1472
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{25}{2}x^5+\frac{575}{16}x^4+\frac{965}{24}x^3-\frac{829}{32}x^2+\frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4795}{32}x+\frac{1331}{128}\log\left(2x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="maxima")
```

output
```
25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arct
an(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{25}{2}\,x^5 + \frac{575}{16}\,x^4 + \frac{965}{24}\,x^3 - \frac{829}{32}\,x^2 + \frac{59895}{1472}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{4795}{32}\,x + \frac{1331}{128}\,\log\left(2\,x^2 - x + 3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="giac")
```

output
```
25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arct
an(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 12.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \frac{1331\,\ln\left(2\,x^2-x+3\right)}{128} - \frac{4795\,x}{32} + \frac{59895\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{1472} - \frac{829\,x^2}{32} + \frac{965\,x^3}{24} + \frac{575\,x^4}{16} + \frac{25\,x^5}{2}$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3),x)
```

output
```
(1331*log(2*x^2 - x + 3))/128 - (4795*x)/32 + (59895*23^(1/2)*atan((4*23^(
1/2)*x)/23 - 23^(1/2)/23))/1472 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/
16 + (25*x^5)/2
```

3.39 $\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx$

3.39.1 Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{51x}{8} + \frac{85x^2}{8} + \frac{25x^3}{6} + \frac{847\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}} - \frac{363}{32}\log\left(3-x+2x^2\right)$$

output
```
51/8*x+85/8*x^2+25/6*x^3-363/32*ln(2*x^2-x+3)+847/368*arctan(1/23*(1-4*x)*
23^(1/2))*23^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{1}{24}x\left(153+255x+100x^2\right) - \frac{847\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{16\sqrt{23}} - \frac{363}{32}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

output
```
(x*(153 + 255*x + 100*x^2))/24 - (847*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16*Sqr
t[23]) - (363*Log[3 - x + 2*x^2])/32
```

3.39.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{2x^2-x+3}\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{25x^2}{2}-\frac{121(3x+1)}{8(2x^2-x+3)}+\frac{85x}{4}+\frac{51}{8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{847\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}+\frac{25x^3}{6}+\frac{85x^2}{8}-\frac{363}{32}\log\left(2x^2-x+3\right)+\frac{51x}{8}$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

output
```
(51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*
Sqrt[23]) - (363*Log[3 - x + 2*x^2])/32
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.39.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\ln(2x^2-x+3)}{32}-\frac{847\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{368}$	44
risch	$\frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\ln(16x^2-8x+24)}{32}-\frac{847\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{368}$	44

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

output
```
25/6*x^3+85/8*x^2+51/8*x-363/32*ln(2*x^2-x+3)-847/368*23^(1/2)*arctan(1/23
*(-1+4*x)*23^(1/2))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="fricas")
```

output
```
25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5
1/8*x - 363/32*log(2*x^2 - x + 3)
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{25x^3}{6}+\frac{85x^2}{8}+\frac{51x}{8}-\frac{363\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32} - \frac{847\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{368}$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3),x)
```

output
```
25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*log(x**2 - x/2 + 3/2)/32 - 847*sqrt(2
3)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/368
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{25}{6}\,x^3 + \frac{85}{8}\,x^2 - \frac{847}{368}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{51}{8}\,x - \frac{363}{32}\,\log\left(2\,x^2 - x + 3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="maxima")
```

output
```
25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5
1/8*x - 363/32*log(2*x^2 - x + 3)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{25}{6}\,x^3 + \frac{85}{8}\,x^2 - \frac{847}{368}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{51}{8}\,x - \frac{363}{32}\,\log\left(2\,x^2 - x + 3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="giac")
```

output
```
25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5
1/8*x - 363/32*log(2*x^2 - x + 3)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 12.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx = \frac{51\,x}{8} - \frac{363\,\ln\left(2\,x^2-x+3\right)}{32} - \frac{847\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{368} + \frac{85\,x^2}{8} + \frac{25\,x^3}{6}$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3),x)
```

output
```
(51*x)/8 - (363*log(2*x^2 - x + 3))/32 - (847*23^(1/2)*atan((4*23^(1/2)*x)
/23 - 23^(1/2)/23))/368 + (85*x^2)/8 + (25*x^3)/6
```

3.40 $\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx$

3.40.1 Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx=\frac{5x}{2}+\frac{33\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(3-x+2x^2\right)$$

output
```
5/2*x+11/8*ln(2*x^2-x+3)+33/92*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx=\frac{5x}{2}-\frac{33\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

output
```
(5*x)/2 - (33*ArcTan[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 
2*x^2])/8
```

3.40.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{2x^2-x+3}\,dx$$
$$\Big\downarrow \text{2188}$$
$$\int \left(\frac{5}{2}-\frac{11(1-x)}{2\left(2x^2-x+3\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{33\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

output
```
(5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2
*x^2])/8
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

3.40.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{5x}{2}+\frac{11\ln\left(2x^{2}-x+3\right)}{8}-\frac{33\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{92}$	34
risch	$\frac{5x}{2}+\frac{11\ln\left(16x^{2}-8x+24\right)}{8}-\frac{33\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{92}$	34

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3),x,method=_RETURNVERBOSE)
```

output
```
5/2*x+11/8*ln(2*x^2-x+3)-33/92*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = -\frac{33}{92}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="fricas")
```

output
```
-33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
 x + 3)
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = \frac{5x}{2}+\frac{11\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{33\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3),x)
```

output
```
5*x/2 + 11*log(x**2 - x/2 + 3/2)/8 - 33*sqrt(23)*atan(4*sqrt(23)*x/23 - sq
rt(23)/23)/92
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = -\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{5}{2}x+\frac{11}{8}\log\left(2x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="maxima")
```

output
```
-33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
 x + 3)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = -\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{5}{2}x+\frac{11}{8}\log\left(2x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="giac")
```

output
```
-33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
 x + 3)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx = \frac{5x}{2}+\frac{11\ln\left(2x^2-x+3\right)}{8}-\frac{33\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3),x)
```

output
```
(5*x)/2 + (11*log(2*x^2 - x + 3))/8 - (33*23^(1/2)*atan((4*23^(1/2)*x)/23
- 23^(1/2)/23))/92
```

3.41 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx$

3.41.1 Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{3\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x+5x^2\right)$$

output
```
-1/44*ln(2*x^2-x+3)+1/44*ln(5*x^2+3*x+2)+3/506*arctan(1/23*(1-4*x)*23^(1/2
))*23^(1/2)+13/682*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = -\frac{3\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x+5x^2\right)$$

input
```
Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqr
t[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

3.41.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1311, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)(5x^2+3x+2)}dx$$

$$\Big\downarrow \text{ 1311}$$

$$\frac{1}{242}\int -\frac{11(2x+1)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{11(5x+8)}{5x^2+3x+2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{22}\int \frac{5x+8}{5x^2+3x+2}dx-\frac{1}{22}\int \frac{2x+1}{2x^2-x+3}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{22}\left(-\frac{3}{2}\int \frac{1}{2x^2-x+3}dx-\frac{1}{2}\int -\frac{1-4x}{2x^2-x+3}dx\right)+\frac{1}{22}\left(\frac{13}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{1}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx-\frac{3}{2}\int \frac{1}{2x^2-x+3}dx\right)+\frac{1}{22}\left(\frac{13}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{1}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx+3\int \frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{1}{22}\left(\frac{1}{2}\int \frac{10x+3}{5x^2+3x+2}dx-13\int \frac{1}{-(10x+3)^2-31}d(10x+3)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{22}\left(\frac{1}{2}\int \frac{1-4x}{2x^2-x+3}dx-\frac{3\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)+\frac{1}{22}\left(\frac{1}{2}\int \frac{10x+3}{5x^2+3x+2}dx+\frac{13\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{22}\left(-\frac{3\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{1}{2}\log\left(2x^2-x+3\right)\right)+\frac{1}{22}\left(\frac{13\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}+\frac{1}{2}\log\left(5x^2+3x+2\right)\right)$$

input
```
Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

output
```
((-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] - Log[3 - x + 2*x^2]/2)/22 + ((
13*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] + Log[2 + 3*x + 5*x^2]/2)/22
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1311
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2
)), x_Symbol] :> With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*
f - a*b*e*f + a^2*f^2}, Simp[1/q   Int[(c^2*d - b*c*e + b^2*f - a*c*f - (c^
2*e - b*c*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*e^2 - c*d*f -
 b*e*f + a*f^2 + (c*e*f - b*f^2)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f,
 0]
```

3.41.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\ln(5x^2+3x+2)}{44}+\frac{13\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{682}-\frac{\ln(2x^2-x+3)}{44}-\frac{3\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{506}$	60
risch	$-\frac{\ln(16x^2-8x+24)}{44}-\frac{3\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{506}+\frac{\ln(100x^2+60x+40)}{44}+\frac{13\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{682}$	60

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/44*ln(5*x^2+3*x+2)+13/682*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)-1/44*l
n(2*x^2-x+3)-3/506*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - \frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{1}{44}\log\left(5\,x^2+3\,x+2\right) - \frac{1}{44}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1
/23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x +
 3)
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = -\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{44}+\frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{44} - \frac{3\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{506} + \frac{13\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{682}$$

input
```
integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2),x)
```

output
```
-log(x**2 - x/2 + 3/2)/44 + log(x**2 + 3*x/5 + 2/5)/44 - 3*sqrt(23)*atan(4
*sqrt(23)*x/23 - sqrt(23)/23)/506 + 13*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*
sqrt(31)/31)/682
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{1}{44}\log\left(5x^2+3x+2\right) - \frac{1}{44}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1
/23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x +
 3)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - \frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{1}{44}\log\left(5\,x^2+3\,x+2\right) - \frac{1}{44}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1
/23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x +
 3)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx = \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{44}+\frac{\sqrt{23}\,3\mathrm{i}}{1012}\right) - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{44}+\frac{\sqrt{31}\,13\mathrm{i}}{1364}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{1}{44}+\frac{\sqrt{31}\,13\mathrm{i}}{1364}\right)$$

input
```
int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)),x)
```

output
```
log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 - 1/44) - log(x + (23^(
1/2)*1i)/4 - 1/4)*((23^(1/2)*3i)/1012 + 1/44) - log(x - (31^(1/2)*1i)/10 +
 3/10)*((31^(1/2)*13i)/1364 - 1/44) + log(x + (31^(1/2)*1i)/10 + 3/10)*((3
1^(1/2)*13i)/1364 + 1/44)
```

3.42 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$

3.42.1 Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{4+65x}{682(2+3x+5x^2)} + \frac{7\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968}\log\left(3-x+2x^2\right) - \frac{3}{968}\log\left(2+3x+5x^2\right)$$

output
```
1/682*(4+65*x)/(5*x^2+3*x+2)+3/968*ln(2*x^2-x+3)-3/968*ln(5*x^2+3*x+2)+7/1
1132*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+2891/465124*arctan(1/31*(3+10*
x)*31^(1/2))*31^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{4+65x}{682(2+3x+5x^2)} - \frac{7\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{484\sqrt{23}} + \frac{2891\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}} + \frac{3}{968}\log\left(3-x+2x^2\right) - \frac{3}{968}\log\left(2+3x+5x^2\right)$$

input
```
Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(4 + 65*x)/(682*(2 + 3*x + 5*x^2)) - (7*ArcTan[(-1 + 4*x)/Sqrt[23]])/(484*
Sqrt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3
 - x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

3.42.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1305, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{65x+4}{682(5x^2+3x+2)}-\frac{\int -\frac{11(130x^2-127x+164)}{(2x^2-x+3)(5x^2+3x+2)}dx}{7502}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{682}\int \frac{130x^2-127x+164}{(2x^2-x+3)(5x^2+3x+2)}dx+\frac{65x+4}{682(5x^2+3x+2)}$$

$$\Big\downarrow \text{2141}$$

$$\frac{1}{682}\left(\frac{1}{242}\int -\frac{341(5-6x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{11(1306-465x)}{5x^2+3x+2}dx\right)+\frac{65x+4}{682(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{682}\left(\frac{1}{22}\int \frac{1306-465x}{5x^2+3x+2}dx-\frac{31}{22}\int \frac{5-6x}{2x^2-x+3}dx\right)+\frac{65x+4}{682(5x^2+3x+2)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(\frac{2891}{2}\int \frac{1}{5x^2+3x+2}dx-\frac{93}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{7}{2}\int \frac{1}{2x^2-x+3}dx-\frac{3}{2}\int -\frac{1-4x}{2x^2-x+3}dx\right)\right)+\frac{65x+4}{682(5x^2+3x+2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(\frac{2891}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{93}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{7}{2}\int\frac{1}{2x^2-x+3}dx+\frac{3}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)\right.$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(-\frac{93}{2}\int\frac{10x+3}{5x^2+3x+2}dx-2891\int\frac{1}{-(10x+3)^2-31}d(10x+3)\right)-\frac{31}{22}\left(\frac{3}{2}\int\frac{1-4x}{2x^2-x+3}dx-7\int\frac{}{-(4}\right.\right.$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(\frac{2891\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{93}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{3}{2}\int\frac{1-4x}{2x^2-x+3}dx+\frac{7\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)\right)+$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{682}\left(\frac{1}{22}\left(\frac{2891\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{93}{2}\log\left(5x^2+3x+2\right)\right)-\frac{31}{22}\left(\frac{7\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{3}{2}\log\left(2x^2-x+3\right)\right)\right)+$$
$$\frac{65x+4}{682\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + ((-31*((7*ArcTan[(-1 + 4*x)/Sqrt[23]]
)/Sqrt[23] - (3*Log[3 - x + 2*x^2])/2))/22 + ((2891*ArcTan[(3 + 10*x)/Sqrt
[31]])/Sqrt[31] - (93*Log[2 + 3*x + 5*x^2])/2)/22)/682
```

3.42. $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.42.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

method	result
risch	$\frac{\frac{13x}{682}+\frac{2}{1705}}{x^2+\frac{3}{5}x+\frac{2}{5}}-\frac{3\ln\left(100x^2+60x+40\right)}{968}+\frac{2891\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{465124}-\frac{7\sqrt{23}\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{11132}+\frac{3\ln\left(16x^2-8x+24\right)}{968}$
default	$-\frac{-\frac{286x}{31}-\frac{88}{155}}{484\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{3\ln\left(5x^2+3x+2\right)}{968}+\frac{2891\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{465124}+\frac{3\ln\left(2x^2-x+3\right)}{968}-\frac{7\sqrt{23}\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{11132}$

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
(13/682*x+2/1705)/(x^2+3/5*x+2/5)-3/968*ln(100*x^2+60*x+40)+2891/465124*ar
ctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)-7/11132*23^(1/2)*arctan(1/23*(-1+4*x
)*23^(1/2))+3/968*ln(16*x^2-8*x+24)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$$
$$= \frac{132986\sqrt{31}(5x^2+3x+2)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-13454\sqrt{23}(5x^2+3x+2)\arctan\left(\frac{1}{23}\sqrt{23}(4x-\right.}{2139570}$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/21395704*(132986*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x +
 3)) - 13454*sqrt(23)*(5*x^2 + 3*x + 2)*arctan(1/23*sqrt(23)*(4*x - 1)) -
66309*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 66309*(5*x^2 + 3*x + 2)*log
(2*x^2 - x + 3) + 2039180*x + 125488)/(5*x^2 + 3*x + 2)
```

3.42.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{65x+4}{3410x^2+2046x+1364} + \frac{3\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968} - \frac{3\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968} - \frac{7\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{11132} + \frac{2891\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{465124}$$

input
```
integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

output
```
(65*x + 4)/(3410*x**2 + 2046*x + 1364) + 3*log(x**2 - x/2 + 3/2)/968 - 3*l
og(x**2 + 3*x/5 + 2/5)/968 - 7*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/11132 + 2891*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/465124
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{65x+4}{682(5x^2+3x+2)} - \frac{3}{968}\log\left(5x^2+3x+2\right) + \frac{3}{968}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*a
rctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/96
8*log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{65x+4}{682(5x^2+3x+2)} - \frac{3}{968}\log\left(5x^2+3x+2\right) + \frac{3}{968}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*a
rctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/96
8*log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 12.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx = \frac{\frac{13\,x}{682}+\frac{2}{1705}}{x^2+\frac{3\,x}{5}+\frac{2}{5}}+\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{3}{968}+\frac{\sqrt{23}\,7\mathrm{i}}{22264}\right) - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{3}{968}+\frac{\sqrt{31}\,2891\mathrm{i}}{930248}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{3}{968}+\frac{\sqrt{31}\,2891\mathrm{i}}{930248}\right)$$

input
```
int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^2),x)
```

output
```
((13*x)/682 + 2/1705)/((3*x)/5 + x^2 + 2/5) + log(x - (23^(1/2)*1i)/4 - 1/
4)*((23^(1/2)*7i)/22264 + 3/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/
2)*7i)/22264 - 3/968) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)
/930248 + 3/968) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2891i)/9302
48 - 3/968)
```

3.43 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx$

3.43.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{4+65x}{1364(2+3x+5x^2)^2}+\frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{45\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{847793\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{10232728\sqrt{31}} - \frac{\log(3-x+2x^2)}{21296}+\frac{\log(2+3x+5x^2)}{21296}$$

output
```
1/1364*(4+65*x)/(5*x^2+3*x+2)^2+1/465124*(7923+21605*x)/(5*x^2+3*x+2)-1/21
296*ln(2*x^2-x+3)+1/21296*ln(5*x^2+3*x+2)-45/244904*arctan(1/23*(1-4*x)*23
^(1/2))*23^(1/2)+847793/317214568*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{45\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{1695586\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)+31\left(\frac{44(17210+89144x+104430x^2+108025x^3)}{(2+3x+5x^2)^2}-961\log(3-x+2x^2)+961\log(2\right.}{634429136}$$

input
```
Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(45*ArcTan[(-1 + 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (1695586*Sqrt[31]*ArcT
an[(3 + 10*x)/Sqrt[31]] + 31*((44*(17210 + 89144*x + 104430*x^2 + 108025*x
^3))/(2 + 3*x + 5*x^2)^2 - 961*Log[3 - x + 2*x^2] + 961*Log[2 + 3*x + 5*x^
2]))/634429136
```

3.43.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1305, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{65x+4}{1364(5x^2+3x+2)^2}-\frac{\int -\frac{11(390x^2-319x+523)}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{15004}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{390x^2-319x+523}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{1364}+\frac{65x+4}{1364(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\int \frac{22(43210x^2-15839x+60010)}{(2x^2-x+3)(5x^2+3x+2)}dx}{7502}+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{341}\int \frac{43210x^2-15839x+60010}{(2x^2-x+3)(5x^2+3x+2)}dx+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{2141}$$

$$\frac{\frac{1}{341}\left(\frac{1}{242}\int \frac{10571(23-2x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{11(4805x+425338)}{5x^2+3x+2}dx\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364(5x^2+3x+2)^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\int\frac{23-2x}{2x^2-x+3}dx+\frac{1}{22}\int\frac{4805x+425338}{5x^2+3x+2}dx\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 1142$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{45}{2}\int\frac{1}{2x^2-x+3}dx-\frac{1}{2}\int-\frac{1-4x}{2x^2-x+3}dx\right)+\frac{1}{22}\left(\frac{847793}{2}\int\frac{1}{5x^2+3x+2}dx+\frac{961}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 25$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{45}{2}\int\frac{1}{2x^2-x+3}dx+\frac{1}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{1}{22}\left(\frac{847793}{2}\int\frac{1}{5x^2+3x+2}dx+\frac{961}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 1083$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{1}{2}\int\frac{1-4x}{2x^2-x+3}dx-45\int\frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{1}{22}\left(\frac{961}{2}\int\frac{10x+3}{5x^2+3x+2}dx-847793\int\frac{1}{-(10x+3)^2-31}d(10x+3\cdots\right.\right.}{1364}$$
$$\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 217$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{1}{2}\int\frac{1-4x}{2x^2-x+3}dx+\frac{45\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)+\frac{1}{22}\left(\frac{961}{2}\int\frac{10x+3}{5x^2+3x+2}dx+\frac{847793\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right)\right)+\frac{21605x+7923}{341(5x^2+3x+2)}}{1364}+\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 1103$$

$$\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{45\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{1}{2}\log\left(2x^2-x+3\right)\right)+\frac{1}{22}\left(\frac{847793\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}+\frac{961}{2}\log\left(5x^2+3x+2\right)\right)\right)+\frac{21605x+\cdots}{341(5x^2+\cdots}}{1364}$$
$$\frac{65x+4}{1364\left(5x^2+3x+2\right)^2}$$

3.43. $\int\frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx$

input
```
Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + ((7923 + 21605*x)/(341*(2 + 3*x +
5*x^2)) + ((961*((45*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] - Log[3 - x + 2
*x^2]/2))/22 + ((847793*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] + (961*Log[2
 + 3*x + 5*x^2])/2)/22)/341)/1364
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.43. $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} \, dx$

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.43.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result
default	$\frac{\frac{108025}{465124}x^3+\frac{52215}{232562}x^2+\frac{2026}{10571}x+\frac{8605}{232562}}{(5x^2+3x+2)^2}+\frac{\ln(5x^2+3x+2)}{21296}+\frac{847793\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{317214568}-\frac{\ln(2x^2-x+3)}{21296}+\frac{45\sqrt{23}\ \mathrm{arctan}}{24}$
risch	$\frac{\frac{108025}{465124}x^3+\frac{52215}{232562}x^2+\frac{2026}{10571}x+\frac{8605}{232562}}{(5x^2+3x+2)^2}+\frac{45\sqrt{23}\ \arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{244904}-\frac{\ln(16x^2-8x+24)}{21296}+\frac{\ln(100x^2+60x+40)}{21296}+\frac{847793\ \mathrm{a}}{}$

input
```
int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
25/10648*(95062/961*x^3+459492/4805*x^2+1961168/24025*x+75724/4805)/(5*x^2
+3*x+2)^2+1/21296*ln(5*x^2+3*x+2)+847793/317214568*arctan(1/31*(10*x+3)*31
^(1/2))*31^(1/2)-1/21296*ln(2*x^2-x+3)+45/244904*23^(1/2)*arctan(1/23*(-1+
4*x)*23^(1/2))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx$$
$$= \frac{3388960300\,x^3 + 38998478\,\sqrt{31}(25\,x^4 + 30\,x^3 + 29\,x^2 + 12\,x + 4)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x + 3)\right) + 2681190\,\sqrt{}}{}$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/14591870128*(3388960300*x^3 + 38998478*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^
2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2681190*sqrt(23)*(25*x^4
+ 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3276177960
*x^2 + 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 + 3*x + 2) -
 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(2*x^2 - x + 3) + 2796625
568*x + 539912120)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{108025x^3+104430x^2+89144x+17210}{11628100x^4+13953720x^3+13488596x^2+5581488x+1860496} - \frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296} + \frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296} + \frac{45\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{244904} + \frac{847793\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{317214568}$$

input
```
integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

output
```
(108025*x**3 + 104430*x**2 + 89144*x + 17210)/(11628100*x**4 + 13953720*x*
*3 + 13488596*x**2 + 5581488*x + 1860496) - log(x**2 - x/2 + 3/2)/21296 +
log(x**2 + 3*x/5 + 2/5)/21296 + 45*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23
)/23)/244904 + 847793*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3172
14568
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{108025x^3+104430x^2+89144x+17210}{465124(25x^4+30x^3+29x^2+12x+4)} + \frac{1}{21296}\log\left(5x^2+3x+2\right) - \frac{1}{21296}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2
+ 89144*x + 17210)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1/21296*log(5*x
^2 + 3*x + 2) - 1/21296*log(2*x^2 - x + 3)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)\,(2+3x+5x^2)^3}\,dx &= \frac{847793}{317214568}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)\\&\quad+\frac{45}{244904}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)\\&\quad+\frac{108025\,x^3+104430\,x^2+89144\,x+17210}{465124\,(5\,x^2+3\,x+2)^2}\\&\quad+\frac{1}{21296}\,\log\left(5\,x^2+3\,x+2\right)-\frac{1}{21296}\,\log\left(2\,x^2-x+3\right)\end{aligned}$$

input
```
integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2
+ 89144*x + 17210)/(5*x^2 + 3*x + 2)^2 + 1/21296*log(5*x^2 + 3*x + 2) - 1/
21296*log(2*x^2 - x + 3)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{\frac{4321\,x^3}{465124}+\frac{10443\,x^2}{1162810}+\frac{2026\,x}{264275}+\frac{1721}{1162810}}{x^4+\frac{6\,x^3}{5}+\frac{29\,x^2}{25}+\frac{12\,x}{25}+\frac{4}{25}}$$
$$+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{21296}+\frac{\sqrt{23}\,45\mathrm{i}}{489808}\right)$$
$$-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{1}{21296}+\frac{\sqrt{31}\,847793\mathrm{i}}{634429136}\right)$$
$$+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{1}{21296}+\frac{\sqrt{31}\,847793\mathrm{i}}{634429136}\right)$$
$$-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{1}{21296}+\frac{\sqrt{23}\,45\mathrm{i}}{489808}\right)$$

input
```
int(1/((2*x^2 - x + 3)*(3*x + 5*x^2 + 2)^3),x)
```

output
```
log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 - 1/21296) - log(x -
 (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*45i)/489808 + 1/21296) - log(x - (31^(1
/2)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 - 1/21296) + log(x + (31^
(1/2)*1i)/10 + 3/10)*((31^(1/2)*847793i)/634429136 + 1/21296) + ((2026*x)/
264275 + (10443*x^2)/1162810 + (4321*x^3)/465124 + 1721/1162810)/((12*x)/2
5 + (29*x^2)/25 + (6*x^3)/5 + x^4 + 4/25)
```

3.44 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx$

3.44.1 Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\,(3-x+2x^2)}-\frac{13292697\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{1472\sqrt{23}}-\frac{30613}{128}\log\left(3-x+2x^2\right)$$

output
```
-89359/64*x-1185/8*x^2+9775/48*x^3+2125/16*x^4+125/4*x^5-14641/2944*(101+7
9*x)/(2*x^2-x+3)-30613/128*ln(2*x^2-x+3)-13292697/33856*arctan(1/23*(1-4*x
)*23^(1/2))*23^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = -\frac{89359x}{64}-\frac{1185x^2}{8}+\frac{9775x^3}{48}+\frac{2125x^4}{16}+\frac{125x^5}{4}-\frac{14641(101+79x)}{2944\,(3-x+2x^2)}+\frac{13292697\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{1472\sqrt{23}}-\frac{30613}{128}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

output
```
(-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
 - (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (13292697*ArcTan[(-1 + 4*
x)/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

3.44.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^2}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{23}\int \frac{460000x^6+1334000x^5+1706600x^4+574540x^3-976534x^2-661181x+832627}{64(2x^2-x+3)}dx-\frac{14641(79x+101)}{2944(2x^2-x+3)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{460000x^6+1334000x^5+1706600x^4+574540x^3-976534x^2-661181x+832627}{2x^2-x+3}dx}{1472}-\frac{14641(79x+101)}{2944(2x^2-x+3)}$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{\int \left(230000x^4+782000x^3+899300x^2-436080x+\frac{2662(2629-529x)}{2x^2-x+3}-2055257\right)dx}{1472}-\frac{14641(79x+101)}{2944(2x^2-x+3)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{13292697\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{\sqrt{23}}+46000x^5+195500x^4+\frac{899300x^3}{3}-218040x^2-\frac{704099}{2}\log\left(2x^2-x+3\right)-2055257x}{1472}-\frac{14641(79x+101)}{2944(2x^2-x+3)}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

output
```
(-14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (-2055257*x - 218040*x^2 +
(899300*x^3)/3 + 195500*x^4 + 46000*x^5 - (13292697*ArcTan[(1 - 4*x)/Sqrt[
23]])/Sqrt[23] - (704099*Log[3 - x + 2*x^2])/2)/1472
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.44.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
risch	$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}+\frac{-\frac{1156639x}{5888}-\frac{1478741}{5888}}{x^2-\frac{1}{2}x+\frac{3}{2}}-\frac{30613\ln\left(16x^2-8x+24\right)}{128}+\frac{13292697\sqrt{23}\ \mathrm{arctan}}{3385}$
default	$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{89359x}{64}-\frac{1331\left(\frac{869x}{92}+\frac{1111}{92}\right)}{64\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}-\frac{30613\ln\left(2x^2-x+3\right)}{128}+\frac{13292697\sqrt{23}\ \arctan\left(\frac{(-}{}\right.}{33856}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
125/4*x^5+2125/16*x^4+9775/48*x^3-1185/8*x^2-89359/64*x+(-1156639/5888*x-1
478741/5888)/(x^2-1/2*x+3/2)-30613/128*ln(16*x^2-8*x+24)+13292697/33856*23
^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx$$
$$= \frac{12696000\,x^7 + 47610000\,x^6 + 74800600\,x^5 - 20609840\,x^4 - 413058012\,x^3 + 79756182\,\sqrt{23}(2\,x^2 - x + 3)}{20313}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="fricas")
```

output
```
1/203136*(12696000*x^7 + 47610000*x^6 + 74800600*x^5 - 20609840*x^4 - 4130
58012*x^3 + 79756182*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x -
1)) + 193356906*x^2 - 48582831*(2*x^2 - x + 3)*log(2*x^2 - x + 3) - 930684
489*x - 102033129)/(2*x^2 - x + 3)
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} + \frac{-1156639x - 1478741}{5888x^2 - 2944x + 8832} - \frac{30613\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{13292697\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{33856}$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**2,x)
```

output
```
125*x**5/4 + 2125*x**4/16 + 9775*x**3/48 - 1185*x**2/8 - 89359*x/64 + (-11
56639*x - 1478741)/(5888*x**2 - 2944*x + 8832) - 30613*log(x**2 - x/2 + 3/
2)/128 + 13292697*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/33856
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{125}{4}\,x^5 + \frac{2125}{16}\,x^4 + \frac{9775}{48}\,x^3 - \frac{1185}{8}\,x^2 + \frac{13292697}{33856}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{89359}{64}\,x - \frac{14641\,(79\,x+101)}{2944\,(2\,x^2-x+3)} - \frac{30613}{128}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="maxima")
```

output
```
125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(2
3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/
(2*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{125}{4}\,x^5 + \frac{2125}{16}\,x^4 + \frac{9775}{48}\,x^3 - \frac{1185}{8}\,x^2 + \frac{13292697}{33856}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{89359}{64}\,x - \frac{14641\,(79\,x+101)}{2944\,(2\,x^2-x+3)} - \frac{30613}{128}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="giac")
```

output
```
125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(2
3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/
(2*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = \frac{13292697\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{33856} - \frac{30613\,\ln\left(2\,x^2-x+3\right)}{128} - \frac{\frac{1156639\,x}{5888}+\frac{1478741}{5888}}{x^2-\frac{x}{2}+\frac{3}{2}} - \frac{89359\,x}{64} - \frac{1185\,x^2}{8} + \frac{9775\,x^3}{48} + \frac{2125\,x^4}{16} + \frac{125\,x^5}{4}$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^2,x)
```

output
```
(13292697*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/33856 - (30613*l
og(2*x^2 - x + 3))/128 - ((1156639*x)/5888 + 1478741/5888)/(x^2 - x/2 + 3/
2) - (89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^
5)/4
```

3.45 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} \, dx$

3.45.1 Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} \, dx = \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}-\frac{1331(17-45x)}{736\,(3-x+2x^2)} + \frac{223971 \arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log\left(3-x+2x^2\right)$$

output
```
915/16*x+175/4*x^2+125/12*x^3-1331/736*(17-45*x)/(2*x^2-x+3)-2057/32*ln(2*
x^2-x+3)+223971/8464*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} \, dx = \frac{915x}{16}+\frac{175x^2}{4}+\frac{125x^3}{12}+\frac{1331(-17+45x)}{736\,(3-x+2x^2)} - \frac{223971 \arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{368\sqrt{23}}-\frac{2057}{32}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

output
```
(915*x)/16 + (175*x^2)/4 + (125*x^3)/12 + (1331*(-17 + 45*x))/(736*(3 - x
+ 2*x^2)) - (223971*ArcTan[(-1 + 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Lo
g[3 - x + 2*x^2])/32
```

3.45.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{23}\int -\frac{-23000x^4-52900x^3-44390x^2+19067x+25195}{16\,(2x^2-x+3)}dx-\frac{1331(17-45x)}{736\,(2x^2-x+3)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{368}\int \frac{-23000x^4-52900x^3-44390x^2+19067x+25195}{2x^2-x+3}dx-\frac{1331(17-45x)}{736\,(2x^2-x+3)}$$

$$\Big\downarrow \text{2188}$$

$$-\frac{1}{368}\int \left(-11500x^2-32200x+\frac{242(391x+365)}{2x^2-x+3}-21045\right)dx-\frac{1331(17-45x)}{736\,(2x^2-x+3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{368}\left(\frac{223971\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{\sqrt{23}}+\frac{11500x^3}{3}+16100x^2-\frac{47311}{2}\log\left(2x^2-x+3\right)+21045x\right)-\frac{1331(17-45x)}{736\,(2x^2-x+3)}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

output
```
(-1331*(17 - 45*x))/(736*(3 - x + 2*x^2)) + (21045*x + 16100*x^2 + (11500*
x^3)/3 + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/Sqrt[23] - (47311*Log[3 - x +
 2*x^2])/2)/368
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.45.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}+\frac{\frac{59895x}{1472}-\frac{22627}{1472}}{x^2-\frac{1}{2}x+\frac{3}{2}}-\frac{2057\ln\left(16x^2-8x+24\right)}{32}-\frac{223971\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{8464}$	60
default	$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}-\frac{121\left(-\frac{495x}{92}+\frac{187}{92}\right)}{16\left(x^2-\frac{1}{2}x+\frac{3}{2}\right)}-\frac{2057\ln\left(2x^2-x+3\right)}{32}-\frac{223971\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{8464}$	61

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
125/12*x^3+175/4*x^2+915/16*x+(59895/1472*x-22627/1472)/(x^2-1/2*x+3/2)-20
57/32*ln(16*x^2-8*x+24)-223971/8464*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2)
)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx$$
$$= \frac{1058000\,x^5 + 3914600\,x^4 + 5173620\,x^3 - 1343826\,\sqrt{23}(2\,x^2 - x + 3)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + 3761190}{50784\,(2\,x^2 - x + 3)}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="fricas")
```

output
```
1/50784*(1058000*x^5 + 3914600*x^4 + 5173620*x^3 - 1343826*sqrt(23)*(2*x^2
 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3761190*x^2 - 3264459*(2*x^2 -
 x + 3)*log(2*x^2 - x + 3) + 12845385*x - 1561263)/(2*x^2 - x + 3)
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{125x^3}{12} + \frac{175x^2}{4} + \frac{915x}{16} + \frac{59895x - 22627}{1472x^2 - 736x + 2208} - \frac{2057\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{223971\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{8464}$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)
```

output
```
125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x
 + 2208) - 2057*log(x**2 - x/2 + 3/2)/32 - 223971*sqrt(23)*atan(4*sqrt(23)
*x/23 - sqrt(23)/23)/8464
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{125}{12}\,x^3 + \frac{175}{4}\,x^2 - \frac{223971}{8464}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{915}{16}\,x + \frac{1331\,(45\,x-17)}{736\,(2\,x^2-x+3)} - \frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="maxima")
```

output
```
125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x -
1)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2
- x + 3)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{125}{12}\,x^3 + \frac{175}{4}\,x^2 - \frac{223971}{8464}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{915}{16}\,x + \frac{1331\,(45\,x-17)}{736\,(2\,x^2-x+3)} - \frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="giac")
```

output
```
125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x -
1)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2
- x + 3)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx = \frac{915\,x}{16} - \frac{2057\,\ln\left(2\,x^2-x+3\right)}{32} + \frac{\frac{59895\,x}{1472}-\frac{22627}{1472}}{x^2-\frac{x}{2}+\frac{3}{2}} - \frac{223971\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{8464} + \frac{175\,x^2}{4} + \frac{125\,x^3}{12}$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^2,x)
```

output
```
(915*x)/16 - (2057*log(2*x^2 - x + 3))/32 + ((59895*x)/1472 - 22627/1472)/
(x^2 - x/2 + 3/2) - (223971*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23)
)/8464 + (175*x^2)/4 + (125*x^3)/12
```

3.46 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx$

3.46.1 Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{25x}{4}+\frac{121(19-7x)}{184\,(3-x+2x^2)}+\frac{1859\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)$$

output
```
25/4*x+121/184*(19-7*x)/(2*x^2-x+3)+55/8*ln(2*x^2-x+3)+1859/2116*arctan(1/
23*(1-4*x)*23^(1/2))*23^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{25x}{4}-\frac{121(-19+7x)}{184\,(3-x+2x^2)}-\frac{1859\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

output
```
(25*x)/4 - (121*(-19 + 7*x))/(184*(3 - x + 2*x^2)) - (1859*ArcTan[(-1 + 4*
x)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

3.46.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{23}\int \frac{1150x^2+1955x+163}{4(2x^2-x+3)}dx+\frac{121(19-7x)}{184(2x^2-x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{92}\int \frac{1150x^2+1955x+163}{2x^2-x+3}dx+\frac{121(19-7x)}{184(2x^2-x+3)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{92}\int \left(575-\frac{22(71-115x)}{2x^2-x+3}\right)dx+\frac{121(19-7x)}{184(2x^2-x+3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{92}\left(\frac{1859\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{\sqrt{23}}+\frac{1265}{2}\log\left(2x^2-x+3\right)+575x\right)+\frac{121(19-7x)}{184(2x^2-x+3)}$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

output
```
(121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (575*x + (1859*ArcTan[(1 - 4*x)/S
qrt[23]])/Sqrt[23] + (1265*Log[3 - x + 2*x^2])/2)/92
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.46.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{25x}{4}+\frac{-\frac{847x}{368}+\frac{2299}{368}}{x^2-\frac{1}{2}x+\frac{3}{2}}+\frac{55\ln\left(16x^2-8x+24\right)}{8}-\frac{1859\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{2116}$	50
default	$\frac{25x}{4}+\frac{-\frac{847x}{368}+\frac{2299}{368}}{x^2-\frac{1}{2}x+\frac{3}{2}}+\frac{55\ln\left(2x^2-x+3\right)}{8}-\frac{1859\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{2116}$	51

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
25/4*x+(-847/368*x+2299/368)/(x^2-1/2*x+3/2)+55/8*ln(16*x^2-8*x+24)-1859/2
116*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{52900\,x^3 - 3718\,\sqrt{23}(2\,x^2-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - 26450\,x^2 + 29095\,(2\,x^2-x+3)\log\left(2\,x^2 \right.}{4232\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="fricas")
```

output
```
1/4232*(52900*x^3 - 3718*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*
x - 1)) - 26450*x^2 + 29095*(2*x^2 - x + 3)*log(2*x^2 - x + 3) + 59869*x +
 52877)/(2*x^2 - x + 3)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{25x}{4} + \frac{2299-847x}{368x^2-184x+552} + \frac{55\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8} - \frac{1859\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)
```

output
```
25*x/4 + (2299 - 847*x)/(368*x**2 - 184*x + 552) + 55*log(x**2 - x/2 + 3/2
)/8 - 1859*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/2116
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = -\frac{1859}{2116}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{25}{4}\,x - \frac{121\,(7\,x-19)}{184\,(2\,x^2-x+3)} + \frac{55}{8}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="maxima")
```

output
```
-1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*
x - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = -\frac{1859}{2116}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{25}{4}\,x-\frac{121\,(7\,x-19)}{184\,(2\,x^2-x+3)}+\frac{55}{8}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="giac")
```

output
```
-1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*
x - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^2}\,dx = \frac{25\,x}{4}+\frac{55\,\ln\left(2\,x^2-x+3\right)}{8}-\frac{\frac{847\,x}{368}-\frac{2299}{368}}{x^2-\frac{x}{2}+\frac{3}{2}}-\frac{1859\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^2,x)
```

output
```
(25*x)/4 + (55*log(2*x^2 - x + 3))/8 - ((847*x)/368 - 2299/368)/(x^2 - x/2
 + 3/2) - (1859*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/2116
```

3.47 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx$

3.47.1 Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = -\frac{11(5+3x)}{46\,(3-x+2x^2)} - \frac{82\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

output
```
-11/46*(5+3*x)/(2*x^2-x+3)-82/529*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = -\frac{11(5+3x)}{46\,(3-x+2x^2)} + \frac{82\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

output
```
(-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(2
3*Sqrt[23])
```

3.47.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{23}\int \frac{41}{2x^2-x+3}dx-\frac{11(3x+5)}{46(2x^2-x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{41}{23}\int \frac{1}{2x^2-x+3}dx-\frac{11(3x+5)}{46(2x^2-x+3)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{82}{23}\int \frac{1}{-(4x-1)^2-23}d(4x-1)-\frac{11(3x+5)}{46(2x^2-x+3)}$$

$$\Big\downarrow \text{217}$$

$$\frac{82\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}}-\frac{11(3x+5)}{46(2x^2-x+3)}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

output
```
(-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(2
3*Sqrt[23])
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.47.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{-\frac{33x}{92}-\frac{55}{92}}{x^{2}-\frac{1}{2}x+\frac{3}{2}}+\frac{82\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{529}$	34
risch	$\frac{-\frac{33x}{92}-\frac{55}{92}}{x^{2}-\frac{1}{2}x+\frac{3}{2}}+\frac{82\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{529}$	34

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-33/92*x-55/92)/(x^2-1/2*x+3/2)+82/529*23^(1/2)*arctan(1/23*(-1+4*x)*23^(
1/2))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = \frac{164\sqrt{23}(2\,x^2-x+3)\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-759\,x-1265}{1058\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="fricas")
```

output
```
1/1058*(164*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 759
*x - 1265)/(2*x^2 - x + 3)
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = \frac{-33x-55}{92x^2-46x+138} + \frac{82\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{529}$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2,x)
```

output
```
(-33*x - 55)/(92*x**2 - 46*x + 138) + 82*sqrt(23)*atan(4*sqrt(23)*x/23 - s
qrt(23)/23)/529
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = \frac{82}{529}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{11\,(3\,x+5)}{46\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="maxima")
```

output
```
82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
 x + 3)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = \frac{82}{529}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{11\,(3x+5)}{46\,(2x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="giac")
```

output
```
82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
 x + 3)
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^2}\,dx = \frac{82\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{529} - \frac{\frac{33x}{92}+\frac{55}{92}}{x^2-\frac{x}{2}+\frac{3}{2}}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^2,x)
```

output
```
(82*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/529 - ((33*x)/92 + 55/
92)/(x^2 - x/2 + 3/2)
```

3.48 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx$

3.48.1 Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{13-6x}{506(3-x+2x^2)} + \frac{241\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968}\log\left(3-x+2x^2\right) + \frac{13}{968}\log\left(2+3x+5x^2\right)$$

output
```
1/506*(13-6*x)/(2*x^2-x+3)-13/968*ln(2*x^2-x+3)+13/968*ln(5*x^2+3*x+2)+241
/256036*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)+69/15004*arctan(1/31*(3+10*
x)*31^(1/2))*31^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{13-6x}{506(3-x+2x^2)} - \frac{241\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{11132\sqrt{23}} + \frac{69\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}} - \frac{13}{968}\log\left(3-x+2x^2\right) + \frac{13}{968}\log\left(2+3x+5x^2\right)$$

input
```
Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(506*(3 - x + 2*x^2)) - (241*ArcTan[(-1 + 4*x)/Sqrt[23]])/(1113
2*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3
- x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

3.48.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1305, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^2(5x^2+3x+2)}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{13-6x}{506(2x^2-x+3)}-\frac{\int -\frac{11(-30x^2+97x+172)}{(2x^2-x+3)(5x^2+3x+2)}dx}{5566}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{506}\int \frac{-30x^2+97x+172}{(2x^2-x+3)(5x^2+3x+2)}dx+\frac{13-6x}{506(2x^2-x+3)}$$

$$\Big\downarrow \text{2141}$$

$$\frac{1}{506}\left(\frac{1}{242}\int \frac{11(29-598x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{253(65x+54)}{5x^2+3x+2}dx\right)+\frac{13-6x}{506(2x^2-x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{506}\left(\frac{1}{22}\int \frac{29-598x}{2x^2-x+3}dx+\frac{23}{22}\int \frac{65x+54}{5x^2+3x+2}dx\right)+\frac{13-6x}{506(2x^2-x+3)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{506}\left(\frac{1}{22}\left(-\frac{241}{2}\int \frac{1}{2x^2-x+3}dx-\frac{299}{2}\int -\frac{1-4x}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{69}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{13}{2}\int \frac{10x+3}{5x^2+3x+\cdots}\right.\right.$$
$$\frac{13-6x}{506(2x^2-x+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{506}\left(\frac{1}{22}\left(\frac{299}{2}\int\frac{1-4x}{2x^2-x+3}dx-\frac{241}{2}\int\frac{1}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{69}{2}\int\frac{1}{5x^2+3x+2}dx+\frac{13}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{506}\left(\frac{1}{22}\left(\frac{299}{2}\int\frac{1-4x}{2x^2-x+3}dx+241\int\frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{23}{22}\left(\frac{13}{2}\int\frac{10x+3}{5x^2+3x+2}dx-69\int\frac{}{-(10}\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{506}\left(\frac{1}{22}\left(\frac{299}{2}\int\frac{1-4x}{2x^2-x+3}dx-\frac{241\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)+\frac{23}{22}\left(\frac{13}{2}\int\frac{10x+3}{5x^2+3x+2}dx+\frac{69\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right)\right)+$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{506}\left(\frac{1}{22}\left(-\frac{241\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{299}{2}\log\left(2x^2-x+3\right)\right)+\frac{23}{22}\left(\frac{69\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}+\frac{13}{2}\log\left(5x^2+3x+2\right)\right)\right)$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(506*(3 - x + 2*x^2)) + (((-241*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sq
rt[23] - (299*Log[3 - x + 2*x^2])/2)/22 + (23*((69*ArcTan[(3 + 10*x)/Sqrt[
31]])/Sqrt[31] + (13*Log[2 + 3*x + 5*x^2])/2))/22)/506
```

3.48. $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx$

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.48.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.81

method	result
risch	$\frac{-\frac{3x}{506}+\frac{13}{1012}}{x^{2}-\frac{1}{2}x+\frac{3}{2}}-\frac{13\ln\left(16x^{2}-8x+24\right)}{968}-\frac{241\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{256036}+\frac{13\ln\left(100x^{2}+60x+40\right)}{968}+\frac{69\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{15004}$
default	$\frac{13\ln\left(5x^{2}+3x+2\right)}{968}+\frac{69\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{15004}-\frac{\frac{66x}{23}-\frac{143}{23}}{484\left(x^{2}-\frac{1}{2}x+\frac{3}{2}\right)}-\frac{13\ln\left(2x^{2}-x+3\right)}{968}-\frac{241\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{256036}$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-3/506*x+13/1012)/(x^2-1/2*x+3/2)-13/968*ln(16*x^2-8*x+24)-241/256036*23^
(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))+13/968*ln(100*x^2+60*x+40)+69/15004*a
rctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{73002\,\sqrt{31}(2\,x^2-x+3)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)-14942\,\sqrt{23}(2\,x^2-x+3)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)}{15874232\,(}$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/15874232*(73002*sqrt(31)*(2*x^2 - x + 3)*arctan(1/31*sqrt(31)*(10*x + 3)
) - 14942*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 21318
7*(2*x^2 - x + 3)*log(5*x^2 + 3*x + 2) - 213187*(2*x^2 - x + 3)*log(2*x^2
- x + 3) - 188232*x + 407836)/(2*x^2 - x + 3)
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{13-6x}{1012x^2-506x+1518} - \frac{13\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968} + \frac{13\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968} - \frac{241\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{256036} + \frac{69\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{15004}$$

input
```
integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

output
```
(13 - 6*x)/(1012*x**2 - 506*x + 1518) - 13*log(x**2 - x/2 + 3/2)/968 + 13*
log(x**2 + 3*x/5 + 2/5)/968 - 241*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)
/23)/256036 + 69*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/15004
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{6x-13}{506(2x^2-x+3)} + \frac{13}{968}\log\left(5x^2+3x+2\right) - \frac{13}{968}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*a
rctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968
*log(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = \frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{6x-13}{506(2x^2-x+3)} + \frac{13}{968}\log\left(5x^2+3x+2\right) - \frac{13}{968}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*a
rctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968
*log(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

3.48.9 Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx = -\frac{\frac{3x}{506}-\frac{13}{1012}}{x^2-\frac{x}{2}+\frac{3}{2}} - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{13}{968}+\frac{\sqrt{31}\,69\mathrm{i}}{30008}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{13}{968}+\frac{\sqrt{31}\,69\mathrm{i}}{30008}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{13}{968}+\frac{\sqrt{23}\,241\mathrm{i}}{512072}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{13}{968}+\frac{\sqrt{23}\,241\mathrm{i}}{512072}\right)$$

input
```
int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)),x)
```

output
```
log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 + 13/968) - log(x -
 (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*69i)/30008 - 13/968) - ((3*x)/506 - 1
3/1012)/(x^2 - x/2 + 3/2) + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i
)/512072 - 13/968) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*241i)/51207
2 + 13/968)
```

3.49 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx$

3.49.1 Optimal result

Integrand size = 25, antiderivative size = 127

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} dx = -\frac{25(117-137x)}{172546(2+3x+5x^2)} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)} + \frac{2769\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}} + \frac{12643\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{165044\sqrt{31}} + \frac{19\log(3-x+2x^2)}{10648} - \frac{19\log(2+3x+5x^2)}{10648}$$

output
```
-25/172546*(117-137*x)/(5*x^2+3*x+2)+1/506*(13-6*x)/(2*x^2-x+3)/(5*x^2+3*x
+2)+19/10648*ln(2*x^2-x+3)-19/10648*ln(5*x^2+3*x+2)+2769/2816396*arctan(1/
23*(1-4*x)*23^(1/2))*23^(1/2)+12643/5116364*arctan(1/31*(3+10*x)*31^(1/2))
*31^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)^2 (2+3x+5x^2)^2} \, dx$$
$$= \frac{\frac{31372(-4342+11154x-9275x^2+6850x^3)}{6+7x+16x^2+x^3+10x^4} - 5322018\sqrt{23}\arctan\left(\frac{-1+4x}{\sqrt{23}}\right) + 13376294\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right) + 965901}{5413113112}$$

input
```
Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

output
```
((31372*(-4342 + 11154*x - 9275*x^2 + 6850*x^3))/(6 + 7*x + 16*x^2 + x^3 +
 10*x^4) - 5322018*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 13376294*Sqrt[31
]*ArcTan[(3 + 10*x)/Sqrt[31]] + 9659011*Log[3 - x + 2*x^2] - 9659011*Log[2
 + 3*x + 5*x^2])/5413113112
```

3.49.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1305, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^2 (5x^2+3x+2)^2} \, dx$$
$$\Big\downarrow \ 1305$$
$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)} - \frac{\int -\frac{11(-90x^2+209x+211)}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{5566}$$
$$\Big\downarrow \ 27$$
$$\frac{1}{506}\int \frac{-90x^2+209x+211}{(2x^2-x+3)(5x^2+3x+2)^2}dx + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$
$$\Big\downarrow \ 2135$$
$$\frac{1}{506}\left(\frac{\int \frac{22(6850x^2-19235x+12538)}{(2x^2-x+3)(5x^2+3x+2)}dx}{7502} - \frac{25(117-137x)}{341(5x^2+3x+2)}\right) + \frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{506}\left(\frac{1}{341}\int \frac{6850x^2-19235x+12538}{(2x^2-x+3)(5x^2+3x+2)}dx-\frac{25(117-137x)}{341(5x^2+3x+2)}\right)+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$$\Big\downarrow \text{2141}$$

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{1}{242}\int -\frac{341(1603-874x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{253(5438-2945x)}{5x^2+3x+2}dx\right)-\frac{25(117-137x)}{341(5x^2+3x+2)}\right)+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\int \frac{5438-2945x}{5x^2+3x+2}dx-\frac{31}{22}\int \frac{1603-874x}{2x^2-x+3}dx\right)-\frac{25(117-137x)}{341(5x^2+3x+2)}\right)+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(\frac{12643}{2}\int \frac{1}{5x^2+3x+2}dx-\frac{589}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{2769}{2}\int \frac{1}{2x^2-x+3}dx-\frac{437}{2}\int -\frac{}{2}\right.\right.\right.$$
$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(\frac{12643}{2}\int \frac{1}{5x^2+3x+2}dx-\frac{589}{2}\int \frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{2769}{2}\int \frac{1}{2x^2-x+3}dx+\frac{437}{2}\int \frac{}{2x^2}\right.\right.\right.$$
$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(-\frac{589}{2}\int \frac{10x+3}{5x^2+3x+2}dx-12643\int \frac{1}{-(10x+3)^2-31}d(10x+3)\right)-\frac{31}{22}\left(\frac{437}{2}\int \frac{1-4x}{2x^2-x+3}dx\right.\right.\right.$$
$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)}$$

$$\Big\downarrow \text{217}$$

3.49. $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx$

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(\frac{12643\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{589}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{437}{2}\int\frac{1-4x}{2x^2-x+3}dx+\frac{2769\arctan\left(\cdots\right.}{\sqrt{23}}\cdots\right.\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}$$

$\big\downarrow$ 1103

$$\frac{1}{506}\left(\frac{1}{341}\left(\frac{23}{22}\left(\frac{12643\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{589}{2}\log\left(5x^2+3x+2\right)\right)-\frac{31}{22}\left(\frac{2769\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{437}{2}\log\left(2x^2-\cdots\right.\right.\right.\right.$$
$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(13 - 6*x)/(506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)) + ((-25*(117 - 137*x))/
(341*(2 + 3*x + 5*x^2)) + ((-31*((2769*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[2
3] - (437*Log[3 - x + 2*x^2])/2))/22 + (23*((12643*ArcTan[(3 + 10*x)/Sqrt[
31]])/Sqrt[31] - (589*Log[2 + 3*x + 5*x^2])/2))/22)/341)/506
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q
, 0]
```

3.49. $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2} \, dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.49. $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx$

3.49.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.74

method	result
default	$-\frac{-\frac{759x}{31}+\frac{1078}{155}}{5324\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{19\ln\left(5x^2+3x+2\right)}{10648}+\frac{12643\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{5116364}+\frac{-\frac{77x}{23}-\frac{341}{46}}{5324x^2-2662x+7986}+\frac{19\ln\left(2x^2-x+3\right)}{10648}-$
risch	$\frac{\frac{3425}{86273}x^3-\frac{9275}{172546}x^2+\frac{507}{7843}x-\frac{2171}{86273}}{(2x^2-x+3)(5x^2+3x+2)}+\frac{19\ln\left(16x^2-8x+24\right)}{10648}-\frac{2769\sqrt{23}\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{2816396}-\frac{19\ln\left(100x^2+60x+40\right)}{10648}+$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5324*(-759/31*x+1078/155)/(x^2+3/5*x+2/5)-19/10648*ln(5*x^2+3*x+2)+1264
3/5116364*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)+1/5324*(-77/23*x-341/46)
/(x^2-1/2*x+3/2)+19/10648*ln(2*x^2-x+3)-2769/2816396*23^(1/2)*arctan(1/23*
(-1+4*x)*23^(1/2))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.31

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx$$
$$= \frac{214898200\,x^3 + 13376294\,\sqrt{31}(10\,x^4 + x^3 + 16\,x^2 + 7\,x + 6)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x + 3)\right) - 5322018\,\sqrt{23}(1}{}$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/5413113112*(214898200*x^3 + 13376294*sqrt(31)*(10*x^4 + x^3 + 16*x^2 + 7
*x + 6)*arctan(1/31*sqrt(31)*(10*x + 3)) - 5322018*sqrt(23)*(10*x^4 + x^3
+ 16*x^2 + 7*x + 6)*arctan(1/23*sqrt(23)*(4*x - 1)) - 290975300*x^2 - 9659
011*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*log(5*x^2 + 3*x + 2) + 9659011*(10*x
^4 + x^3 + 16*x^2 + 7*x + 6)*log(2*x^2 - x + 3) + 349923288*x - 136217224)
/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx = \frac{6850x^3-9275x^2+11154x-4342}{1725460x^4+172546x^3+2760736x^2+1207822x+1035276} + \frac{19\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{10648} - \frac{19\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{10648} - \frac{2769\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2816396} + \frac{12643\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{5116364}$$

input
```
integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

output
```
(6850*x**3 - 9275*x**2 + 11154*x - 4342)/(1725460*x**4 + 172546*x**3 + 276
0736*x**2 + 1207822*x + 1035276) + 19*log(x**2 - x/2 + 3/2)/10648 - 19*log
(x**2 + 3*x/5 + 2/5)/10648 - 2769*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)
/23)/2816396 + 12643*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/51163
64
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx = \frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{6850x^3-9275x^2+11154x-4342}{172546(10x^4+x^3+16x^2+7x+6)} - \frac{19}{10648}\log\left(5x^2+3x+2\right) + \frac{19}{10648}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 11
154*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x
 + 2) + 19/10648*log(2*x^2 - x + 3)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^2}\,dx = \frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{6850x^3-9275x^2+11154x-4342}{172546(10x^4+x^3+16x^2+7x+6)} - \frac{19}{10648}\log\left(5x^2+3x+2\right) + \frac{19}{10648}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output

```
12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 11
154*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x
 + 2) + 19/10648*log(2*x^2 - x + 3)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{1}{(3-x+2x^2)^2 (2+3x+5x^2)^2} \, dx = \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{19}{10648}+\frac{\sqrt{23}\,2769\mathrm{i}}{5632792}\right) - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{19}{10648}+\frac{\sqrt{31}\,12643\mathrm{i}}{10232728}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{19}{10648}+\frac{\sqrt{31}\,12643\mathrm{i}}{10232728}\right) + \frac{\frac{685\,x^3}{172546}-\frac{1855\,x^2}{345092}+\frac{507\,x}{78430}-\frac{2171}{862730}}{x^4+\frac{x^3}{10}+\frac{8\,x^2}{5}+\frac{7\,x}{10}+\frac{3}{5}}$$

input
```
int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^2),x)
```

output
```
log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 + 19/10648) - log
(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2769i)/5632792 - 19/10648) - log(x
- (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 + 19/10648) + log(x
 + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*12643i)/10232728 - 19/10648) + ((50
7*x)/78430 - (1855*x^2)/345092 + (685*x^3)/172546 - 2171/862730)/((7*x)/10
 + (8*x^2)/5 + x^3/10 + x^4 + 3/5)
```

3.50 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx$

3.50.1 Optimal result

Integrand size = 25, antiderivative size = 148

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3} dx = \frac{-9446+5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599+3996965x}{235352744(2+3x+5x^2)} - \frac{25557\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{5387888\sqrt{23}} + \frac{4464079\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{225120016\sqrt{31}} + \frac{97\log(3-x+2x^2)}{468512} - \frac{97\log(2+3x+5x^2)}{468512}$$

output
```
1/690184*(-9446+5765*x)/(5*x^2+3*x+2)^2+1/506*(13-6*x)/(2*x^2-x+3)/(5*x^2+
3*x+2)^2+1/235352744*(1765599+3996965*x)/(5*x^2+3*x+2)+97/468512*ln(2*x^2-
x+3)-97/468512*ln(5*x^2+3*x+2)-25557/123921424*arctan(1/23*(1-4*x)*23^(1/2
))*23^(1/2)+4464079/6978720496*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.92

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx = \frac{-11+90x}{244904(3-x+2x^2)}+\frac{-98+345x}{30008(2+3x+5x^2)^2}+\frac{67573+164380x}{10232728(2+3x+5x^2)}+\frac{25557\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{5387888\sqrt{23}}+\frac{4464079\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{225120016\sqrt{31}}+\frac{97\log(3-x+2x^2)}{468512}-\frac{97\log(2+3x+5x^2)}{468512}$$

input
```
Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(-11 + 90*x)/(244904*(3 - x + 2*x^2)) + (-98 + 345*x)/(30008*(2 + 3*x + 5*
x^2)^2) + (67573 + 164380*x)/(10232728*(2 + 3*x + 5*x^2)) + (25557*ArcTan[
(-1 + 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3 + 10*x)/Sqrt
[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/468512 - (97*Log[2 +
 3*x + 5*x^2])/468512
```

3.50.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {1305, 27, 2135, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^2(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)^2}-\frac{\int -\frac{11(-150x^2+321x+250)}{(2x^2-x+3)(5x^2+3x+2)^3}dx}{5566}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{506}\int \frac{-150x^2+321x+250}{(2x^2-x+3)(5x^2+3x+2)^3}dx+\frac{13-6x}{506(2x^2-x+3)(5x^2+3x+2)^2}$$

$$\Big\downarrow 2135$$

$$\frac{1}{506}\left(\frac{\int \frac{11(34590x^2-106699x+68191)}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{15004}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{1}{506}\left(\frac{\int \frac{34590x^2-106699x+68191}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 2135$$

$$\frac{1}{506}\left(\frac{\frac{\int \frac{22\left(7993930x^2-1730927x+7580866\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}dx}{7502}+\frac{3996965x+1765599}{341(5x^2+3x+2)}}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{1}{506}\left(\frac{\frac{1}{341}\int \frac{7993930x^2-1730927x+7580866}{(2x^2-x+3)(5x^2+3x+2)}dx+\frac{3996965x+1765599}{341(5x^2+3x+2)}}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 2141$$

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{1}{242}\int \frac{10571(4462x+11663)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{253(2092214-466085x)}{5x^2+3x+2}dx\right)+\frac{3996965x+1765599}{341(5x^2+3x+2)}}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\int \frac{4462x+11663}{2x^2-x+3}dx+\frac{23}{22}\int \frac{2092214-466085x}{5x^2+3x+2}dx\right)+\frac{3996965x+1765599}{341(5x^2+3x+2)}}{1364}-\frac{9446-5765x}{1364\left(5x^2+3x+2\right)^2}\right)+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow 1142$$

3.50. $\displaystyle\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{25557}{2}\int\frac{1}{2x^2-x+3}dx+\frac{2231}{2}\int-\frac{1-4x}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{4464079}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{93217}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)\right)+3}{1364}\right.$$

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{25557}{2}\int\frac{1}{2x^2-x+3}dx-\frac{2231}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{4464079}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{93217}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)\right)+\frac{399}{34}}{1364}\right.$$

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 1083

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(-\frac{2231}{2}\int\frac{1-4x}{2x^2-x+3}dx-25557\int\frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{23}{22}\left(-\frac{93217}{2}\int\frac{10x+3}{5x^2+3x+2}dx-4464079\int\frac{}{-(10}\right.\right.}{1364}\right.$$

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 217

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{25557\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{2231}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{23}{22}\left(\frac{4464079\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{93217}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)\right)+\frac{399}{34}}{1364}\right.$$

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{1}{506}\left(\frac{\frac{1}{341}\left(\frac{961}{22}\left(\frac{25557\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}+\frac{2231}{2}\log\left(2x^2-x+3\right)\right)+\frac{23}{22}\left(\frac{4464079\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{93217}{2}\log\left(5x^2+3x+2\right)\right.\right.}{1364}\right.$$

$$\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

3.50. $\int\frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$

output
```
(13 - 6*x)/(506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (-1/1364*(9446 - 57
65*x)/(2 + 3*x + 5*x^2)^2 + ((1765599 + 3996965*x)/(341*(2 + 3*x + 5*x^2))
 + ((961*((25557*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] + (2231*Log[3 - x +
 2*x^2])/2))/22 + (23*((4464079*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] - (9
3217*Log[2 + 3*x + 5*x^2])/2))/22)/341)/1364)/506
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.50. $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.50.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.72

method	result
default	$-\frac{25\left(-\frac{723272}{961}x^3-\frac{3656422}{4805}x^2-\frac{14280728}{24025}x-\frac{2238016}{24025}\right)}{234256(5x^2+3x+2)^2}-\frac{97\ln\left(5x^2+3x+2\right)}{468512}+\frac{4464079\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{6978720496}+\frac{\frac{990x}{23}}{234256x^2-11}$
risch	$\frac{\frac{19984825}{117676372}x^5+\frac{21652955}{235352744}x^4+\frac{69648769}{235352744}x^3+\frac{23910151}{117676372}x^2+\frac{5333615}{29419093}x+\frac{158567}{5348926}}{(2x^2-x+3)(5x^2+3x+2)^2}-\frac{97\ln\left(100x^2+60x+40\right)}{468512}+\frac{4464079\arctan\left(\frac{(10x+3}{31}\right.}{6978720496}$

input
```
int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
-25/234256*(-723272/961*x^3-3656422/4805*x^2-14280728/24025*x-2238016/2402
5)/(5*x^2+3*x+2)^2-97/468512*ln(5*x^2+3*x+2)+4464079/6978720496*arctan(1/3
1*(10*x+3)*31^(1/2))*31^(1/2)+1/234256*(990/23*x-121/23)/(x^2-1/2*x+3/2)+9
7/468512*ln(2*x^2-x+3)+25557/123921424*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1
/2))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.60

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$$
$$= \frac{1253927859800\,x^5+679296504260\,x^4+2185021181068\,x^3+4722995582\,\sqrt{31}(50\,x^6+35\,x^5+103\,x^4+8}{}$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/7383486284768*(1253927859800*x^5 + 679296504260*x^4 + 2185021181068*x^3
+ 4722995582*sqrt(31)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x
+ 12)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1522737174*sqrt(23)*(50*x^6 + 35*
x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 1500218514344*x^2 - 1528665583*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 +
83*x^2 + 32*x + 12)*log(5*x^2 + 3*x + 2) + 1528665583*(50*x^6 + 35*x^5 + 1
03*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*log(2*x^2 - x + 3) + 1338609358240*x
 + 218880812656)/(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

3.50.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.97

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$$
$$= \frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{11767637200x^6+8237346040x^5+24241332632x^4+20004983240x^3+19534277752x^2+7531287808x+}$$
$$+\frac{97\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{468512}-\frac{97\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{468512}$$
$$+\frac{25557\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{123921424}+\frac{4464079\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{6978720496}$$

input
```
integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

output
```
(39969650*x**5 + 21652955*x**4 + 69648769*x**3 + 47820302*x**2 + 42668920*
x + 6976948)/(11767637200*x**6 + 8237346040*x**5 + 24241332632*x**4 + 2000
4983240*x**3 + 19534277752*x**2 + 7531287808*x + 2824232928) + 97*log(x**2
 - x/2 + 3/2)/468512 - 97*log(x**2 + 3*x/5 + 2/5)/468512 + 25557*sqrt(23)*
atan(4*sqrt(23)*x/23 - sqrt(23)/23)/123921424 + 4464079*sqrt(31)*atan(10*s
qrt(31)*x/31 + 3*sqrt(31)/31)/6978720496
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx = \frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{235352744(50x^6+35x^5+103x^4+85x^3+83x^2+32x+12)} - \frac{97}{468512}\log\left(5x^2+3x+2\right) + \frac{97}{468512}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/12392
1424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5
+ 21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/(50*x
^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) - 97/468512*log(5*x^2
 + 3*x + 2) + 97/468512*log(2*x^2 - x + 3)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx = \frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{235352744(5x^2+3x+2)^2(2x^2-x+3)} - \frac{97}{468512}\log\left(5x^2+3x+2\right) + \frac{97}{468512}\log\left(2x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/12392
1424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5
+ 21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/((5*x
^2 + 3*x + 2)^2*(2*x^2 - x + 3)) - 97/468512*log(5*x^2 + 3*x + 2) + 97/468
512*log(2*x^2 - x + 3)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$$
$$= \frac{\frac{799393\,x^5}{235352744}+\frac{4330591\,x^4}{2353527440}+\frac{69648769\,x^3}{11767637200}+\frac{23910151\,x^2}{5883818600}+\frac{1066723\,x}{294190930}+\frac{158567}{267446300}}{x^6+\frac{7\,x^5}{10}+\frac{103\,x^4}{50}+\frac{17\,x^3}{10}+\frac{83\,x^2}{50}+\frac{16\,x}{25}+\frac{6}{25}}$$
$$+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{97}{468512}+\frac{\sqrt{23}\,25557\mathrm{i}}{247842848}\right)$$
$$-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{97}{468512}+\frac{\sqrt{23}\,25557\mathrm{i}}{247842848}\right)$$
$$-\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{97}{468512}+\frac{\sqrt{31}\,4464079\mathrm{i}}{13957440992}\right)$$
$$+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{97}{468512}+\frac{\sqrt{31}\,4464079\mathrm{i}}{13957440992}\right)$$

input
```
int(1/((2*x^2 - x + 3)^2*(3*x + 5*x^2 + 2)^3),x)
```

output
```
log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 + 97/468512) -
 log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*25557i)/247842848 - 97/468512)
+ ((1066723*x)/294190930 + (23910151*x^2)/5883818600 + (69648769*x^3)/1176
7637200 + (4330591*x^4)/2353527440 + (799393*x^5)/235352744 + 158567/26744
6300)/((16*x)/25 + (83*x^2)/50 + (17*x^3)/10 + (103*x^4)/50 + (7*x^5)/10 +
 x^6 + 6/25) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i)/13957
440992 + 97/468512) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*4464079i
)/13957440992 - 97/468512)
```

3.51 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx$

3.51.1 Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\,(3-x+2x^2)^2}+\frac{1331(5229+76420x)}{135424\,(3-x+2x^2)}+\frac{63799791\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{16928\sqrt{23}}-\frac{13915}{64}\log\left(3-x+2x^2\right)$$

output
```
2725/8*x+4875/32*x^2+625/24*x^3-14641/5888*(101+79*x)/(2*x^2-x+3)^2+1331/1
35424*(5229+76420*x)/(2*x^2-x+3)-13915/64*ln(2*x^2-x+3)+63799791/389344*ar
ctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\,(3-x+2x^2)^2}+\frac{1331(5229+76420x)}{135424\,(3-x+2x^2)}-\frac{63799791\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{16928\sqrt{23}}-\frac{13915}{64}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

output
```
(2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3
- x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) - (6379
9791*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*
x^2])/64
```

3.51.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2191, 27, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(5x^2+3x+2\right)^4}{\left(2x^2-x+3\right)^3}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{46}\int \frac{1840000x^6+5336000x^5+6826400x^4+2298160x^3-3906136x^2-2644724x+2173869}{128\left(2x^2-x+3\right)^2}dx-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1840000x^6+5336000x^5+6826400x^4+2298160x^3-3906136x^2-2644724x+2173869}{\left(2x^2-x+3\right)^2}dx}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{\frac{1}{23}\int -\frac{16\left(-1322500x^4-4496500x^3-5170975x^2+2507460x+5460539\right)}{2x^2-x+3}dx+\frac{1331(76420x+5229)}{23(2x^2-x+3)}}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1331(76420x+5229)}{23(2x^2-x+3)}-\frac{16}{23}\int \frac{-1322500x^4-4496500x^3-5170975x^2+2507460x+5460539}{2x^2-x+3}dx}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{\frac{1331(76420x+5229)}{23(2x^2-x+3)}-\frac{16}{23}\int\left(-661250x^2-2578875x+\frac{121(60835x+116609)}{2x^2-x+3}-2883050\right)dx}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1331(76420x+5229)}{23(2x^2-x+3)}-\frac{16}{23}\left(-\frac{63799791\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{23}}-\frac{661250x^3}{3}-\frac{2578875x^2}{2}+\frac{7361035}{4}\log\left(2x^2-x+3\right)-2883050x\right)}{5888}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

output
```
(-14641*(101 + 79*x))/(5888*(3 - x + 2*x^2)^2) + ((1331*(5229 + 76420*x))/
(23*(3 - x + 2*x^2)) - (16*(-2883050*x - (2578875*x^2)/2 - (661250*x^3)/3
- (63799791*ArcTan[(1 - 4*x)/Sqrt[23]])/(2*Sqrt[23]) + (7361035*Log[3 - x
+ 2*x^2])/4))/23)/5888
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.51.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

method	result
default	$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}-\frac{121\left(-\frac{210155}{4232}x^3+\frac{362791}{16928}x^2-\frac{561121}{8464}x+\frac{54263}{16928}\right)}{4(2x^2-x+3)^2}-\frac{13915\ln(2x^2-x+3)}{64}-\frac{63799791\sqrt{23}\arctan\left(\right.}{389344}$
risch	$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8}+\frac{\frac{25428755}{16928}x^3-\frac{43897711}{67712}x^2+\frac{67895641}{33856}x-\frac{6565823}{67712}}{(2x^2-x+3)^2}-\frac{13915\ln(16x^2-8x+24)}{64}-\frac{63799791\sqrt{23}\,\text{arcta}}{3893}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
625/24*x^3+4875/32*x^2+2725/8*x-121/4*(-210155/4232*x^3+362791/16928*x^2-5
61121/8464*x+54263/16928)/(2*x^2-x+3)^2-13915/64*ln(2*x^2-x+3)-63799791/38
9344*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.31

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx$$
$$=\frac{486680000\,x^7+2360398000\,x^6+5100406400\,x^5+2157209100\,x^4+24531516180\,x^3-765597492\,\sqrt{23}(4}{}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="fricas")
```

output
```
1/4672128*(486680000*x^7 + 2360398000*x^6 + 5100406400*x^5 + 2157209100*x^
4 + 24531516180*x^3 - 765597492*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 6171678159*x^2 - 1015822830*(4*x^4 - 4
*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 23692590858*x - 453041787)/(
4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.51.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{2725x}{8} + \frac{101715020x^3-43897711x^2+135791282x-6565823}{270848x^4-270848x^3+880256x^2-406272x+609408} - \frac{13915\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{64} - \frac{63799791\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{389344}$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**3,x)
```

output
```
625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 +
135791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x
 + 609408) - 13915*log(x**2 - x/2 + 3/2)/64 - 63799791*sqrt(23)*atan(4*sqr
t(23)*x/23 - sqrt(23)/23)/389344
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{625}{24}x^3+\frac{4875}{32}x^2-\frac{63799791}{389344}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2725}{8}x + \frac{1331\,(76420\,x^3-32981\,x^2+102022\,x-4933)}{67712\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)} - \frac{13915}{64}\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="maxima")
```

output
```
625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(
4*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933
)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 13915/64*log(2*x^2 - x + 3)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{625}{24}\,x^3 + \frac{4875}{32}\,x^2 - \frac{63799791}{389344}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{2725}{8}\,x + \frac{1331\,(76420\,x^3 - 32981\,x^2 + 102022\,x - 4933)}{67712\,(2\,x^2 - x + 3)^2} - \frac{13915}{64}\,\log\left(2\,x^2 - x + 3\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="giac")
```

output
```
625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(
4*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933
)/(2*x^2 - x + 3)^2 - 13915/64*log(2*x^2 - x + 3)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3}\,dx = \frac{2725\,x}{8} - \frac{13915\,\ln\left(2\,x^2 - x + 3\right)}{64} - \frac{63799791\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23} - \frac{\sqrt{23}}{23}\right)}{389344} + \frac{4875\,x^2}{32} + \frac{625\,x^3}{24} + \frac{\frac{25428755\,x^3}{67712} - \frac{43897711\,x^2}{270848} + \frac{67895641\,x}{135424} - \frac{6565823}{270848}}{x^4 - x^3 + \frac{13\,x^2}{4} - \frac{3\,x}{2} + \frac{9}{4}}$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^3,x)
```

output
```
(2725*x)/8 - (13915*log(2*x^2 - x + 3))/64 - (63799791*23^(1/2)*atan((4*23
^(1/2)*x)/23 - 23^(1/2)/23))/389344 + (4875*x^2)/32 + (625*x^3)/24 + ((678
95641*x)/135424 - (43897711*x^2)/270848 + (25428755*x^3)/67712 - 6565823/2
70848)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 + 9/4)
```

3.52 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} dx$

3.52.1 Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} dx = \frac{125x}{8} - \frac{1331(17-45x)}{1472(3-x+2x^2)^2} + \frac{121(21193-12828x)}{33856(3-x+2x^2)} + \frac{165099\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{825}{32}\log\left(3-x+2x^2\right)$$

output
```
125/8*x-1331/1472*(17-45*x)/(2*x^2-x+3)^2+121/33856*(21193-12828*x)/(2*x^2
-x+3)+825/32*ln(2*x^2-x+3)+165099/194672*arctan(1/23*(1-4*x)*23^(1/2))*23^
(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3} dx = \frac{125x}{8} + \frac{1331(-17+45x)}{1472(3-x+2x^2)^2} - \frac{121(-21193+12828x)}{33856(3-x+2x^2)} - \frac{165099\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{8464\sqrt{23}} + \frac{825}{32}\log\left(3-x+2x^2\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

output
```
(125*x)/8 + (1331*(-17 + 45*x))/(1472*(3 - x + 2*x^2)^2) - (121*(-21193 +
12828*x))/(33856*(3 - x + 2*x^2)) - (165099*ArcTan[(-1 + 4*x)/Sqrt[23]])/(
8464*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

3.52.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2191, 27, 2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(5x^2+3x+2\right)^3}{\left(2x^2-x+3\right)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{46}\int -\frac{-92000x^4-211600x^3-177560x^2+76268x+40885}{32\left(2x^2-x+3\right)^2}dx-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-92000x^4-211600x^3-177560x^2+76268x+40885}{\left(2x^2-x+3\right)^2}dx}{1472}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{121(21193-12828x)}{23(2x^2-x+3)}-\frac{1}{23}\int -\frac{16\left(66125x^2+185150x+23997\right)}{2x^2-x+3}dx}{1472}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{16}{23}\int \frac{66125x^2+185150x+23997}{2x^2-x+3}dx+\frac{121(21193-12828x)}{23(2x^2-x+3)}}{1472}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\frac{16}{23}\int \left(\frac{66125}{2}-\frac{33(4557-13225x)}{2(2x^2-x+3)}\right)dx+\frac{121(21193-12828x)}{23(2x^2-x+3)}}{1472}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{16}{23}\left(\frac{165099\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{436425}{8}\log\left(2x^2-x+3\right)+\frac{66125x}{2}\right)+\frac{121(21193-12828x)}{23(2x^2-x+3)}}{1472}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

output
```
(-1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + ((121*(21193 - 12828*x))/(2
3*(3 - x + 2*x^2)) + (16*((66125*x)/2 + (165099*ArcTan[(1 - 4*x)/Sqrt[23]]
)/(4*Sqrt[23]) + (436425*Log[3 - x + 2*x^2])/8))/23)/1472
```

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.52.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{125x}{8}+\frac{-\frac{388047}{4232}x^3+\frac{3340447}{16928}x^2-\frac{1460833}{8464}x+\frac{3586319}{16928}}{(2x^2-x+3)^2}+\frac{825\ln(2x^2-x+3)}{32}-\frac{165099\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{194672}$	63
risch	$\frac{125x}{8}+\frac{-\frac{388047}{4232}x^3+\frac{3340447}{16928}x^2-\frac{1460833}{8464}x+\frac{3586319}{16928}}{(2x^2-x+3)^2}+\frac{825\ln(16x^2-8x+24)}{32}-\frac{165099\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{194672}$	63

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
125/8*x+11/2*(-35277/2116*x^3+303677/8464*x^2-132803/4232*x+326029/8464)/(
2*x^2-x+3)^2+825/32*ln(2*x^2-x+3)-165099/194672*23^(1/2)*arctan(1/23*(-1+4
*x)*23^(1/2))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = \frac{24334000\,x^5 - 24334000\,x^4 + 43385176\,x^3 - 330198\,\sqrt{23}(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9)\arctan\left(\frac{1}{23}\sqrt{23}(4\right.}{389344\,(4\,x^4 - 4}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="fricas")
```

output
```
1/389344*(24334000*x^5 - 24334000*x^4 + 43385176*x^3 - 330198*sqrt(23)*(4*
x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) + 40329281
*x^2 + 10037775*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) - 12
446818*x + 82485337)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = \frac{125x}{8} + \frac{-1552188x^3+3340447x^2-2921666x+3586319}{67712x^4-67712x^3+220064x^2-101568x+152352} + \frac{825\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32} - \frac{165099\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{194672}$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**3,x)
```

output
```
125*x/8 + (-1552188*x**3 + 3340447*x**2 - 2921666*x + 3586319)/(67712*x**4
 - 67712*x**3 + 220064*x**2 - 101568*x + 152352) + 825*log(x**2 - x/2 + 3/
2)/32 - 165099*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/194672
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = -\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{125}{8}x - \frac{121\,(12828\,x^3-27607\,x^2+24146\,x-29639)}{16928\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)} + \frac{825}{32}\log\left(2\,x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="maxima")
```

output
```
-165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/16
928*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(4*x^4 - 4*x^3 + 13*x^2 - 6*
x + 9) + 825/32*log(2*x^2 - x + 3)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = -\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{125}{8}x - \frac{121\left(12828x^3-27607x^2+24146x-29639\right)}{16928\left(2x^2-x+3\right)^2} + \frac{825}{32}\log\left(2x^2-x+3\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="giac")
```

output
```
-165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/16
928*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(2*x^2 - x + 3)^2 + 825/32*l
og(2*x^2 - x + 3)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^3}\,dx = \frac{125x}{8}+\frac{825\ln\left(2x^2-x+3\right)}{32}-\frac{165099\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{194672} - \frac{\frac{388047x^3}{16928}-\frac{3340447x^2}{67712}+\frac{1460833x}{33856}-\frac{3586319}{67712}}{x^4-x^3+\frac{13x^2}{4}-\frac{3x}{2}+\frac{9}{4}}$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^3,x)
```

output
```
(125*x)/8 + (825*log(2*x^2 - x + 3))/32 - (165099*23^(1/2)*atan((4*23^(1/2
)*x)/23 - 23^(1/2)/23))/194672 - ((1460833*x)/33856 - (3340447*x^2)/67712
+ (388047*x^3)/16928 - 3586319/67712)/((13*x^2)/4 - (3*x)/2 - x^3 + x^4 +
9/4)
```

3.53 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} \, dx$

3.53.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} \, dx = \frac{121(19-7x)}{368(3-x+2x^2)^2} - \frac{55(975+332x)}{8464(3-x+2x^2)} - \frac{4330\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

output
```
121/368*(19-7*x)/(2*x^2-x+3)^2-55/8464*(975+332*x)/(2*x^2-x+3)-4330/12167*
arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} \, dx = -\frac{11(4909+938x+4045x^2+1660x^3)}{4232(-3+x-2x^2)^2} + \frac{4330\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

output
```
(-11*(4909 + 938*x + 4045*x^2 + 1660*x^3))/(4232*(-3 + x - 2*x^2)^2) + (43
30*ArcTan[(-1 + 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

3.53.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2191, 27, 2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{46}\int -\frac{5(-920x^2-1564x+39)}{8(2x^2-x+3)^2}dx+\frac{121(19-7x)}{368(2x^2-x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\int \frac{-920x^2-1564x+39}{(2x^2-x+3)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\left(\frac{1}{23}\int -\frac{6928}{2x^2-x+3}dx+\frac{11(332x+975)}{23(2x^2-x+3)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\left(\frac{11(332x+975)}{23(2x^2-x+3)}-\frac{6928}{23}\int \frac{1}{2x^2-x+3}dx\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\left(\frac{13856}{23}\int \frac{1}{-(4x-1)^2-23}d(4x-1)+\frac{11(332x+975)}{23(2x^2-x+3)}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{5}{368}\left(\frac{11(332x+975)}{23(2x^2-x+3)}-\frac{13856\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}}\right)$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

output
```
(121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (5*((11*(975 + 332*x))/(23*(3 -
 x + 2*x^2)) - (13856*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23*Sqrt[23])))/368
```

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{-\frac{4565}{1058}x^3-\frac{44495}{4232}x^2-\frac{5159}{2116}x-\frac{53999}{4232}}{(2x^2-x+3)^2}+\frac{4330\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{12167}$	47
risch	$\frac{-\frac{4565}{1058}x^3-\frac{44495}{4232}x^2-\frac{5159}{2116}x-\frac{53999}{4232}}{(2x^2-x+3)^2}+\frac{4330\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{12167}$	47

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
4*(-4565/4232*x^3-44495/16928*x^2-5159/8464*x-53999/16928)/(2*x^2-x+3)^2+4
330/12167*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = -\frac{419980\,x^3 - 34640\,\sqrt{23}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + 1023385\,x^2 + 237314\,x + 1241977}{97336\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="fricas")
```

output
```
-1/97336*(419980*x^3 - 34640*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*a
rctan(1/23*sqrt(23)*(4*x - 1)) + 1023385*x^2 + 237314*x + 1241977)/(4*x^4
- 4*x^3 + 13*x^2 - 6*x + 9)
```

3.53.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = \frac{-18260x^3-44495x^2-10318x-53999}{16928x^4-16928x^3+55016x^2-25392x+38088} + \frac{4330\sqrt{23}\,\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**3,x)
```

output
```
(-18260*x**3 - 44495*x**2 - 10318*x - 53999)/(16928*x**4 - 16928*x**3 + 55
016*x**2 - 25392*x + 38088) + 4330*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23
)/23)/12167
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = \frac{4330}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{11\,(1660\,x^3+4045\,x^2+938\,x+4909)}{4232\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="maxima")
```

output
```
4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 +
4045*x^2 + 938*x + 4909)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = \frac{4330}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{11\,(1660\,x^3+4045\,x^2+938\,x+4909)}{4232\,(2\,x^2-x+3)^2}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="giac")
```

output
```
4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 +
4045*x^2 + 938*x + 4909)/(2*x^2 - x + 3)^2
```

3.53.9 Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = \frac{4330\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167} - \frac{\frac{4565\,x^3}{4232}+\frac{44495\,x^2}{16928}+\frac{5159\,x}{8464}+\frac{53999}{16928}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^3,x)
```

output
```
(4330*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 - ((5159*x)/84
64 + (44495*x^2)/16928 + (4565*x^3)/4232 + 53999/16928)/((13*x^2)/4 - (3*x
)/2 - x^3 + x^4 + 9/4)
```

3.53. $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx$

3.54 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx$

3.54.1 Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = -\frac{11(5+3x)}{92\,(3-x+2x^2)^2} - \frac{131(1-4x)}{2116\,(3-x+2x^2)} - \frac{262\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

output
```
-11/92*(5+3*x)/(2*x^2-x+3)^2-131/2116*(1-4*x)/(2*x^2-x+3)-262/12167*arctan
(1/23*(1-4*x)*23^(1/2))*23^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{\frac{46(-829+472x-393x^2+524x^3)}{(-3+x-2x^2)^2} + 1048\sqrt{23}\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{48668}$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

output
```
((46*(-829 + 472*x - 393*x^2 + 524*x^3))/(-3 + x - 2*x^2)^2 + 1048*Sqrt[23
]*ArcTan[(-1 + 4*x)/Sqrt[23]])/48668
```

3.54.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2191, 27, 1086, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{46}\int \frac{131}{2(2x^2-x+3)^2}dx-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{131}{92}\int \frac{1}{(2x^2-x+3)^2}dx-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{131}{92}\left(\frac{4}{23}\int \frac{1}{2x^2-x+3}dx-\frac{1-4x}{23(2x^2-x+3)}\right)-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{131}{92}\left(-\frac{8}{23}\int \frac{1}{-(4x-1)^2-23}d(4x-1)-\frac{1-4x}{23(2x^2-x+3)}\right)-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{131}{92}\left(\frac{8\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}}-\frac{1-4x}{23(2x^2-x+3)}\right)-\frac{11(3x+5)}{92(2x^2-x+3)^2}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

output
```
(-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) + (131*(-1/23*(1 - 4*x)/(3 - x + 2*
x^2) + (8*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23*Sqrt[23])))/92
```

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.54.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{262}{529}x^3-\frac{393}{1058}x^2+\frac{236}{529}x-\frac{829}{1058}}{(2x^2-x+3)^2}+\frac{262\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{12167}$	47
risch	$\frac{\frac{262}{529}x^3-\frac{393}{1058}x^2+\frac{236}{529}x-\frac{829}{1058}}{(2x^2-x+3)^2}+\frac{262\sqrt{23}\,\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)}{12167}$	47

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
4*(131/1058*x^3-393/4232*x^2+59/529*x-829/4232)/(2*x^2-x+3)^2+262/12167*23
^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{12052\,x^3+524\,\sqrt{23}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)-9039\,x^2+10856\,x-19067}{24334\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="fricas")
```

output
```
1/24334*(12052*x^3 + 524*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 9039*x^2 + 10856*x - 19067)/(4*x^4 - 4*x^3 +
13*x^2 - 6*x + 9)
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{524x^3-393x^2+472x-829}{4232x^4-4232x^3+13754x^2-6348x+9522} + \frac{262\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**3,x)
```

output
```
(524*x**3 - 393*x**2 + 472*x - 829)/(4232*x**4 - 4232*x**3 + 13754*x**2 -
6348*x + 9522) + 262*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/12167
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{524\,x^3-393\,x^2+472\,x-829}{1058\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="maxima")
```

output
```
262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393
*x^2 + 472*x - 829)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{524\,x^3-393\,x^2+472\,x-829}{1058\,(2\,x^2-x+3)^2}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="giac")
```

output
```
262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393
*x^2 + 472*x - 829)/(2*x^2 - x + 3)^2
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^3}\,dx = \frac{262\,\sqrt{23}\,\mathrm{atan}\left(\frac{4\,\sqrt{23}\,x}{23}-\frac{\sqrt{23}}{23}\right)}{12167} + \frac{\frac{131\,x^3}{1058}-\frac{393\,x^2}{4232}+\frac{59\,x}{529}-\frac{829}{4232}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^3,x)
```

output
```
(262*23^(1/2)*atan((4*23^(1/2)*x)/23 - 23^(1/2)/23))/12167 + ((59*x)/529 -
 (393*x^2)/4232 + (131*x^3)/1058 - 829/4232)/((13*x^2)/4 - (3*x)/2 - x^3 +
 x^4 + 9/4)
```

3.54. $\int \frac{2+3x+5x^2}{(3-x+2x^2)^3} \, dx$

3.55 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx$

3.55.1 Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = \frac{13-6x}{1012(3-x+2x^2)^2}+\frac{3625-746x}{256036(3-x+2x^2)} - \frac{53403\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}+\frac{247\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{10648\sqrt{31}} - \frac{119\log(3-x+2x^2)}{21296}+\frac{119\log(2+3x+5x^2)}{21296}$$

output
```
1/1012*(13-6*x)/(2*x^2-x+3)^2+1/256036*(3625-746*x)/(2*x^2-x+3)-119/21296*
ln(2*x^2-x+3)+119/21296*ln(5*x^2+3*x+2)-53403/129554216*arctan(1/23*(1-4*x
)*23^(1/2))*23^(1/2)+247/330088*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = \frac{3310986\sqrt{23}\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)+6010498\sqrt{31}\arctan\left(\frac{3+10x}{\sqrt{31}}\right)+713\left(-\frac{44(-14164+7381x-7996x^2+1492x^3)}{(-3+x-2x^2)^2}-6295\right.}{8032361392}$$

input
```
Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

output
```
(3310986*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 6010498*Sqrt[31]*ArcTan[(3
 + 10*x)/Sqrt[31]] + 713*((-44*(-14164 + 7381*x - 7996*x^2 + 1492*x^3))/(-
3 + x - 2*x^2)^2 - 62951*Log[3 - x + 2*x^2] + 62951*Log[2 + 3*x + 5*x^2]))
/8032361392
```

3.55.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1305, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^3(5x^2+3x+2)}dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{13-6x}{1012(2x^2-x+3)^2}-\frac{\int -\frac{22(-45x^2+88x+166)}{(2x^2-x+3)^2(5x^2+3x+2)}dx}{11132}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{506}\int \frac{-45x^2+88x+166}{(2x^2-x+3)^2(5x^2+3x+2)}dx+\frac{13-6x}{1012(2x^2-x+3)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{506}\left(\frac{\int \frac{11(-3730x^2+32147x+27074)}{(2x^2-x+3)(5x^2+3x+2)}dx}{5566}+\frac{3625-746x}{506(2x^2-x+3)}\right)+\frac{13-6x}{1012(2x^2-x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{506}\left(\frac{1}{506}\int \frac{-3730x^2+32147x+27074}{(2x^2-x+3)(5x^2+3x+2)}dx+\frac{3625-746x}{506(2x^2-x+3)}\right)+\frac{13-6x}{1012(2x^2-x+3)^2}$$

$$\Big\downarrow \text{2141}$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{1}{242}\int \frac{77(8311-17986x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{5819(595x+302)}{5x^2+3x+2}dx\right)+\frac{3625-746x}{506(2x^2-x+3)}\right)+\frac{13-6x}{1012(2x^2-x+3)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\int \frac{8311-17986x}{2x^2-x+3}dx+\frac{529}{22}\int \frac{595x+302}{5x^2+3x+2}dx\right)+\frac{3625-746x}{506\left(2x^2-x+3\right)}\right)+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{7629}{2}\int \frac{1}{2x^2-x+3}dx-\frac{8993}{2}\int -\frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{247}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{119}{2}\int \frac{}{5x}\right.\right.\right.\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{7629}{2}\int \frac{1}{2x^2-x+3}dx+\frac{8993}{2}\int \frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{247}{2}\int \frac{1}{5x^2+3x+2}dx+\frac{119}{2}\int \frac{1(}{5x^2}\right.\right.\right.\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{8993}{2}\int \frac{1-4x}{2x^2-x+3}dx-7629\int \frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{529}{22}\left(\frac{119}{2}\int \frac{10x+3}{5x^2+3x+2}dx-\right.\right.\right.\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{8993}{2}\int \frac{1-4x}{2x^2-x+3}dx+\frac{7629\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)+\frac{529}{22}\left(\frac{119}{2}\int \frac{10x+3}{5x^2+3x+2}dx+\frac{247\arctan\left(\right.}{\sqrt{31}}\right.\right.\right.\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{506}\left(\frac{1}{506}\left(\frac{7}{22}\left(\frac{7629\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{8993}{2}\log\left(2x^2-x+3\right)\right)+\frac{529}{22}\left(\frac{247\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}+\frac{119}{2}\log\left(5x^2+3\right.\right.\right.\right.\frac{13-6x}{1012\left(2x^2-x+3\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

3.55. $\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)}\,dx$

output
```
(13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + ((3625 - 746*x)/(506*(3 - x + 2*x^2)
) + ((7*((7629*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] - (8993*Log[3 - x + 2
*x^2])/2))/22 + (529*((247*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] + (119*Lo
g[2 + 3*x + 5*x^2])/2))/22)/506)/506
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.55. $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx$

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.55.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result
default	$\frac{119\ln(5x^2+3x+2)}{21296}+\frac{247\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{330088}-\frac{\frac{8206}{529}x^3-\frac{43978}{529}x^2+\frac{81191}{1058}x-\frac{77902}{529}}{2662(2x^2-x+3)^2}-\frac{119\ln(2x^2-x+3)}{21296}+\frac{53403\sqrt{23}\text{ ar}}{12}$
risch	$\frac{-\frac{373}{64009}x^3+\frac{1999}{64009}x^2-\frac{61}{2116}x+\frac{3541}{64009}}{(2x^2-x+3)^2}+\frac{119\ln(100x^2+60x+40)}{21296}+\frac{247\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{330088}-\frac{119\ln(16x^2-8x+24)}{21296}+\frac{534}{}$

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
119/21296*ln(5*x^2+3*x+2)+247/330088*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/
2)-1/2662*(8206/529*x^3-43978/529*x^2+81191/1058*x-77902/529)/(2*x^2-x+3)^
2-119/21296*ln(2*x^2-x+3)+53403/129554216*23^(1/2)*arctan(1/23*(-1+4*x)*23
^(1/2))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = -\frac{46807024\,x^3 - 6010498\,\sqrt{31}(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - 3310986\,\sqrt{23}(4}{}$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
-1/8032361392*(46807024*x^3 - 6010498*sqrt(31)*(4*x^4 - 4*x^3 + 13*x^2 - 6
*x + 9)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3310986*sqrt(23)*(4*x^4 - 4*x^3
 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) - 250850512*x^2 - 448
84063*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(5*x^2 + 3*x + 2) + 44884063*(
4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 231556732*x - 44435
3008)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx &= \frac{-1492x^3+7996x^2-7381x+14164}{1024144x^4-1024144x^3+3328468x^2-1536216x+2304324} \\ &\quad - \frac{119\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296} + \frac{119\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296} \\ &\quad + \frac{53403\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{129554216} + \frac{247\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{330088}\end{aligned}$$

input
```
integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

output
```
(-1492*x**3 + 7996*x**2 - 7381*x + 14164)/(1024144*x**4 - 1024144*x**3 + 3
328468*x**2 - 1536216*x + 2304324) - 119*log(x**2 - x/2 + 3/2)/21296 + 119
*log(x**2 + 3*x/5 + 2/5)/21296 + 53403*sqrt(23)*atan(4*sqrt(23)*x/23 - sqr
t(23)/23)/129554216 + 247*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/
330088
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx &= \frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) \\ &\quad + \frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) \\ &\quad - \frac{1492x^3-7996x^2+7381x-14164}{256036(4x^4-4x^3+13x^2-6x+9)} \\ &\quad + \frac{119}{21296}\log\left(5x^2+3x+2\right) - \frac{119}{21296}\log\left(2x^2-x+3\right)\end{aligned}$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 73
81*x - 14164)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) + 119/21296*log(5*x^2 + 3
*x + 2) - 119/21296*log(2*x^2 - x + 3)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = \frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) + \frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{1492\,x^3-7996\,x^2+7381\,x-14164}{256036\,(2\,x^2-x+3)^2} + \frac{119}{21296}\,\log\left(5\,x^2+3\,x+2\right) - \frac{119}{21296}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqr
t(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 73
81*x - 14164)/(2*x^2 - x + 3)^2 + 119/21296*log(5*x^2 + 3*x + 2) - 119/212
96*log(2*x^2 - x + 3)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx = -\ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{119}{21296}+\frac{\sqrt{31}\,247\mathrm{i}}{660176}\right)+\ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{119}{21296}+\frac{\sqrt{31}\,247\mathrm{i}}{660176}\right)-\ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{119}{21296}+\frac{\sqrt{23}\,53403\mathrm{i}}{259108432}\right)+\ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{119}{21296}+\frac{\sqrt{23}\,53403\mathrm{i}}{259108432}\right)-\frac{\frac{373\,x^3}{256036}-\frac{1999\,x^2}{256036}+\frac{61\,x}{8464}-\frac{3541}{256036}}{x^4-x^3+\frac{13\,x^2}{4}-\frac{3\,x}{2}+\frac{9}{4}}$$

input
```
int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)),x)
```

output
```
log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 + 119/21296) - lo
g(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*247i)/660176 - 119/21296) - log(
x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 + 119/21296) + log
(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*53403i)/259108432 - 119/21296) - ((
61*x)/8464 - (1999*x^2)/256036 + (373*x^3)/256036 - 3541/256036)/((13*x^2)
/4 - (3*x)/2 - x^3 + x^4 + 9/4)
```

3.56 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx$

3.56.1 Optimal result

Integrand size = 25, antiderivative size = 160

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx &= \frac{-2328909-252815x}{174616552(2+3x+5x^2)} \\ &+ \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)} \\ &+ \frac{9665-1446x}{512072(3-x+2x^2)(2+3x+5x^2)} \\ &+ \frac{2038497\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{123921424\sqrt{23}} + \frac{246757\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{7261936\sqrt{31}} \\ &+ \frac{181\log(3-x+2x^2)}{468512} - \frac{181\log(2+3x+5x^2)}{468512}\end{aligned}$$

output
```
1/174616552*(-2328909-252815*x)/(5*x^2+3*x+2)+1/1012*(13-6*x)/(2*x^2-x+3)^
2/(5*x^2+3*x+2)+1/512072*(9665-1446*x)/(2*x^2-x+3)/(5*x^2+3*x+2)+181/46851
2*ln(2*x^2-x+3)-181/468512*ln(5*x^2+3*x+2)+2038497/2850192752*arctan(1/23*
(1-4*x)*23^(1/2))*23^(1/2)+246757/225120016*arctan(1/31*(3+10*x)*31^(1/2))
*31^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^2} dx = \frac{-31-14x}{22264 (3-x+2x^2)^2} + \frac{-1782-2923x}{1408198 (3-x+2x^2)} + \frac{-1474+1235x}{330088 (2+3x+5x^2)} - \frac{2038497 \arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{123921424\sqrt{23}} + \frac{246757 \arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{7261936\sqrt{31}} + \frac{181 \log(3-x+2x^2)}{468512} - \frac{181 \log(2+3x+5x^2)}{468512}$$

input
```
Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(-31 - 14*x)/(22264*(3 - x + 2*x^2)^2) + (-1782 - 2923*x)/(1408198*(3 - x
+ 2*x^2)) + (-1474 + 1235*x)/(330088*(2 + 3*x + 5*x^2)) - (2038497*ArcTan[
(-1 + 4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3 + 10*x)/Sqr
t[31]])/(7261936*Sqrt[31]) + (181*Log[3 - x + 2*x^2])/468512 - (181*Log[2
+ 3*x + 5*x^2])/468512
```

3.56.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {1305, 27, 2135, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^3 (5x^2+3x+2)^2} dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{13-6x}{1012 (2x^2-x+3)^2 (5x^2+3x+2)} - \frac{\int -\frac{11(-150x^2+288x+371)}{(2x^2-x+3)^2(5x^2+3x+2)^2}dx}{11132}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-150x^2+288x+371}{(2x^2-x+3)^2(5x^2+3x+2)^2}dx}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\int \frac{11\left(-21690x^2+181009x+88019\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}dx}{5566}+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{506}\int \frac{-21690x^2+181009x+88019}{(2x^2-x+3)(5x^2+3x+2)^2}dx+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{1}{506}\left(\frac{\int \frac{22\left(-505630x^2-8759443x+5285594\right)}{\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}dx}{7502}-\frac{252815x+2328909}{341(5x^2+3x+2)}\right)+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\int \frac{-505630x^2-8759443x+5285594}{(2x^2-x+3)(5x^2+3x+2)}dx-\frac{252815x+2328909}{341(5x^2+3x+2)}\right)+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2141}$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{1}{242}\int -\frac{341(1067123-191498x)}{2x^2-x+3}dx+\frac{1}{242}\int \frac{5819(114962-28055x)}{5x^2+3x+2}dx\right)-\frac{252815x+2328909}{341(5x^2+3x+2)}\right)+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\int \frac{114962-28055x}{5x^2+3x+2}dx-\frac{31}{22}\int \frac{1067123-191498x}{2x^2-x+3}dx\right)-\frac{252815x+2328909}{341(5x^2+3x+2)}\right)+\frac{9665-1446x}{506(2x^2-x+3)(5x^2+3x+2)}}{1012}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1142}$$

3.56. $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(\frac{246757}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{5611}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{2038497}{2}\int\frac{1}{2x^2-x+3}dx-\frac{95749}{2}\int-\frac{1-4x}{2x^2-x+3}dx\right)\right)-\frac{25}{3}\right.}{1012}$$

$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(\frac{246757}{2}\int\frac{1}{5x^2+3x+2}dx-\frac{5611}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{2038497}{2}\int\frac{1}{2x^2-x+3}dx+\frac{95749}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)\right)-\frac{2528}{341}\right.}{1012}$$

$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(-\frac{5611}{2}\int\frac{10x+3}{5x^2+3x+2}dx-246757\int\frac{1}{-(10x+3)^2-31}d(10x+3)\right)-\frac{31}{22}\left(\frac{95749}{2}\int\frac{1-4x}{2x^2-x+3}dx-2038497\int\frac{}{-(4x\right.\right.\right.}{1012}$$

$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(\frac{246757\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{5611}{2}\int\frac{10x+3}{5x^2+3x+2}dx\right)-\frac{31}{22}\left(\frac{95749}{2}\int\frac{1-4x}{2x^2-x+3}dx+\frac{2038497\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}\right)\right)-\frac{2528}{341}\right.}{1012}$$

$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{1}{506}\left(\frac{1}{341}\left(\frac{529}{22}\left(\frac{246757\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}-\frac{5611}{2}\log\left(5x^2+3x+2\right)\right)-\frac{31}{22}\left(\frac{2038497\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{95749}{2}\log\left(2x^2-x+3\right)\right.\right.\right.}{1012}$$

$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

3.56. $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx$

output
```
(13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + ((9665 - 1446*x)/(
506*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)) + (-1/341*(2328909 + 252815*x)/(2 +
 3*x + 5*x^2) + ((-31*((2038497*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] - (9
5749*Log[3 - x + 2*x^2])/2))/22 + (529*((246757*ArcTan[(3 + 10*x)/Sqrt[31]
])/Sqrt[31] - (5611*Log[2 + 3*x + 5*x^2])/2))/22)/341)/506)/1012
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.56. $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx$

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.56.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

method	result
default	$-\frac{-\frac{5434x}{31}+\frac{32428}{155}}{234256\left(x^2+\frac{3}{5}x+\frac{2}{5}\right)}-\frac{181\ln\left(5x^2+3x+2\right)}{468512}+\frac{246757\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{225120016}+\frac{-\frac{128612}{529}x^3-\frac{14102}{529}x^2-\frac{173195}{529}x-\frac{321497}{1058}}{58564\left(2x^2-x+3\right)^2}+$
risch	$\frac{-\frac{252815}{43654138}x^5-\frac{1038047}{21827069}x^4+\frac{5042869}{174616552}x^3-\frac{21674311}{174616552}x^2+\frac{1471955}{43654138}x-\frac{200677}{3968558}}{\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}-\frac{181\ln\left(100x^2+60x+40\right)}{468512}+\frac{246757\arctan\left(\frac{(10x+3)}{31}\right.}{225120016}$

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/234256*(-5434/31*x+32428/155)/(x^2+3/5*x+2/5)-181/468512*ln(5*x^2+3*x+2
)+246757/225120016*arctan(1/31*(10*x+3)*31^(1/2))*31^(1/2)+1/58564*(-12861
2/529*x^3-14102/529*x^2-173195/529*x-321497/1058)/(2*x^2-x+3)^2+181/468512
*ln(2*x^2-x+3)-2038497/2850192752*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.42

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx =$$
$$-\frac{31725248720\,x^5+260524883872\,x^4-158204886268\,x^3-6004584838\sqrt{31}(20\,x^6-8\,x^5+61\,x^4+x^3+}{}$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/5478070469344*(31725248720*x^5 + 260524883872*x^4 - 158204886268*x^3 -
6004584838*sqrt(31)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*a
rctan(1/31*sqrt(31)*(10*x + 3)) + 3917991234*sqrt(23)*(20*x^6 - 8*x^5 + 61
*x^4 + x^3 + 53*x^2 + 15*x + 18)*arctan(1/23*sqrt(23)*(4*x - 1)) + 6799664
84692*x^2 + 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18
)*log(5*x^2 + 3*x + 2) - 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^
2 + 15*x + 18)*log(2*x^2 - x + 3) - 184712689040*x + 277008109136)/(20*x^6
 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx = \frac{-1011260x^5-8304376x^4+5042869x^3-21674311x^2+5887820x-8829788}{3492331040x^6-1396932416x^5+10651609672x^4+174616552x^3+9254677256x^2+2619248280x+314\ldots} + \frac{181\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{468512} - \frac{181\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{468512} - \frac{2038497\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2850192752} + \frac{246757\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{225120016}$$

input
```
integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

output
```
(-1011260*x**5 - 8304376*x**4 + 5042869*x**3 - 21674311*x**2 + 5887820*x -
 8829788)/(3492331040*x**6 - 1396932416*x**5 + 10651609672*x**4 + 17461655
2*x**3 + 9254677256*x**2 + 2619248280*x + 3143097936) + 181*log(x**2 - x/2
 + 3/2)/468512 - 181*log(x**2 + 3*x/5 + 2/5)/468512 - 2038497*sqrt(23)*ata
n(4*sqrt(23)*x/23 - sqrt(23)/23)/2850192752 + 246757*sqrt(31)*atan(10*sqrt
(31)*x/31 + 3*sqrt(31)/31)/225120016
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.72

$$\int \frac{1}{(3-x+2x^2)^3\,(2+3x+5x^2)^2}\,dx = \frac{246757}{225120016}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) - \frac{2038497}{2850192752}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{1011260\,x^5+8304376\,x^4-5042869\,x^3+21674311\,x^2-5887820\,x+8829788}{174616552\,(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18)} - \frac{181}{468512}\,\log\left(5\,x^2+3\,x+2\right) + \frac{181}{468512}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/28501
92752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5
+ 8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/(20*x^6
- 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) - 181/468512*log(5*x^2 + 3*x
+ 2) + 181/468512*log(2*x^2 - x + 3)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \frac{1}{(3-x+2x^2)^3\,(2+3x+5x^2)^2}\,dx = \frac{246757}{225120016}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) - \frac{2038497}{2850192752}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{1011260\,x^5+8304376\,x^4-5042869\,x^3+21674311\,x^2-5887820\,x+8829788}{174616552\,(5\,x^2+3\,x+2)(2\,x^2-x+3)^2} - \frac{181}{468512}\,\log\left(5\,x^2+3\,x+2\right) + \frac{181}{468512}\,\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/28501
92752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5
+ 8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/((5*x^2
+ 3*x + 2)*(2*x^2 - x + 3)^2) - 181/468512*log(5*x^2 + 3*x + 2) + 181/4685
12*log(2*x^2 - x + 3)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.85

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^2}\,dx = -\frac{\frac{50563\,x^5}{174616552}+\frac{1038047\,x^4}{436541380}-\frac{5042869\,x^3}{3492331040}+\frac{21674311\,x^2}{3492331040}-\frac{294391\,x}{174616552}+\frac{200677}{79371160}}{x^6-\frac{2\,x^5}{5}+\frac{61\,x^4}{20}+\frac{x^3}{20}+\frac{53\,x^2}{20}+\frac{3\,x}{4}+\frac{9}{10}} - \ln\left(x+\frac{3}{10}-\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{181}{468512}+\frac{\sqrt{31}\,246757\mathrm{i}}{450240032}\right) + \ln\left(x+\frac{3}{10}+\frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{181}{468512}+\frac{\sqrt{31}\,246757\mathrm{i}}{450240032}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{181}{468512}+\frac{\sqrt{23}\,2038497\mathrm{i}}{5700385504}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{181}{468512}+\frac{\sqrt{23}\,2038497\mathrm{i}}{5700385504}\right)$$

input
```
int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^2),x)
```

output
```
log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 - 181/46851
2) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*246757i)/450240032 + 181/
468512) - ((21674311*x^2)/3492331040 - (294391*x)/174616552 - (5042869*x^3
)/3492331040 + (1038047*x^4)/436541380 + (50563*x^5)/174616552 + 200677/79
371160)/((3*x)/4 + (53*x^2)/20 + x^3/20 + (61*x^4)/20 - (2*x^5)/5 + x^6 +
9/10) + log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/5700385504 + 1
81/468512) - log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*2038497i)/570038550
4 - 181/468512)
```

3.57 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$

3.57.1 Optimal result

Integrand size = 25, antiderivative size = 181

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx =& -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2}\\ &+\frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2}\\ &+\frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2}\\ &+\frac{15(2618306+7140435x)}{14886061058(2+3x+5x^2)}\\ &-\frac{880575\arctan\left(\frac{1-4x}{\sqrt{23}}\right)}{340783916\sqrt{23}}+\frac{2768835\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{619080044\sqrt{31}}\\ &+\frac{405\log(3-x+2x^2)}{1288408}-\frac{405\log(2+3x+5x^2)}{1288408}\end{aligned}$$

output
```
-5/87308276*(223707+77020*x)/(5*x^2+3*x+2)^2+1/1012*(13-6*x)/(2*x^2-x+3)^2
/(5*x^2+3*x+2)^2+5/64009*(302-35*x)/(2*x^2-x+3)/(5*x^2+3*x+2)^2+15/1488606
1058*(2618306+7140435*x)/(5*x^2+3*x+2)+405/1288408*ln(2*x^2-x+3)-405/12884
08*ln(5*x^2+3*x+2)-880575/7838030068*arctan(1/23*(1-4*x)*23^(1/2))*23^(1/2
)+2768835/19191481364*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx = \frac{-4342+11154x-9275x^2+6850x^3}{345092(6+7x+16x^2+x^3+10x^4)^2} + \frac{5(14085977+51156233x-5711469x^2+42842610x^3)}{14886061058(6+7x+16x^2+x^3+10x^4)} + \frac{880575\arctan\left(\frac{-1+4x}{\sqrt{23}}\right)}{340783916\sqrt{23}} + \frac{2768835\arctan\left(\frac{3+10x}{\sqrt{31}}\right)}{619080044\sqrt{31}} + \frac{405\log(3-x+2x^2)}{1288408} - \frac{405\log(2+3x+5x^2)}{1288408}$$

input
```
Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(-4342 + 11154*x - 9275*x^2 + 6850*x^3)/(345092*(6 + 7*x + 16*x^2 + x^3 +
10*x^4)^2) + (5*(14085977 + 51156233*x - 5711469*x^2 + 42842610*x^3))/(148
86061058*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)) + (880575*ArcTan[(-1 + 4*x)/Sq
rt[23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619
080044*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5
*x^2])/1288408
```

3.57.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1305, 27, 2135, 27, 2135, 27, 2135, 27, 2141, 27, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^3(5x^2+3x+2)^3}\,dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2} - \frac{\int -\frac{110(-21x^2+40x+41)}{(2x^2-x+3)^2(5x^2+3x+2)^3}dx}{11132}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{506}\int \frac{-21x^2+40x+41}{(2x^2-x+3)^2(5x^2+3x+2)^3}dx+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \textcolor{blue}{2135}$$

$$\frac{5}{506}\left(\frac{\int \frac{22(-1750x^2+17315x+6819)}{(2x^2-x+3)(5x^2+3x+2)^3}dx}{5566}+\frac{2(302-35x)}{253(2x^2-x+3)(5x^2+3x+2)^2}\right)+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{5}{506}\left(\frac{1}{253}\int \frac{-1750x^2+17315x+6819}{(2x^2-x+3)(5x^2+3x+2)^3}dx+\frac{2(302-35x)}{253(2x^2-x+3)(5x^2+3x+2)^2}\right)+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \textcolor{blue}{2135}$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{\int \frac{264(-38510x^2-114479x+45248)}{(2x^2-x+3)(5x^2+3x+2)^2}dx}{15004}-\frac{77020x+223707}{682(5x^2+3x+2)^2}\right)+\frac{2(302-35x)}{253(2x^2-x+3)(5x^2+3x+2)^2}\right)+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\int \frac{-38510x^2-114479x+45248}{(2x^2-x+3)(5x^2+3x+2)^2}dx-\frac{77020x+223707}{682(5x^2+3x+2)^2}\right)+\frac{2(302-35x)}{253(2x^2-x+3)(5x^2+3x+2)^2}\right)+\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \textcolor{blue}{2135}$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{\int \frac{11(14280870x^2-5235733x+5790640)}{(2x^2-x+3)(5x^2+3x+2)}dx}{7502}+\frac{7140435x+2618306}{682(5x^2+3x+2)}\right)-\frac{77020x+223707}{682(5x^2+3x+2)^2}\right)+\frac{}{253(2}\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$$\Big\downarrow \textcolor{blue}{27}$$

3.57. $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\int\frac{14280870x^2-5235733x+5790640}{(2x^2-x+3)(5x^2+3x+2)}dx+\frac{7140435x+2618306}{682(5x^2+3x+2)}\right)-\frac{77020x+223707}{682(5x^2+3x+2)^2}\right)\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$\downarrow$ 2141

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{1}{242}\int\frac{10571(28566x+22211)}{2x^2-x+3}dx+\frac{1}{242}\int\frac{5819(53374-129735x)}{5x^2+3x+2}dx\right)+\frac{7140435x+26}{682(5x^2+3x}\right.\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$\downarrow$ 27

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\int\frac{28566x+22211}{2x^2-x+3}dx+\frac{529}{22}\int\frac{53374-129735x}{5x^2+3x+2}dx\right)+\frac{7140435x+2618306}{682(5x^2+3x+2)}\right)-\frac{77}{682}\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$\downarrow$ 1142

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(\frac{58705}{2}\int\frac{1}{2x^2-x+3}dx+\frac{14283}{2}\int-\frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{184589}{2}\int\frac{1}{5x^2+3x}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$\downarrow$ 25

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(\frac{58705}{2}\int\frac{1}{2x^2-x+3}dx-\frac{14283}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{184589}{2}\int\frac{1}{5x^2+3x+}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$\downarrow$ 1083

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(-\frac{14283}{2}\int\frac{1-4x}{2x^2-x+3}dx-58705\int\frac{1}{-(4x-1)^2-23}d(4x-1)\right)+\frac{529}{22}\left(-\frac{25947}{2}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012(2x^2-x+3)^2(5x^2+3x+2)^2}$$

$\downarrow$ 217

3.57. $\int\frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(\frac{58705\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}-\frac{14283}{2}\int\frac{1-4x}{2x^2-x+3}dx\right)+\frac{529}{22}\left(\frac{184589\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 1103$$

$$\frac{5}{506}\left(\frac{1}{253}\left(\frac{6}{341}\left(\frac{1}{682}\left(\frac{961}{22}\left(\frac{58705\arctan\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{23}}+\frac{14283}{2}\log\left(2x^2-x+3\right)\right)+\frac{529}{22}\left(\frac{184589\arctan\left(\frac{10x+3}{\sqrt{31}}\right)}{\sqrt{31}}\right.\right.\right.\right.\right.$$
$$\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(13 - 6*x)/(1012*(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*((2*(302 - 35
*x))/(253*(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2) + (-1/682*(223707 + 77020*x
)/(2 + 3*x + 5*x^2)^2 + (6*((2618306 + 7140435*x)/(682*(2 + 3*x + 5*x^2))
+ ((961*((58705*ArcTan[(-1 + 4*x)/Sqrt[23]])/Sqrt[23] + (14283*Log[3 - x +
 2*x^2])/2))/22 + (529*((184589*ArcTan[(3 + 10*x)/Sqrt[31]])/Sqrt[31] - (2
5947*Log[2 + 3*x + 5*x^2])/2))/22)/682))/341)/253))/506
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

3.57. $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3} \, dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.57. $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$

3.57.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.65

method	result
default	$-\frac{25\left(-\frac{3013197}{961}x^3-\frac{14516062}{4805}x^2-\frac{51193868}{24025}x-\frac{5423968}{24025}\right)}{2576816(5x^2+3x+2)^2}-\frac{405\ln\left(5x^2+3x+2\right)}{1288408}+\frac{2768835\arctan\left(\frac{(10x+3)\sqrt{31}}{31}\right)\sqrt{31}}{19191481364}+\frac{\frac{302907}{529}x^3-\cdots}{6\cdots}$
risch	$\frac{\frac{1071065250}{7443030529}x^7-\frac{35680200}{7443030529}x^6+\frac{5956663105}{14886061058}x^5+\frac{2002653845}{14886061058}x^4+\frac{5543790435}{14886061058}x^3+\frac{4691822415}{29772122116}x^2+\frac{1254420353}{7443030529}x+\frac{235280627}{14886061058}}{(2x^2-x+3)^2(5x^2+3x+2)^2}+\frac{405\ln(\cdots}{\cdots}$

input
```
int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
-25/2576816*(-3013197/961*x^3-14516062/4805*x^2-51193868/24025*x-5423968/2
4025)/(5*x^2+3*x+2)^2-405/1288408*ln(5*x^2+3*x+2)+2768835/19191481364*arct
an(1/31*(10*x+3)*31^(1/2))*31^(1/2)+1/644204*(302907/529*x^3-368291/529*x^
2+2501587/2116*x-665819/1058)/(2*x^2-x+3)^2+405/1288408*ln(2*x^2-x+3)+8805
75/7838030068*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.64

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx = \frac{67202918046000\,x^7-2238718468800\,x^6+186872434930060\,x^5+62827256425340\,x^4+17391979352682\cdots}{\cdots}$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/467005507511576*(67202918046000*x^7 - 2238718468800*x^6 + 18687243493006
0*x^5 + 62827256425340*x^4 + 173919793526820*x^3 + 67376830890*sqrt(31)*(1
00*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x +
 36)*arctan(1/31*sqrt(31)*(10*x + 3)) + 52466419650*sqrt(23)*(100*x^8 + 20
*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*arctan
(1/23*sqrt(23)*(4*x - 1)) + 73595926401690*x^2 - 146799174285*(100*x^8 + 2
0*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(5
*x^2 + 3*x + 2) + 146799174285*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390
*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(2*x^2 - x + 3) + 78707350628632*
x + 7381223830244)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x
^3 + 241*x^2 + 84*x + 36)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^3}\,dx$$
$$= \frac{4284261000x^7 - 142720800x^6 + 11913326210x^5 + 4005307690x^4 + 11087580870}{2977212211600x^8 + 595442442320x^7 + 9556851199236x^6 + 5120805003952x^5 + 11611127625240x^4 + 70}$$
$$+ \frac{405\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{1288408} - \frac{405\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{1288408}$$
$$+ \frac{880575\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{7838030068} + \frac{2768835\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{19191481364}$$

input
```
integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

output
```
(4284261000*x**7 - 142720800*x**6 + 11913326210*x**5 + 4005307690*x**4 + 1
1087580870*x**3 + 4691822415*x**2 + 5017681412*x + 470561254)/(29772122116
00*x**8 + 595442442320*x**7 + 9556851199236*x**6 + 5120805003952*x**5 + 11
611127625240*x**4 + 7026220819376*x**3 + 7175081429956*x**2 + 250085825774
4*x + 1071796396176) + 405*log(x**2 - x/2 + 3/2)/1288408 - 405*log(x**2 +
3*x/5 + 2/5)/1288408 + 880575*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)
/7838030068 + 2768835*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/1919
1481364
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^3}\,dx = \frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)$$
$$+ \frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)$$
$$+ \frac{4284261000\,x^7 - 142720800\,x^6 + 11913326210\,x^5 + 4005307690\,x^4 + 11087580870\,x^3 + 4691822415\,x^2}{29772122116\,(100\,x^8 + 20\,x^7 + 321\,x^6 + 172\,x^5 + 390\,x^4 + 236\,x^3 + 241\,x^2 + 84}$$
$$- \frac{405}{1288408}\log\left(5\,x^2 + 3\,x + 2\right) + \frac{405}{1288408}\log\left(2\,x^2 - x + 3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/783
8030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(42842610
00*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^
3 + 4691822415*x^2 + 5017681412*x + 470561254)/(100*x^8 + 20*x^7 + 321*x^6
 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) - 405/1288408*log(5*
x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^3}\,dx = \frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)$$
$$+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)$$
$$+\frac{4284261000\,x^7-142720800\,x^6+11913326210\,x^5+4005307690\,x^4+11087580870\,x^3+4691822415\,x^2}{29772122116\left(10\,x^4+x^3+16\,x^2+7\,x+6\right)^2}$$
$$-\frac{405}{1288408}\log\left(5\,x^2+3\,x+2\right)+\frac{405}{1288408}\log\left(2\,x^2-x+3\right)$$

input
```
integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/783
8030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(42842610
00*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^
3 + 4691822415*x^2 + 5017681412*x + 470561254)/(10*x^4 + x^3 + 16*x^2 + 7*
x + 6)^2 - 405/1288408*log(5*x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x +
3)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.86

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx$$
$$= \frac{\frac{21421305\,x^7}{14886061058} - \frac{356802\,x^6}{7443030529} + \frac{1191332621\,x^5}{297721221160} + \frac{400530769\,x^4}{297721221160} + \frac{1108758087\,x^3}{297721221160} + \frac{938364483\,x^2}{595442442320} + \frac{1254420353\,x}{744303052900} + \frac{235280627}{1488606105800}}{x^8 + \frac{x^7}{5} + \frac{321\,x^6}{100} + \frac{43\,x^5}{25} + \frac{39\,x^4}{10} + \frac{59\,x^3}{25} + \frac{241\,x^2}{100} + \frac{21\,x}{25} + \frac{9}{25}}$$
$$+ \ln\left(x - \frac{1}{4} + \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(\frac{405}{1288408} + \frac{\sqrt{23}\,880575\mathrm{i}}{15676060136}\right)$$
$$- \ln\left(x + \frac{3}{10} - \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(\frac{405}{1288408} + \frac{\sqrt{31}\,2768835\mathrm{i}}{38382962728}\right)$$
$$+ \ln\left(x + \frac{3}{10} + \frac{\sqrt{31}\,1\mathrm{i}}{10}\right)\left(-\frac{405}{1288408} + \frac{\sqrt{31}\,2768835\mathrm{i}}{38382962728}\right)$$
$$- \ln\left(x - \frac{1}{4} - \frac{\sqrt{23}\,1\mathrm{i}}{4}\right)\left(-\frac{405}{1288408} + \frac{\sqrt{23}\,880575\mathrm{i}}{15676060136}\right)$$

input
```
int(1/((2*x^2 - x + 3)^3*(3*x + 5*x^2 + 2)^3),x)
```

output
```
log(x + (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 + 405/12884
08) - log(x - (23^(1/2)*1i)/4 - 1/4)*((23^(1/2)*880575i)/15676060136 - 405
/1288408) - log(x - (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)/38382962
728 + 405/1288408) + log(x + (31^(1/2)*1i)/10 + 3/10)*((31^(1/2)*2768835i)
/38382962728 - 405/1288408) + ((1254420353*x)/744303052900 + (938364483*x^
2)/595442442320 + (1108758087*x^3)/297721221160 + (400530769*x^4)/29772122
1160 + (1191332621*x^5)/297721221160 - (356802*x^6)/7443030529 + (21421305
*x^7)/14886061058 + 235280627/1488606105800)/((21*x)/25 + (241*x^2)/100 +
(59*x^3)/25 + (39*x^4)/10 + (43*x^5)/25 + (321*x^6)/100 + x^7/5 + x^8 + 9/
25)
```

3.58 $\int \sqrt{3-x+2x^2}(2+3x+5x^2)^4 \, dx$

3.58.1 Optimal result

Integrand size = 27, antiderivative size = 208

$$\begin{aligned}\int \sqrt{3-x+2x^2}(2+3x+5x^2)^4 \, dx &= -\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541(3-x+2x^2)^{3/2}}{440401920}\\ &+\frac{804243809x(3-x+2x^2)^{3/2}}{36700160}-\frac{83948353x^2(3-x+2x^2)^{3/2}}{2293760}\\ &+\frac{8325631x^3(3-x+2x^2)^{3/2}}{1032192}+\frac{4796405x^4(3-x+2x^2)^{3/2}}{43008}+\frac{233225x^5(3-x+2x^2)^{3/2}}{1536}\\ &+\frac{14125}{144}x^6(3-x+2x^2)^{3/2}+\frac{125}{4}x^7(3-x+2x^2)^{3/2}-\frac{8267844569\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{134217728\sqrt{2}}\end{aligned}$$

output
```
27185733541/440401920*(2*x^2-x+3)^(3/2)+804243809/36700160*x*(2*x^2-x+3)^(
3/2)-83948353/2293760*x^2*(2*x^2-x+3)^(3/2)+8325631/1032192*x^3*(2*x^2-x+3
)^(3/2)+4796405/43008*x^4*(2*x^2-x+3)^(3/2)+233225/1536*x^5*(2*x^2-x+3)^(3
/2)+14125/144*x^6*(2*x^2-x+3)^(3/2)+125/4*x^7*(2*x^2-x+3)^(3/2)-8267844569
/268435456*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-359471503/67108864*(1-4*
x)*(2*x^2-x+3)^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.46

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$$
$$= \frac{4\sqrt{3-x+2x^2}(3801512106459+537752185764x-174418077792x^2+2211683657856x^3+53547419914$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(3801512106459 + 537752185764*x - 174418077792*x^2
+ 2211683657856*x^3 + 5354741991424*x^4 + 7612808028160*x^5 + 772596203520
0*x^6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9) - 26043
71039235*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/84557168640
```

3.58.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^4\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{20}\int \frac{5}{2}\sqrt{2x^2-x+3}\left(14125x^7+13550x^6+18720x^5+14088x^4+7488x^3+3008x^2+768x+128\right)dx+\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \sqrt{2x^2-x+3}\left(14125x^7+13550x^6+18720x^5+14088x^4+7488x^3+3008x^2+768x+128\right)dx+\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{18}\int \frac{3}{2}\sqrt{2x^2-x+3}\left(233225x^6+55140x^5+169056x^4+89856x^3+36096x^2+9216x+1536\right)dx+\frac{14125}{18}(2\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\int \sqrt{2x^2-x+3}\left(233225x^6+55140x^5+169056x^4+89856x^3+36096x^2+9216x+1536\right)dx+\frac{14125}{18}(2x^2\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{16}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(4796405x^5-1586958x^4+2875392x^3+1155072x^2+294912x+49152\right)dx+\frac{2332}{16}\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\int \sqrt{2x^2-x+3}\left(4796405x^5-1586958x^4+2875392x^3+1155072x^2+294912x+49152\right)dx+\frac{23322}{16}\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{14}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(8325631x^4-34602744x^3+32342016x^2+8257536x+1376256\right)dx+\frac{479640}{14}\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\int \sqrt{2x^2-x+3}\left(8325631x^4-34602744x^3+32342016x^2+8257536x+1376256\right)dx+\frac{4796405}{14}(\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{12}\int \frac{9}{2}\sqrt{2x^2-x+3}\left(-83948353x^3+69594114x^2+22020096x+3670016\right)dx+\frac{8325631}{12}(2x^2\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

3.58. $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$

$$\Big\downarrow 27$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\int \sqrt{2x^2-x+3}\left(-83948353x^3+69594114x^2+22020096x+3670016\right)dx+\frac{8325631}{12}\left(2x^2-\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow 2192$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{10}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(804243809x^2+1447782156x+73400320\right)dx-\frac{83948353}{10}x^2\left(2x^2-x\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow 27$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\int \sqrt{2x^2-x+3}\left(804243809x^2+1447782156x+73400320\right)dx-\frac{83948353}{10}x^2\left(2x^2-x+\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow 2192$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{8}\int -\frac{1}{2}(3651057734-27185733541x)\sqrt{2x^2-x+3}dx+\frac{804243809}{8}x\left(2x^2-x+3\right)^{3/2}\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow 27$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{804243809}{8}x\left(2x^2-x+3\right)^{3/2}-\frac{1}{16}\int (3651057734-27185733541x)\sqrt{2x^2-x+3}dx\right)\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow 1160$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{12581502605}{4}\int \sqrt{2x^2-x+3}dx+\frac{27185733541}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{804243809}{8}x\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$$\Big\downarrow 1087$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{12581502605}{4}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{27185733541}{6}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

3.58. $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\, dx$

$\Big\downarrow$ 1090

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{12581502605}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

$\Big\downarrow$ 222

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{28}\left(\frac{3}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{12581502605}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{27185733541}{6}(2\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(125*x^7*(3 - x + 2*x^2)^(3/2))/4 + ((14125*x^6*(3 - x + 2*x^2)^(3/2))/18
+ ((233225*x^5*(3 - x + 2*x^2)^(3/2))/16 + ((4796405*x^4*(3 - x + 2*x^2)^(
3/2))/14 + ((8325631*x^3*(3 - x + 2*x^2)^(3/2))/12 + (3*((-83948353*x^2*(3
 - x + 2*x^2)^(3/2))/10 + ((804243809*x*(3 - x + 2*x^2)^(3/2))/8 + ((27185
733541*(3 - x + 2*x^2)^(3/2))/6 + (12581502605*(-1/8*((1 - 4*x)*Sqrt[3 - x
 + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/4)/16)/20))/
8)/28)/32)/12)/8
```

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.58.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.36

method	result
risch	$\frac{(1321205760000x^9+3486515200000x^8+6327795712000x^7+7725962035200x^6+7612808028160x^5+5354741991424x^4+22116836578}{21139292160}$
trager	$\left(\frac{125}{2}x^9+\frac{11875}{72}x^8+\frac{689675}{2304}x^7+\frac{7859255}{21504}x^6+\frac{185859571}{516096}x^5+\frac{373517159}{1474560}x^4+\frac{5759592859}{55050240}x^3-\frac{259550711}{31457280}x^2+\right.$
default	$\frac{359471503(-1+4x)\sqrt{2x^2-x+3}}{67108864}+\frac{8267844569\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{268435456}+\frac{27185733541\left(2x^2-x+3\right)^{\frac{3}{2}}}{440401920}+\frac{125x^7\left(2x^2-x+3\right)^{\frac{3}{2}}}{4}$

input
```
int((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21139292160*(1321205760000*x^9+3486515200000*x^8+6327795712000*x^7+77259
62035200*x^6+7612808028160*x^5+5354741991424*x^4+2211683657856*x^3-1744180
77792*x^2+537752185764*x+3801512106459)*(2*x^2-x+3)^(1/2)+8267844569/26843
5456*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.47

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$$
$$= \frac{1}{21139292160}\left(1321205760000\,x^9 + 3486515200000\,x^8 + 6327795712000\,x^7 + 7725962035200\,x^6 + 761280\right.$$
$$+ \frac{8267844569}{536870912}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1) - 32x^2 + 16x - 25\right)$$

input
```
integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/21139292160*(1321205760000*x^9 + 3486515200000*x^8 + 6327795712000*x^7 +
 7725962035200*x^6 + 7612808028160*x^5 + 5354741991424*x^4 + 2211683657856
*x^3 - 174418077792*x^2 + 537752185764*x + 3801512106459)*sqrt(2*x^2 - x +
 3) + 8267844569/536870912*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25)
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.47

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{125x^9}{2} + \frac{11875x^8}{72} + \frac{689675x^7}{2304} + \frac{7859255x^6}{21504} + \frac{185859571x^5}{516096} + \frac{373517159x^4}{1474560} + \frac{5759592859x^3}{55050240} - \frac{259550711x^2}{31457280} + \frac{44812682147x}{1761607680} + \frac{422390234051}{2348810240}\right) + \frac{8267844569\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{268435456}$$

input
```
integrate((5*x**2+3*x+2)**4*(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(125*x**9/2 + 11875*x**8/72 + 689675*x**7/2304 + 7859
255*x**6/21504 + 185859571*x**5/516096 + 373517159*x**4/1474560 + 57595928
59*x**3/55050240 - 259550711*x**2/31457280 + 44812682147*x/1761607680 + 42
2390234051/2348810240) + 8267844569*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)
/268435456
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.85

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx = \frac{125}{4}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^7+\frac{14125}{144}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^6 + \frac{233225}{1536}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^5 + \frac{4796405}{43008}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^4 + \frac{8325631}{1032192}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^3 - \frac{83948353}{2293760}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2 + \frac{804243809}{36700160}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x + \frac{27185733541}{440401920}\left(2\,x^2-x+3\right)^{\frac{3}{2}} + \frac{359471503}{16777216}\sqrt{2\,x^2-x+3}x + \frac{8267844569}{268435456}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{359471503}{67108864}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
125/4*(2*x^2 - x + 3)^(3/2)*x^7 + 14125/144*(2*x^2 - x + 3)^(3/2)*x^6 + 23
3225/1536*(2*x^2 - x + 3)^(3/2)*x^5 + 4796405/43008*(2*x^2 - x + 3)^(3/2)*
x^4 + 8325631/1032192*(2*x^2 - x + 3)^(3/2)*x^3 - 83948353/2293760*(2*x^2
- x + 3)^(3/2)*x^2 + 804243809/36700160*(2*x^2 - x + 3)^(3/2)*x + 27185733
541/440401920*(2*x^2 - x + 3)^(3/2) + 359471503/16777216*sqrt(2*x^2 - x +
3)*x + 8267844569/268435456*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 359
471503/67108864*sqrt(2*x^2 - x + 3)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.45

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)^4\,dx$$
$$= \frac{1}{21139292160}\,(4\,(8\,(4\,(16\,(20\,(40\,(140\,(160\,(36\,x+95)x+27587)x+4715553)x+185859571)x+2614620$$
$$-\frac{8267844569}{268435456}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36*x + 95)*x + 27587)*x + 47
15553)*x + 185859571)*x + 2614620113)*x + 17278778577)*x - 5450564931)*x +
 134438046441)*x + 3801512106459)*sqrt(2*x^2 - x + 3) - 8267844569/2684354
56*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx = \frac{8325631\,x^3\,\left(2\,x^2-x+3\right)^{3/2}}{1032192} - \frac{83948353\,x^2\,\left(2\,x^2-x+3\right)^{3/2}}{2293760} + \frac{4796405\,x^4\,\left(2\,x^2-x+3\right)^{3/2}}{43008} + \frac{233225\,x^5\,\left(2\,x^2-x+3\right)^{3/2}}{1536} + \frac{14125\,x^6\,\left(2\,x^2-x+3\right)^{3/2}}{144} + \frac{125\,x^7\,\left(2\,x^2-x+3\right)^{3/2}}{4} - \frac{41987163941\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{1174405120} - \frac{1825528867\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{36700160} + \frac{27185733541\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{7046430720} + \frac{804243809\,x\,\left(2\,x^2-x+3\right)^{3/2}}{36700160} + \frac{625271871443\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{9395240960}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^4,x)
```

output
```
(8325631*x^3*(2*x^2 - x + 3)^(3/2))/1032192 - (83948353*x^2*(2*x^2 - x + 3
)^(3/2))/2293760 + (4796405*x^4*(2*x^2 - x + 3)^(3/2))/43008 + (233225*x^5
*(2*x^2 - x + 3)^(3/2))/1536 + (14125*x^6*(2*x^2 - x + 3)^(3/2))/144 + (12
5*x^7*(2*x^2 - x + 3)^(3/2))/4 - (41987163941*2^(1/2)*log((2*x^2 - x + 3)^
(1/2) + (2^(1/2)*(2*x - 1/2))/2))/1174405120 - (1825528867*(x/2 - 1/8)*(2*
x^2 - x + 3)^(1/2))/36700160 + (27185733541*(2*x^2 - x + 3)^(1/2)*(32*x^2
- 4*x + 45))/7046430720 + (804243809*x*(2*x^2 - x + 3)^(3/2))/36700160 + (
625271871443*2^(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))
/9395240960
```

3.59 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$

3.59.1 Optimal result

Integrand size = 27, antiderivative size = 166

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx &= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}\\&\quad-\frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880}+\frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680}+\frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752}\\&\quad+\frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2}+\frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2}-\frac{155620231\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}\end{aligned}$$

output

```
-22548119/4587520*(2*x^2-x+3)^(3/2)-9627393/1146880*x*(2*x^2-x+3)^(3/2)+53
1681/71680*x^2*(2*x^2-x+3)^(3/2)+247435/10752*x^3*(2*x^2-x+3)^(3/2)+8825/4
48*x^4*(2*x^2-x+3)^(3/2)+125/16*x^5*(2*x^2-x+3)^(3/2)-155620231/8388608*ar
csinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-6766097/2097152*(1-4*x)*(2*x^2-x+3)^(
1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$$
$$=\frac{4\sqrt{3-x+2x^2}(-3957369321-1621307916x+4583812128x^2+9872163456x^3+11212171264x^4+1095\ldots}{88080384\ldots}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-3957369321 - 1621307916*x + 4583812128*x^2 + 9872
163456*x^3 + 11212171264*x^4 + 10958233600*x^5 + 6955008000*x^6 + 34406400
00*x^7) - 16340124255*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/8808
03840
```

3.59.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.519, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^3dx$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{16}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(8825x^5+5370x^4+6624x^3+3648x^2+1152x+256\right)dx+\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\int \sqrt{2x^2-x+3}\left(8825x^5+5370x^4+6624x^3+3648x^2+1152x+256\right)dx+\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{32}\left(\frac{1}{14}\int \frac{1}{2}\sqrt{2x^2-x+3}\left(247435x^4-26328x^3+102144x^2+32256x+7168\right)dx+\frac{8825}{14}\left(2x^2-x+3\right)^{3/2}x^4\right)+\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\int \sqrt{2x^2-x+3}\left(247435x^4-26328x^3+102144x^2+32256x+7168\right)dx+\frac{8825}{14}\left(2x^2-x+3\right)^{3/2}x^4\right)+\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{12}\int \frac{3}{2}\sqrt{2x^2-x+3}\left(531681x^3-667458x^2+258048x+57344\right)dx+\frac{247435}{12}\left(2x^2-x+3\right)^{3/2}x^3\right)+\frac{8}{}\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\int \sqrt{2x^2-x+3}\left(531681x^3-667458x^2+258048x+57344\right)dx+\frac{247435}{12}\left(2x^2-x+3\right)^{3/2}x^3\right)+\frac{882}{14}\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{10}\int \frac{1}{2}\left(-9627393x^2-1219212x+1146880\right)\sqrt{2x^2-x+3}dx+\frac{531681}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{247}{1}\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\int \left(-9627393x^2-1219212x+1146880\right)\sqrt{2x^2-x+3}dx+\frac{531681}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{2474}{12}\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{8}\int \frac{1}{2}(76114438-67644357x)\sqrt{2x^2-x+3}dx-\frac{9627393}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{531681}{10}\left(2x^2-\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

3.59. $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\int(76114438-67644357x)\sqrt{2x^2-x+3}dx-\frac{9627393}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{531681}{10}\left(2x^2-\right.\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \ 1160$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{236813395}{4}\int\sqrt{2x^2-x+3}dx-\frac{22548119}{2}\left(2x^2-x+3\right)^{3/2}\right)-\frac{9627393}{8}x\left(2x^2-x+3\right)^{3/2}\right.\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{236813395}{4}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{22548119}{2}\left(2x^2-x+3\right)^{3}\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{236813395}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{22548}{2}\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

$$\Big\downarrow \ 222$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{236813395}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{22548119}{2}\left(2x^2-x+3\right)^{3/2}\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(125*x^5*(3 - x + 2*x^2)^(3/2))/16 + ((8825*x^4*(3 - x + 2*x^2)^(3/2))/14
+ ((247435*x^3*(3 - x + 2*x^2)^(3/2))/12 + ((531681*x^2*(3 - x + 2*x^2)^(3
/2))/10 + ((-9627393*x*(3 - x + 2*x^2)^(3/2))/8 + ((-22548119*(3 - x + 2*x
^2)^(3/2))/2 + (236813395*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcS
inh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/4)/16)/20)/8)/28)/32
```

3.59. $\int\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$

3.59.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.59.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$\frac{(3440640000x^7+6955008000x^6+10958233600x^5+11212171264x^4+9872163456x^3+4583812128x^2-1621307916x-3957369321)\sqrt{2x}}{220200960}$
trager	$\left(\frac{125}{8}x^7+\frac{7075}{224}x^6+\frac{267535}{5376}x^5+\frac{782099}{15360}x^4+\frac{25708759}{573440}x^3+\frac{6821149}{327680}x^2-\frac{135108993}{18350080}x-\frac{1319123107}{73400320}\right)\sqrt{2x^2-}$
default	$\frac{6766097(-1+4x)\sqrt{2x^2-x+3}}{2097152}+\frac{155620231\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}-\frac{22548119(2x^2-x+3)^{\frac{3}{2}}}{4587520}+\frac{125x^5(2x^2-x+3)^{\frac{3}{2}}}{16}+\frac{882}{}$

input
```
int((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/220200960*(3440640000*x^7+6955008000*x^6+10958233600*x^5+11212171264*x^4
+9872163456*x^3+4583812128*x^2-1621307916*x-3957369321)*(2*x^2-x+3)^(1/2)+
155620231/8388608*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.53

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$$
$$=\frac{1}{220200960}\left(3440640000\,x^7+6955008000\,x^6+10958233600\,x^5+11212171264\,x^4+9872163456\,x^3+458\right.$$
$$+\frac{155620231}{16777216}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/220200960*(3440640000*x^7 + 6955008000*x^6 + 10958233600*x^5 + 112121712
64*x^4 + 9872163456*x^3 + 4583812128*x^2 - 1621307916*x - 3957369321)*sqrt
(2*x^2 - x + 3) + 155620231/16777216*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.50

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)^3\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{125x^7}{8}+\frac{7075x^6}{224}+\frac{267535x^5}{5376}+\frac{782099x^4}{15360}+\frac{25708759x^3}{573440}+\frac{6821149x^2}{327680}-\frac{135108993x}{18350080}-\frac{1319123107}{73400320}\right)+\frac{155620231\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}$$

input
```
integrate((5*x**2+3*x+2)**3*(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(125*x**7/8 + 7075*x**6/224 + 267535*x**5/5376 + 7820
99*x**4/15360 + 25708759*x**3/573440 + 6821149*x**2/327680 - 135108993*x/1
8350080 - 1319123107/73400320) + 155620231*sqrt(2)*asinh(4*sqrt(23)*(x - 1
/4)/23)/8388608
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)^3\,dx = \frac{125}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^5+\frac{8825}{448}\left(2x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{247435}{10752}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{531681}{71680}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{9627393}{1146880}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{22548119}{4587520}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{6766097}{524288}\sqrt{2x^2-x+3}x+\frac{155620231}{8388608}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{6766097}{2097152}\sqrt{2x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
125/16*(2*x^2 - x + 3)^(3/2)*x^5 + 8825/448*(2*x^2 - x + 3)^(3/2)*x^4 + 24
7435/10752*(2*x^2 - x + 3)^(3/2)*x^3 + 531681/71680*(2*x^2 - x + 3)^(3/2)*
x^2 - 9627393/1146880*(2*x^2 - x + 3)^(3/2)*x - 22548119/4587520*(2*x^2 -
x + 3)^(3/2) + 6766097/524288*sqrt(2*x^2 - x + 3)*x + 155620231/8388608*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6766097/2097152*sqrt(2*x^2 - x +
3)
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.50

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3 \, dx$$
$$= \frac{1}{220200960}\,(4\,(8\,(4\,(16\,(100\,(120\,(140\,x+283)x+53507)x+5474693)x+77126277)x+143244129)x-$$
$$-\frac{155620231}{8388608}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/220200960*(4*(8*(4*(16*(100*(120*(140*x + 283)*x + 53507)*x + 5474693)*x
 + 77126277)*x + 143244129)*x - 405326979)*x - 3957369321)*sqrt(2*x^2 - x
+ 3) - 155620231/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
x + 3)) + 1)
```

3.59.9 Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)^3\,dx = \frac{531681\,x^2\,(2\,x^2-x+3)^{3/2}}{71680} + \frac{247435\,x^3\,(2\,x^2-x+3)^{3/2}}{10752} + \frac{8825\,x^4\,(2\,x^2-x+3)^{3/2}}{448} + \frac{125\,x^5\,(2\,x^2-x+3)^{3/2}}{16} + \frac{875316037\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,(2\,x-\frac{1}{2})}{2}\right)}{36700160} + \frac{38057219\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{1146880} - \frac{22548119\,\sqrt{2\,x^2-x+3}\,(32\,x^2-4\,x+45)}{73400320} - \frac{9627393\,x\,(2\,x^2-x+3)^{3/2}}{1146880} - \frac{1555820211\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,(4\,x-1)}{2}\right)}{293601280}$$

input

```
int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3,x)
```

output

```
(531681*x^2*(2*x^2 - x + 3)^(3/2))/71680 + (247435*x^3*(2*x^2 - x + 3)^(3/
2))/10752 + (8825*x^4*(2*x^2 - x + 3)^(3/2))/448 + (125*x^5*(2*x^2 - x + 3
)^(3/2))/16 + (875316037*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x
 - 1/2))/2))/36700160 + (38057219*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/11468
80 - (22548119*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/73400320 - (9627
393*x*(2*x^2 - x + 3)^(3/2))/1146880 - (1555820211*2^(1/2)*log(2*(2*x^2 -
x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/293601280
```

3.60 $\int \sqrt{3-x+2x^2}(2+3x+5x^2)^2\,dx$

3.60.1 Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)^2\,dx = \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384}-\frac{2107(3-x+2x^2)^{3/2}}{3072}+\frac{769}{256}x(3-x+2x^2)^{3/2}+\frac{63}{16}x^2(3-x+2x^2)^{3/2}+\frac{25}{12}x^3(3-x+2x^2)^{3/2}+\frac{284533\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}$$

output
```
-2107/3072*(2*x^2-x+3)^(3/2)+769/256*x*(2*x^2-x+3)^(3/2)+63/16*x^2*(2*x^2-
x+3)^(3/2)+25/12*x^3*(2*x^2-x+3)^(3/2)+284533/65536*arcsinh(1/23*(1-4*x)*2
3^(1/2))*2^(1/2)+12371/16384*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)^2\,dx = \frac{4\sqrt{3-x+2x^2}(-64023+328204x+365536x^2+408960x^3+284672x^4+204800x^5)+853599\sqrt{2}\log(1}{196608}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-64023 + 328204*x + 365536*x^2 + 408960*x^3 + 2846
72*x^4 + 204800*x^5) + 853599*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2
]])/196608
```

3.60.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2 dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{12}\int \frac{3}{2}\sqrt{2x^2-x+3}\left(315x^3+82x^2+96x+32\right)dx+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \sqrt{2x^2-x+3}\left(315x^3+82x^2+96x+32\right)dx+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{10}\int \frac{5}{2}\sqrt{2x^2-x+3}\left(769x^2-372x+128\right)dx+\frac{63}{2}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{4}\int \sqrt{2x^2-x+3}\left(769x^2-372x+128\right)dx+\frac{63}{2}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\int -\frac{1}{2}(2107x+2566)\sqrt{2x^2-x+3}dx+\frac{769}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{63}{2}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{769}{8}x\left(2x^2-x+3\right)^{3/2}-\frac{1}{16}\int(2107x+2566)\sqrt{2x^2-x+3}dx\right)+\frac{63}{2}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{16}\left(-\frac{12371}{4}\int\sqrt{2x^2-x+3}dx-\frac{2107}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{769}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{63}{2}\left(2x^2-x+3\right)^{3\cdots}\right.$$
$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{16}\left(-\frac{12371}{4}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{2107}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{769}{8}x\left(2x^2\cdots\right.\right.\right.$$
$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{16}\left(-\frac{12371}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{2107}{6}\left(2x^2-x+3\right)\cdots\right.\right.\right.$$
$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{16}\left(-\frac{12371}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{2107}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{769}{8}x\left(2x^2-x\cdots\right.\right.\right.$$
$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(25*x^3*(3 - x + 2*x^2)^(3/2))/12 + ((63*x^2*(3 - x + 2*x^2)^(3/2))/2 + ((
769*x*(3 - x + 2*x^2)^(3/2))/8 + ((-2107*(3 - x + 2*x^2)^(3/2))/6 - (12371
*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(16*Sqrt[2])))/4)/16)/4)/8
```

3.60. $\int\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx$

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.60.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023)\sqrt{2x^2-x+3}}{49152} - \frac{284533\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$
trager	$\left(\frac{25}{6}x^5+\frac{139}{24}x^4+\frac{1065}{128}x^3+\frac{11423}{1536}x^2+\frac{82051}{12288}x-\frac{21341}{16384}\right)\sqrt{2x^2-x+3}+\frac{284533\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{Root}\right.}{}$
default	$-\frac{12371(-1+4x)\sqrt{2x^2-x+3}}{16384} - \frac{284533\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536} - \frac{2107(2x^2-x+3)^{\frac{3}{2}}}{3072} + \frac{25x^3(2x^2-x+3)^{\frac{3}{2}}}{12} + \frac{63x^2(2x^2-}{16}$

input
```
int((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/49152*(204800*x^5+284672*x^4+408960*x^3+365536*x^2+328204*x-64023)*(2*x^
2-x+3)^(1/2)-284533/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx$$
$$= \frac{1}{49152}\left(204800\,x^5+284672\,x^4+408960\,x^3+365536\,x^2+328204\,x-64023\right)\sqrt{2\,x^2-x+3} + \frac{284533}{131072}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/49152*(204800*x^5 + 284672*x^4 + 408960*x^3 + 365536*x^2 + 328204*x - 64
023)*sqrt(2*x^2 - x + 3) + 284533/131072*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2
- x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{25x^5}{6}+\frac{139x^4}{24}+\frac{1065x^3}{128}+\frac{11423x^2}{1536}+\frac{82051x}{12288}-\frac{21341}{16384}\right)-\frac{284533\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$$

input
```
integrate((5*x**2+3*x+2)**2*(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(25*x**5/6 + 139*x**4/24 + 1065*x**3/128 + 11423*x**2
/1536 + 82051*x/12288 - 21341/16384) - 284533*sqrt(2)*asinh(4*sqrt(23)*(x
- 1/4)/23)/65536
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx &= \frac{25}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2 \\ &\quad+\frac{769}{256}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{2107}{3072}\left(2x^2-x+3\right)^{\frac{3}{2}} \\ &\quad-\frac{12371}{4096}\sqrt{2x^2-x+3}x \\ &\quad-\frac{284533}{65536}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) \\ &\quad+\frac{12371}{16384}\sqrt{2x^2-x+3}\end{aligned}$$

input
```
integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
25/12*(2*x^2 - x + 3)^(3/2)*x^3 + 63/16*(2*x^2 - x + 3)^(3/2)*x^2 + 769/25
6*(2*x^2 - x + 3)^(3/2)*x - 2107/3072*(2*x^2 - x + 3)^(3/2) - 12371/4096*s
qrt(2*x^2 - x + 3)*x - 284533/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1
)) + 12371/16384*sqrt(2*x^2 - x + 3)
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx$$
$$=\frac{1}{49152}\left(4\left(8\left(4\left(16\left(100\,x+139\right)x+3195\right)x+11423\right)x+82051\right)x-64023\right)\sqrt{2\,x^2-x+3}$$
$$+\frac{284533}{65536}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/49152*(4*(8*(4*(16*(100*x + 139)*x + 3195)*x + 11423)*x + 82051)*x - 640
23)*sqrt(2*x^2 - x + 3) + 284533/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3)) + 1)
```

3.60.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx=\frac{63\,x^2\left(2\,x^2-x+3\right)^{3/2}}{16}+\frac{25\,x^3\left(2\,x^2-x+3\right)^{3/2}}{12}$$
$$-\frac{29509\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{8192}$$
$$-\frac{1283\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2\,x^2-x+3}}{256}$$
$$-\frac{2107\sqrt{2\,x^2-x+3}\left(32\,x^2-4\,x+45\right)}{49152}$$
$$+\frac{769\,x\left(2\,x^2-x+3\right)^{3/2}}{256}$$
$$-\frac{48461\sqrt{2}\,\ln\left(2\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,(4\,x-1)}{2}\right)}{65536}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2,x)
```

output
```
(63*x^2*(2*x^2 - x + 3)^(3/2))/16 + (25*x^3*(2*x^2 - x + 3)^(3/2))/12 - (2
9509*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/8192 -
(1283*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/256 - (2107*(2*x^2 - x + 3)^(1/2)
*(32*x^2 - 4*x + 45))/49152 + (769*x*(2*x^2 - x + 3)^(3/2))/256 - (48461*2
^(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/65536
```

3.60. $\int \sqrt{3-x+2x^2}(2+3x+5x^2)^2\,dx$

3.61 $\int \sqrt{3-x+2x^2}(2+3x+5x^2)\,dx$

3.61.1 Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)\,dx = -\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}(3-x+2x^2)^{3/2}+\frac{5}{8}x(3-x+2x^2)^{3/2}-\frac{1863\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

output
```
73/96*(2*x^2-x+3)^(3/2)+5/8*x*(2*x^2-x+3)^(3/2)-1863/2048*arcsinh(1/23*(1-
4*x)*23^(1/2))*2^(1/2)-81/512*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)\,dx = \frac{4\sqrt{3-x+2x^2}(3261+2684x+1376x^2+1920x^3)-5589\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{6144}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(3261 + 2684*x + 1376*x^2 + 1920*x^3) - 5589*Sqrt[2
]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/6144
```

3.61.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right)\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\int \frac{1}{2}(73x+2)\sqrt{2x^2-x+3}dx+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int (73x+2)\sqrt{2x^2-x+3}dx+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{16}\left(\frac{81}{4}\int \sqrt{2x^2-x+3}dx+\frac{73}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{16}\left(\frac{81}{4}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{73}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{16}\left(\frac{81}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{73}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{16}\left(\frac{81}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{73}{6}\left(2x^2-x+3\right)^{3/2}\right)+\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

output
```
(5*x*(3 - x + 2*x^2)^(3/2))/8 + ((73*(3 - x + 2*x^2)^(3/2))/6 + (81*(-1/8*
((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sq
rt[2])))/4)/16
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.61.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result
risch	$\frac{(1920x^3+1376x^2+2684x+3261)\sqrt{2x^2-x+3}}{1536}+\frac{1863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{2048}$
default	$\frac{81(-1+4x)\sqrt{2x^2-x+3}}{512}+\frac{1863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{2048}+\frac{73\left(2x^2-x+3\right)^{\frac{3}{2}}}{96}+\frac{5x\left(2x^2-x+3\right)^{\frac{3}{2}}}{8}$
trager	$\left(\frac{5}{4}x^3+\frac{43}{48}x^2+\frac{671}{384}x+\frac{1087}{512}\right)\sqrt{2x^2-x+3}+\frac{1863\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\text{Roo}\right.}{2048}$

input
```
int((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(1920*x^3+1376*x^2+2684*x+3261)*(2*x^2-x+3)^(1/2)+1863/2048*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx = \frac{1}{1536}\left(1920\,x^3+1376\,x^2+2684\,x+3261\right)\sqrt{2\,x^2-x+3} + \frac{1863}{4096}\,\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/1536*(1920*x^3 + 1376*x^2 + 2684*x + 3261)*sqrt(2*x^2 - x + 3) + 1863/40
96*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x -
25)
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{5x^3}{4}+\frac{43x^2}{48}+\frac{671x}{384}+\frac{1087}{512}\right)+\frac{1863\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}$$

input
```
integrate((5*x**2+3*x+2)*(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**3/4 + 43*x**2/48 + 671*x/384 + 1087/512) + 1863
*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/2048
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \sqrt{3-x+2x^2}(2+3x+5x^2)\,dx = \frac{5}{8}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{73}{96}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{81}{128}\sqrt{2x^2-x+3}x+\frac{1863}{2048}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{81}{512}\sqrt{2x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
5/8*(2*x^2 - x + 3)^(3/2)*x + 73/96*(2*x^2 - x + 3)^(3/2) + 81/128*sqrt(2*
x^2 - x + 3)*x + 1863/2048*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 81/5
12*sqrt(2*x^2 - x + 3)
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx = \frac{1}{1536}\,(4\,(8\,(60\,x+43)x+671)x+3261)\sqrt{2\,x^2-x+3} - \frac{1863}{2048}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/1536*(4*(8*(60*x + 43)*x + 671)*x + 3261)*sqrt(2*x^2 - x + 3) - 1863/204
8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.61.9 Mupad [B] (verification not implemented)

Time = 13.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx = \frac{23\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{256} + \frac{\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{8} + \frac{73\,\sqrt{2\,x^2-x+3}\,(32\,x^2-4\,x+45)}{1536} + \frac{5\,x\,{(2\,x^2-x+3)}^{3/2}}{8} + \frac{1679\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,(4\,x-1)}{2}\right)}{2048}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2),x)
```

output
```
(23*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/256 + ((
x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/8 + (73*(2*x^2 - x + 3)^(1/2)*(32*x^2 -
4*x + 45))/1536 + (5*x*(2*x^2 - x + 3)^(3/2))/8 + (1679*2^(1/2)*log(2*(2*x
^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/2048
```

3.62 $\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$

3.62.1 Optimal result

Integrand size = 27, antiderivative size = 174

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx &= -\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right) \\ &\quad+\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(6+7\sqrt{2}+\left(20+13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) \\ &\quad-\frac{1}{5}\sqrt{\frac{11}{31}\left(-13+10\sqrt{2}\right)}\text{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(-13+10\sqrt{2}\right)}}\left(6-7\sqrt{2}+\left(20-13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)\end{aligned}$$

output
```
-1/5*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/155*arctanh(1/62*(6+x*(20-13
*2^(1/2))-7*2^(1/2))*682^(1/2)/(-13+10*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(
-4433+3410*2^(1/2))^(1/2)+1/155*arctan(1/62*(6+7*2^(1/2)+x*(20+13*2^(1/2))
)*682^(1/2)/(13+10*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(4433+3410*2^(1/2))^(
1/2)
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$$
$$=\frac{1}{5}\left(-\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)+11\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3\right.\right.$$
$$-5\#1^4\&,\frac{-2\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+2\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1+\log\left(-\right.}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

output
```
(-(Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]) + 11*RootSum[-56 - 26*S
qrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2*Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1] + 2*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1]*#1 + Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[
2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/5
```

3.62.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1320, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}}{5x^2+3x+2}dx$$
$$\Big\downarrow \text{1320}$$
$$\frac{2}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{5}\int -\frac{11(1-x)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{2}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{11}{5}\int \frac{1-x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx$$

$$\Big\downarrow \; 1090$$

$$\frac{11}{5}\int \frac{1-x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{1}{5}\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)$$

$$\Big\downarrow \; 222$$

$$\frac{11}{5}\int \frac{1-x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 1368$$

$$\frac{11}{5}\left(\frac{\int -\frac{11\left(\sqrt{2}x-\sqrt{2}+2\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\sqrt{2}x+\sqrt{2}+2\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{11}{5}\left(\frac{\int \frac{-\sqrt{2}x+\sqrt{2}+2}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}x-\sqrt{2}+2}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 1362$$

$$\frac{11}{5}\left(\sqrt{2}\left(13-10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6\right)^2}{2x^2-x+3}-62\left(13-10\sqrt{2}\right)}d\frac{\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6}{\sqrt{2x^2-x+3}}-\sqrt{2}\left(13+10\sqrt{\right.\right.$$

$$\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{11}{5}\left(\sqrt{2}\left(13-10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6\right)^2}{2x^2-x+3}-62\left(13-10\sqrt{2}\right)}d\frac{\left(20-13\sqrt{2}\right)x-7\sqrt{2}+6}{\sqrt{2x^2-x+3}}+\sqrt{\frac{1}{341}\left(13+1\right.}\right.$$

$$\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)$$

$$\Big\downarrow \; 219$$

3.62. $\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$

$$\frac{1}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+$$

$$\frac{11}{5}\left(\sqrt{\frac{1}{341}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(13-10\sqrt{2}\right)\text{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(10\sqrt{\right.}}}}{}\right.}{\sqrt{341\left(10\sqrt{\right.}}}\right.$$

input
```
Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

output
```
(Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/5 + (11*(Sqrt[(13 + 10*Sqrt[2])/341
]*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2
])*x))/Sqrt[3 - x + 2*x^2]] + ((13 - 10*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-13
 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/Sqrt[341*(-13 + 10*Sqrt[2])]))/5
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1320
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f
  Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.62.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.40 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.78

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\operatorname{RootOf}\left(\_Z^2-2\right)\right)}{5}-\frac{\operatorname{RootOf}\left(\_Z^2+24025\operatorname{RootOf}\left(24025\_Z^4+\right.\right.}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/5*RootOf(_Z^2-2)*ln(4*RootOf(_Z^2-2)*x+4*(2*x^2-x+3)^(1/2)-RootOf(_Z^2-2
))-1/155*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)*ln(-(5
194205*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)*RootOf(2
4025*_Z^4+4433*_Z^2+242)^4*x+446710*RootOf(24025*_Z^4+4433*_Z^2+242)^2*Roo
tOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)*x+66745294*(2*x^2-
x+3)^(1/2)*RootOf(24025*_Z^4+4433*_Z^2+242)^2-641080*RootOf(24025*_Z^4+443
3*_Z^2+242)^2*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)-3
8115*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)^2+4433)*x+6024106*
(2*x^2-x+3)^(1/2)+33880*RootOf(_Z^2+24025*RootOf(24025*_Z^4+4433*_Z^2+242)
^2+4433))/(775*x*RootOf(24025*_Z^4+4433*_Z^2+242)^2+55*x-22))-RootOf(24025
*_Z^4+4433*_Z^2+242)*ln(-(649275625*x*RootOf(24025*_Z^4+4433*_Z^2+242)^5+1
83764900*RootOf(24025*_Z^4+4433*_Z^2+242)^3*x+80135000*RootOf(24025*_Z^4+4
433*_Z^2+242)^3-53826850*(2*x^2-x+3)^(1/2)*RootOf(24025*_Z^4+4433*_Z^2+242
)^2+7037844*RootOf(24025*_Z^4+4433*_Z^2+242)*x+19021200*RootOf(24025*_Z^4+
4433*_Z^2+242)-5073772*(2*x^2-x+3)^(1/2))/(775*x*RootOf(24025*_Z^4+4433*_Z
^2+242)^2+88*x+22))
```

3.62.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$$
$$= \frac{1}{620}\sqrt{31}\sqrt{22i\sqrt{31}-286}\log\left(-\frac{\sqrt{2x^2-x+3}(\sqrt{31}-3i)\sqrt{22i\sqrt{31}-286}+5\sqrt{31}(-ix+6i)-95x+}{x}\right.$$
$$-\frac{1}{620}\sqrt{31}\sqrt{22i\sqrt{31}-286}\log\left(\frac{\sqrt{2x^2-x+3}(\sqrt{31}-3i)\sqrt{22i\sqrt{31}-286}-5\sqrt{31}(-ix+6i)+95x-}{x}\right.$$
$$+\frac{1}{620}\sqrt{31}\sqrt{-22i\sqrt{31}-286}\log\left(-\frac{\sqrt{2x^2-x+3}(\sqrt{31}+3i)\sqrt{-22i\sqrt{31}-286}+5\sqrt{31}(ix-6i)-95}{x}\right.$$
$$-\frac{1}{620}\sqrt{31}\sqrt{-22i\sqrt{31}-286}\log\left(\frac{\sqrt{2x^2-x+3}(\sqrt{31}+3i)\sqrt{-22i\sqrt{31}-286}-5\sqrt{31}(ix-6i)+95x}{x}\right.$$
$$+\frac{1}{10}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/620*sqrt(31)*sqrt(22*I*sqrt(31) - 286)*log(-(sqrt(2*x^2 - x + 3)*(sqrt(3
1) - 3*I)*sqrt(22*I*sqrt(31) - 286) + 5*sqrt(31)*(-I*x + 6*I) - 95*x + 110
)/x) - 1/620*sqrt(31)*sqrt(22*I*sqrt(31) - 286)*log((sqrt(2*x^2 - x + 3)*(
sqrt(31) - 3*I)*sqrt(22*I*sqrt(31) - 286) - 5*sqrt(31)*(-I*x + 6*I) + 95*x
 - 110)/x) + 1/620*sqrt(31)*sqrt(-22*I*sqrt(31) - 286)*log(-(sqrt(2*x^2 -
x + 3)*(sqrt(31) + 3*I)*sqrt(-22*I*sqrt(31) - 286) + 5*sqrt(31)*(I*x - 6*I
) - 95*x + 110)/x) - 1/620*sqrt(31)*sqrt(-22*I*sqrt(31) - 286)*log((sqrt(2
*x^2 - x + 3)*(sqrt(31) + 3*I)*sqrt(-22*I*sqrt(31) - 286) - 5*sqrt(31)*(I*
x - 6*I) + 95*x - 110)/x) + 1/10*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3
)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.62.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \int \frac{\sqrt{2x^2-x+3}}{5x^2+3x+2}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2),x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2), x)
```

3.62.7 Maxima [F]

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \int \frac{\sqrt{2\,x^2-x+3}}{5\,x^2+3\,x+2}\,dx$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2), x)
```

3.62.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx = \int \frac{\sqrt{2\,x^2-x+3}}{5\,x^2+3\,x+2}\,dx$$

input
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2), x)
```

3.63 $\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx$

3.63.1 Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{31(2+3x+5x^2)} + \frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(419+277\sqrt{2}+\left(973+696\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \frac{1}{62}\sqrt{\frac{1}{682}\left(-70517+49942\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-70517+49942\sqrt{2}\right)}}\left(419-277\sqrt{2}+\left(973-696\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
1/31*(3+10*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/42284*arctanh(1/31*(419+x*
(973-696*2^(1/2))-277*2^(1/2))*341^(1/2)/(-70517+49942*2^(1/2))^(1/2)/(2*x
^2-x+3)^(1/2))*(-48092594+34060444*2^(1/2))^(1/2)+1/42284*arctan(1/31*(419
+277*2^(1/2)+x*(973+696*2^(1/2)))*341^(1/2)/(70517+49942*2^(1/2))^(1/2)/(2
*x^2-x+3)^(1/2))*(48092594+34060444*2^(1/2))^(1/2)
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.49 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx$$
$$= \frac{\frac{50(3+10x)\sqrt{3-x+2x^2}}{2+3x+5x^2} - 6151\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\right.}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-}\right.}{}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((50*(3 + 10*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) - 6151*RootSum[-56
- 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) +
 Sqrt[3 - x + 2*x^2] - #1]/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3
) & ] + 124*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^
4 & , (49*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2
]*#1^2 - 10*#1^3) & ] - 10*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[
2]*#1^3 - 5*#1^4 & , (191*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1]*#1 + 55*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(13*Sqrt[2
] - 17*#1 - 9*Sqrt[2]*#1^2 + 10*#1^3) & ])/1550
```

3.63.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1302, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}}{(5x^2+3x+2)^2}dx$$
$$\Big\downarrow \text{1302}$$
$$\frac{(10x+3)\sqrt{2x^2-x+3}}{31(5x^2+3x+2)} - \frac{1}{31}\int -\frac{63-22x}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{62}\int \frac{63-22x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{\sqrt{2x^2-x+3}(10x+3)}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \ 1368$$

$$\frac{1}{62}\left(\frac{\int -\frac{11\left(-\left(\left(41-22\sqrt{2}\right)x\right)-63\sqrt{2}+85\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(41+22\sqrt{2}\right)x\right)+63\sqrt{2}+85\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{62}\left(\frac{\int \frac{-\left(\left(41+22\sqrt{2}\right)x\right)+63\sqrt{2}+85}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(41-22\sqrt{2}\right)x\right)-63\sqrt{2}+85}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(10x+3)}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \ 1362$$

$$\frac{1}{62}\left(\frac{(70517-49942\sqrt{2})\int \frac{1}{-\frac{11\left(\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419\right)^2}{2x^2-x+3}-31\left(70517-49942\sqrt{2}\right)}d\frac{\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{(70517+499\ldots}{}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{62}\left(\frac{(70517-49942\sqrt{2})\int \frac{1}{-\frac{11\left(\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419\right)^2}{2x^2-x+3}-31\left(70517-49942\sqrt{2}\right)}d\frac{\left(973-696\sqrt{2}\right)x-277\sqrt{2}+419}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}\left(7051\ldots\right.}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31\,(5x^2+3x+2)}$$

$$\Big\downarrow \ 219$$

3.63. $\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx$

$$\frac{1}{62}\left(\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\sqrt{2}+419\right)}{\sqrt{2x^2-x+3}}\right)+\frac{(70517-49\ldots}{\ldots}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}$$

input
```
Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 4
9942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))]*(419 + 27
7*Sqrt[2] + (973 + 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((70517 - 49942
*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))]*(419 - 277*Sqrt[
2] + (973 - 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-70517 + 4994
2*Sqrt[2])])/62
```

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.63.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.92 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.52

method	result
trager	$\frac{(10x+3)\sqrt{2x^2-x+3}}{155x^2+93x+62} - \frac{2\,\mathrm{RootOf}\left(59535872\_Z^4+384740752\_Z^2+623550841\right)\ln\left(-\frac{150744827904x\,\mathrm{RootOf}\left(59535872\_Z^4+384740\right.}{}\right.}{}$
risch	$\frac{(10x+3)\sqrt{2x^2-x+3}}{155x^2+93x+62} + \frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(26569\sqrt{-775687+549362\sqrt{2}}\,\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\arctan\right.}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/31*(10*x+3)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-2/31*RootOf(59535872*_Z^4+38
4740752*_Z^2+623550841)*ln(-(150744827904*x*RootOf(59535872*_Z^4+384740752
*_Z^2+623550841)^5+232524550016*RootOf(59535872*_Z^4+384740752*_Z^2+623550
841)^3*x+2424286162144*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)^2*(2
*x^2-x+3)^(1/2)-85650470400*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)
^3-988525310334*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)*x+781092138
3613*(2*x^2-x+3)^(1/2)+163005849200*RootOf(59535872*_Z^4+384740752*_Z^2+62
3550841))/(10912*x*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)^2+34189*
x-1426))-1/42284*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384740752*_Z^2+6
23550841)^2+48092594)*ln((-18843103488*RootOf(_Z^2+7441984*RootOf(59535872
*_Z^4+384740752*_Z^2+623550841)^2+48092594)*RootOf(59535872*_Z^4+384740752
*_Z^2+623550841)^4*x-214475327264*RootOf(59535872*_Z^4+384740752*_Z^2+6235
50841)^2*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384740752*_Z^2+623550841
)^2+48092594)*x+826681581291104*RootOf(59535872*_Z^4+384740752*_Z^2+623550
841)^2*(2*x^2-x+3)^(1/2)-10706308800*RootOf(59535872*_Z^4+384740752*_Z^2+6
23550841)^2*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384740752*_Z^2+623550
841)^2+48092594)-475524326173*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384
740752*_Z^2+623550841)^2+48092594)*x+2678769161185106*(2*x^2-x+3)^(1/2)-89
563485700*RootOf(_Z^2+7441984*RootOf(59535872*_Z^4+384740752*_Z^2+62355084
1)^2+48092594))/(5456*x*RootOf(59535872*_Z^4+384740752*_Z^2+623550841)^...
```

3.63. $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$

3.63.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx =$$

$$-\frac{\sqrt{341}(5\,x^2+3\,x+2)\sqrt{713i\,\sqrt{31}-70517}\log\left(\frac{\sqrt{341}\sqrt{2\,x^2-x+3}\sqrt{713i\,\sqrt{31}-70517}\left(419i\,\sqrt{31}+2635\right)-774101\,\sqrt{31}(i\,x-6i}{x}\right.}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/84568*(sqrt(341)*(5*x^2 + 3*x + 2)*sqrt(713*I*sqrt(31) - 70517)*log((sq
rt(341)*sqrt(2*x^2 - x + 3)*sqrt(713*I*sqrt(31) - 70517)*(419*I*sqrt(31) +
 2635) - 774101*sqrt(31)*(I*x - 6*I) + 14707919*x - 17030222)/x) - sqrt(34
1)*(5*x^2 + 3*x + 2)*sqrt(713*I*sqrt(31) - 70517)*log((sqrt(341)*sqrt(2*x^
2 - x + 3)*sqrt(713*I*sqrt(31) - 70517)*(-419*I*sqrt(31) - 2635) - 774101*
sqrt(31)*(I*x - 6*I) + 14707919*x - 17030222)/x) - sqrt(341)*(5*x^2 + 3*x
+ 2)*sqrt(-713*I*sqrt(31) - 70517)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(419
*I*sqrt(31) - 2635)*sqrt(-713*I*sqrt(31) - 70517) - 774101*sqrt(31)*(-I*x
+ 6*I) + 14707919*x - 17030222)/x) + sqrt(341)*(5*x^2 + 3*x + 2)*sqrt(-713
*I*sqrt(31) - 70517)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(-419*I*sqrt(31) +
 2635)*sqrt(-713*I*sqrt(31) - 70517) - 774101*sqrt(31)*(-I*x + 6*I) + 1470
7919*x - 17030222)/x) - 2728*sqrt(2*x^2 - x + 3)*(10*x + 3))/(5*x^2 + 3*x
+ 2)
```

3.63.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx = \int \frac{\sqrt{2x^2-x+3}}{(5x^2+3x+2)^2}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**2, x)
```

3.63.7 Maxima [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx=\int \frac{\sqrt{2\,x^2-x+3}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^2, x)
```

3.63.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^2}\,dx=\int \frac{\sqrt{2\,x^2-x+3}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^2, x)
```

3.64 $\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx$

3.64.1 Optimal result

Integrand size = 27, antiderivative size = 223

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{62\,(2+3x+5x^2)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\,(2+3x+5x^2)}$$

$$+ \frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(112285869463+79399380740\sqrt{2}\right)}}\left(509587+362788\sqrt{2}+\left(1235163+87\ldots\right.\right.}{\sqrt{3-x+2x^2}}\right.}{169136}$$

$$- \frac{\sqrt{\frac{1}{682}\left(-112285869463+79399380740\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-112285869463+79399380740\sqrt{2}\right)}}\left(509587-362788\sqrt{2}+\left(123516\ldots\right.\right.}{\sqrt{3-x+2x^2}}\right.}{169136}$$

output
```
1/62*(3+10*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2+1/84568*(3464+13665*x)*(2*
x^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/115350752*arctanh(1/31*(509587+x*(1235163-8
72375*2^(1/2))-362788*2^(1/2))*341^(1/2)/(-112285869463+79399380740*2^(1/2
))^(1/2)/(2*x^2-x+3)^(1/2))*(-76578962973766+54150377664680*2^(1/2))^(1/2)
+1/115350752*arctan(1/31*(509587+362788*2^(1/2)+x*(1235163+872375*2^(1/2))
)*341^(1/2)/(112285869463+79399380740*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(7
6578962973766+54150377664680*2^(1/2))^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx = \frac{\frac{661250\sqrt{3-x+2x^2}(11020+51362x+58315x^2+68325x^3)}{(2+3x+5x^2)^2} + \text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{-5}{}\right.}{}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((661250*Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3))/(2
 + 3*x + 5*x^2)^2 + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3
 - 5*#1^4 & , (-537295920831*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]
+ 120146195680*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 4
5923442075*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2]
 + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 248*RootSum[-56 - 26*Sqrt[2]*#1
 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-2139373897*Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1] + 277937160*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1]*#1 - 228643025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1
]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/55920590000
```

3.64.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1302, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}}{(5x^2+3x+2)^3}\,dx$$

$$\Big\downarrow \ 1302$$

$$\frac{(10x+3)\sqrt{2x^2-x+3}}{62\,(5x^2+3x+2)^2} - \frac{1}{62}\int -\frac{80x^2-62x+183}{2\sqrt{2x^2-x+3}\,(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{124}\int \frac{80x^2-62x+183}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}dx+\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 2135$$

$$\frac{1}{124}\left(\frac{\int \frac{11(77456-32605x)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{7502}+\frac{\sqrt{2x^2-x+3}(13665x+3464)}{682\left(5x^2+3x+2\right)}\right)+\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{124}\left(\frac{\int \frac{77456-32605x}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{1364}+\frac{\sqrt{2x^2-x+3}(13665x+3464)}{682\left(5x^2+3x+2\right)}\right)+\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 1368$$

$$\frac{1}{124}\left(\frac{\frac{\int -\frac{11\left(-\left(\left(44851-32605\sqrt{2}\right)x\right)-77456\sqrt{2}+110061\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(44851+32605\sqrt{2}\right)x\right)+77456\sqrt{2}+110061\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}}{1364}+\frac{\sqrt{2x^2-x+3}(13665x \cdots}{682\left(5x^2+3x+\cdots\right.}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{124}\left(\frac{\frac{\int \frac{-\left(\left(44851+32605\sqrt{2}\right)x\right)+77456\sqrt{2}+110061}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(44851-32605\sqrt{2}\right)x\right)-77456\sqrt{2}+110061}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}}{1364}+\frac{\sqrt{2x^2-x+3}(13665x+3464)}{682\left(5x^2+3x+2\right)}\right)$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \ 1362$$

$$\frac{1}{124}\left(\frac{\frac{\left(112285869463-79399380740\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(1235163-872375\sqrt{2}\right)x-362788\sqrt{2}+509587\right)^2}{2x^2-x+3}-31\left(112285869463-79399380740\sqrt{2}\right)}d\frac{\left(1235163-872375\sqrt{2}\cdots\right.}{\sqrt{2x\cdots}}\cdots}{\sqrt{2}}\cdots}{\cdots}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

3.64. $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx$

$\downarrow$ 217

$$\frac{1}{124}\left(\frac{\frac{\left(112285869463-79399380740\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(1235163-872375\sqrt{2}\right)x-362788\sqrt{2}+509587\right)^2}{2x^2-x+3}-31\left(112285869463-79399380740\sqrt{2}\right)}d\frac{\left(1235163-872375\sqrt{2}\right.}{\sqrt{2x}}}{\sqrt{2}}}{}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 219

$$\frac{1}{124}\left(\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(112285869463+79399380740\sqrt{2}\right)}}\left(\left(1235163+872375\sqrt{2}\right)x+362788\sqrt{2}\right.}{\sqrt{2x^2-x+3}}\right.}{}\right.$$

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}$$

input
```
Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + (((3464 + 1366
5*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 +
79399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*
Sqrt[2]))]*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x))/Sqrt[
3 - x + 2*x^2]] + ((112285869463 - 79399380740*Sqrt[2])*ArcTanh[(Sqrt[11/(
31*(-112285869463 + 79399380740*Sqrt[2]))]*(509587 - 362788*Sqrt[2] + (123
5163 - 872375*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-112285869463 +
 79399380740*Sqrt[2])])/1364)/124
```

3.64. $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx$

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

3.64. $\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx$

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.64. $\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx$

3.64.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.31 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.17

method	result
trager	Expression too large to display
risch	$\frac{\left(68325 x^{3}+58315 x^{2}+51362 x+11020\right) \sqrt{2 x^{2}-x+3}}{84568 \left(5 x^{2}+3 x+2\right)^{2}}+\frac{\sqrt{\frac{8 \left(\sqrt{2}-1+x\right)^{2}}{\left(\sqrt{2}+1-x\right)^{2}}+\frac{3 \sqrt{2}\, \left(\sqrt{2}-1+x\right)^{2}}{\left(\sqrt{2}+1-x\right)^{2}}+8-3 \sqrt{2}}\, \sqrt{2}\, \left(33504619 \sqrt{-775687+549362\cdots}\right.}{}$
default	Expression too large to display

input

```
int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output

```
1/84568*(68325*x^3+58315*x^2+51362*x+11020)/(5*x^2+3*x+2)^2*(2*x^2-x+3)^(1
/2)-1/115350752*RootOf(_Z^2+29767936*RootOf(238143488*_Z^4+612631703790128
*_Z^2+394016353868467684225)^2+76578962973766)*ln(-(8901118918912*RootOf(_
Z^2+29767936*RootOf(238143488*_Z^4+612631703790128*_Z^2+394016353868467684
225)^2+76578962973766)*RootOf(238143488*_Z^4+612631703790128*_Z^2+39401635
3868467684225)^4*x+21996649948054194864*RootOf(238143488*_Z^4+612631703790
128*_Z^2+394016353868467684225)^2*RootOf(_Z^2+29767936*RootOf(238143488*_Z
^4+612631703790128*_Z^2+394016353868467684225)^2+76578962973766)*x+1055850
3141216967088325712*RootOf(238143488*_Z^4+612631703790128*_Z^2+39401635386
8467684225)^2*(2*x^2-x+3)^(1/2)-203149073871924400*RootOf(238143488*_Z^4+6
12631703790128*_Z^2+394016353868467684225)^2*RootOf(_Z^2+29767936*RootOf(2
38143488*_Z^4+612631703790128*_Z^2+394016353868467684225)^2+76578962973766
)+13558387834041967792583352*RootOf(_Z^2+29767936*RootOf(238143488*_Z^4+61
2631703790128*_Z^2+394016353868467684225)^2+76578962973766)*x+135771321202
55119256152874400047*(2*x^2-x+3)^(1/2)-238972679575677054341025*RootOf(_Z^
2+29767936*RootOf(238143488*_Z^4+612631703790128*_Z^2+39401635386846768422
5)^2+76578962973766))/(21824*x*RootOf(238143488*_Z^4+612631703790128*_Z^2+
394016353868467684225)^2+27985547479*x-114559849))+1/21142*RootOf(23814348
8*_Z^4+612631703790128*_Z^2+394016353868467684225)*ln((1139343221620736*x*
RootOf(238143488*_Z^4+612631703790128*_Z^2+394016353868467684225)^5+304...
```

3.64.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx$$

$$= \frac{\sqrt{341}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\sqrt{114559849i\,\sqrt{31}-112285869463}\log\left(\frac{\sqrt{341}\sqrt{2\,x^2-x+3}\sqrt{114559849i\,\sqrt{}}}{}\right.}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/230701504*(sqrt(341)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(11455984
9*I*sqrt(31) - 112285869463)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(11455
9849*I*sqrt(31) - 112285869463)*(509587*I*sqrt(31) - 3411891) - 1230690401
470*sqrt(31)*(-I*x + 6*I) + 23383117627930*x - 27075188832340)/x) - sqrt(3
41)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(114559849*I*sqrt(31) - 1122
85869463)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(114559849*I*sqrt(31) - 1
12285869463)*(-509587*I*sqrt(31) + 3411891) - 1230690401470*sqrt(31)*(-I*x
 + 6*I) + 23383117627930*x - 27075188832340)/x) - sqrt(341)*(25*x^4 + 30*x
^3 + 29*x^2 + 12*x + 4)*sqrt(-114559849*I*sqrt(31) - 112285869463)*log((sq
rt(341)*sqrt(2*x^2 - x + 3)*(509587*I*sqrt(31) + 3411891)*sqrt(-114559849*
I*sqrt(31) - 112285869463) - 1230690401470*sqrt(31)*(I*x - 6*I) + 23383117
627930*x - 27075188832340)/x) + sqrt(341)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
 + 4)*sqrt(-114559849*I*sqrt(31) - 112285869463)*log((sqrt(341)*sqrt(2*x^2
 - x + 3)*(-509587*I*sqrt(31) - 3411891)*sqrt(-114559849*I*sqrt(31) - 1122
85869463) - 1230690401470*sqrt(31)*(I*x - 6*I) + 23383117627930*x - 270751
88832340)/x) + 2728*(68325*x^3 + 58315*x^2 + 51362*x + 11020)*sqrt(2*x^2 -
 x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

3.64.6 Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx = \int \frac{\sqrt{2x^2-x+3}}{(5x^2+3x+2)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**3, x)
```

3.64.7 Maxima [F]

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx=\int \frac{\sqrt{2\,x^2-x+3}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^3, x)
```

3.64.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}}{(2+3x+5x^2)^3}\,dx=\int \frac{\sqrt{2\,x^2-x+3}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(1/2)/(3*x + 5*x^2 + 2)^3, x)
```

3.65 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx$

3.65.1 Optimal result

Integrand size = 27, antiderivative size = 231

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx = -\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648} - \frac{382121949(1-4x)(3-x+2x^2)^{3/2}}{134217728} + \frac{2124689283(3-x+2x^2)^{5/2}}{146800640} + \frac{48669967x(3-x+2x^2)^{5/2}}{22020096} - \frac{56422489x^2(3-x+2x^2)^{5/2}}{8257536} + \frac{10444117x^3(3-x+2x^2)^{5/2}}{294912} + \frac{941905x^4(3-x+2x^2)^{5/2}}{9216} + \frac{95165}{768}x^5(3-x+2x^2)^{5/2} + \frac{7625}{96}x^6(3-x+2x^2)^{5/2} + \frac{625}{24}x^7(3-x+2x^2)^{5/2} - \frac{606427533063\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4294967296\sqrt{2}}$$

output
```
-382121949/134217728*(1-4*x)*(2*x^2-x+3)^(3/2)+2124689283/146800640*(2*x^2
-x+3)^(5/2)+48669967/22020096*x*(2*x^2-x+3)^(5/2)-56422489/8257536*x^2*(2*
x^2-x+3)^(5/2)+10444117/294912*x^3*(2*x^2-x+3)^(5/2)+941905/9216*x^4*(2*x^
2-x+3)^(5/2)+95165/768*x^5*(2*x^2-x+3)^(5/2)+7625/96*x^6*(2*x^2-x+3)^(5/2)
+625/24*x^7*(2*x^2-x+3)^(5/2)-606427533063/8589934592*arcsinh(1/23*(1-4*x)
*23^(1/2))*2^(1/2)-26366414481/2147483648*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.45

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx = \frac{4\sqrt{3-x+2x^2}(74032009514181+12971175524316x+65151998063712x^2+23902118422310}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(74032009514181 + 12971175524316*x + 65151998063712
*x^2 + 239021184223104*x^3 + 451581382260736*x^4 + 675479464714240*x^5 + 7
65087080448000*x^6 + 745133229998080*x^7 + 534038708224000*x^8 + 349379651
174400*x^9 + 144451829760000*x^10 + 70464307200000*x^11) - 191024672914845
*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/2705829396480
```

3.65.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.19, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.704$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2-x+3)^{3/2} (5x^2+3x+2)^4 \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{24}\int \frac{1}{2}(2x^2-x+3)^{3/2} (83875x^7+86550x^6+112320x^5+84528x^4+44928x^3+18048x^2+4608x+768)\, dx + \frac{625}{24}(2x^2-x+3)^{5/2} x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\int (2x^2-x+3)^{3/2} (83875x^7+86550x^6+112320x^5+84528x^4+44928x^3+18048x^2+4608x+768)\, dx + \frac{625}{24}(2x^2-x+3)^{5/2} x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{22}\int \frac{11}{2}\left(2x^2-x+3\right)^{3/2}\left(475825x^6+174780x^5+338112x^4+179712x^3+72192x^2+18432x+3072\right)dx+\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\int \left(2x^2-x+3\right)^{3/2}\left(475825x^6+174780x^5+338112x^4+179712x^3+72192x^2+18432x+3072\right)dx+\frac{762}{2}\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{1}{20}\int \frac{15}{2}\left(2x^2-x+3\right)^{3/2}\left(941905x^5-50018x^4+479232x^3+192512x^2+49152x+8192\right)dx+\frac{95165}{4}(2\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\int \left(2x^2-x+3\right)^{3/2}\left(941905x^5-50018x^4+479232x^3+192512x^2+49152x+8192\right)dx+\frac{95165}{4}(2x^2-\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{18}\int \frac{1}{2}\left(2x^2-x+3\right)^{3/2}\left(10444117x^4-5353368x^3+6930432x^2+1769472x+294912\right)dx+\frac{941905}{18}\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\int \left(2x^2-x+3\right)^{3/2}\left(10444117x^4-5353368x^3+6930432x^2+1769472x+294912\right)dx+\frac{941905}{18}(2\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{16}\int \frac{1}{2}\left(2x^2-x+3\right)^{3/2}\left(-56422489x^3+33779718x^2+56623104x+9437184\right)dx+\frac{10444117}{16}(2\right.\right.\right.\right.$$
$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

3.65. $\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^4\,dx$

$$\downarrow 27$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\int \left(2x^2-x+3\right)^{3/2}\left(-56422489x^3+33779718x^2+56623104x+9437184\right)dx+\frac{10444117}{16}\left(2x\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\downarrow 2192$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{1}{14}\int \frac{3}{2}\left(2x^2-x+3\right)^{3/2}\left(146009901x^2+754172260x+88080384\right)dx-\frac{56422489}{14}x^2\left(2x^2-\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\downarrow 27$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\int \left(2x^2-x+3\right)^{3/2}\left(146009901x^2+754172260x+88080384\right)dx-\frac{56422489}{14}x^2\left(2x^2-x\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\downarrow 2192$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{12}\int \frac{27}{2}(708229761x+45847030)\left(2x^2-x+3\right)^{3/2}dx+\frac{48669967}{4}x\left(2x^2-x+3\right)^{5/2}\right)\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\downarrow 27$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\int (708229761x+45847030)\left(2x^2-x+3\right)^{3/2}dx+\frac{48669967}{4}x\left(2x^2-x+3\right)^{5/2}\right)-\frac{56}{}\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\downarrow 1160$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\int \left(2x^2-x+3\right)^{3/2}dx+\frac{708229761}{10}\left(2x^2-x+3\right)^{5/2}\right)+\frac{48669967}{4}x\left(2x^2\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\downarrow 1087$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{708229761}{10}\left(2\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

3.65. $\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^4\,dx$

$$\Big\downarrow \ 1087$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\,(\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

$$\Big\downarrow \ 222$$

$$\frac{1}{48}\left(\frac{1}{4}\left(\frac{3}{8}\left(\frac{1}{36}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{9}{8}\left(\frac{891617881}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\,(2x\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(625*x^7*(3 - x + 2*x^2)^(5/2))/24 + ((7625*x^6*(3 - x + 2*x^2)^(5/2))/2 +
 ((95165*x^5*(3 - x + 2*x^2)^(5/2))/4 + (3*((941905*x^4*(3 - x + 2*x^2)^(5
/2))/18 + ((10444117*x^3*(3 - x + 2*x^2)^(5/2))/16 + ((-56422489*x^2*(3 -
x + 2*x^2)^(5/2))/14 + (3*((48669967*x*(3 - x + 2*x^2)^(5/2))/4 + (9*((708
229761*(3 - x + 2*x^2)^(5/2))/10 + (891617881*(-1/16*((1 - 4*x)*(3 - x + 2
*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1
 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/4))/8))/28)/32)/36))/8)/4)/48
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.65.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.37

method	result
risch	$\frac{(70464307200000x^{11}+144451829760000x^{10}+349379651174400x^{9}+534038708224000x^{8}+745133229998080x^{7}+765087080448000x^{6}}{6764}$
trager	$\left(\frac{625}{6}x^{11}+\frac{5125}{24}x^{10}+\frac{33055}{64}x^{9}+\frac{1818925}{2304}x^{8}+\frac{81213077}{73728}x^{7}+\frac{778286825}{688128}x^{6}+\frac{16491197869}{16515072}x^{5}+\frac{31499817401}{47185920}x^{4}\right.$
default	$\frac{382121949(-1+4x)\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{134217728}+\frac{26366414481(-1+4x)\sqrt{2x^{2}-x+3}}{2147483648}+\frac{606427533063\sqrt{2}\,\text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8589934592}+\frac{21246892}{1}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/676457349120*(70464307200000*x^11+144451829760000*x^10+349379651174400*x
^9+534038708224000*x^8+745133229998080*x^7+765087080448000*x^6+67547946471
4240*x^5+451581382260736*x^4+239021184223104*x^3+65151998063712*x^2+129711
75524316*x+74032009514181)*(2*x^2-x+3)^(1/2)+606427533063/8589934592*2^(1/
2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.47

$$\int (3-x+2x^2)^{3/2} (2+3x +5x^2)^4 \, dx = \frac{1}{676457349120} (70464307200000\, x^{11} + 144451829760000\, x^{10} + 349379651174400\, x^9 + 5340387 + \frac{606427533063}{17179869184} \sqrt{2} \log \left(-4\,\sqrt{2}\sqrt{2\,x^2 - x + 3}(4\,x - 1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1/676457349120*(70464307200000*x^11 + 144451829760000*x^10 + 3493796511744
00*x^9 + 534038708224000*x^8 + 745133229998080*x^7 + 765087080448000*x^6 +
 675479464714240*x^5 + 451581382260736*x^4 + 239021184223104*x^3 + 6515199
8063712*x^2 + 12971175524316*x + 74032009514181)*sqrt(2*x^2 - x + 3) + 606
427533063/17179869184*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1)
 - 32*x^2 + 16*x - 25)
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.48

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^4 \, dx = \sqrt{2x^2-x+3} \cdot \left(\frac{625x^{11}}{6} + \frac{5125x^{10}}{24} + \frac{33055x^9}{64} + \frac{1818925x^8}{2304} + \frac{81213077x^7}{73728} + \frac{778286825x^6}{688128} + \frac{16491197869x^5}{16515072} + \frac{31499817401x^4}{47185920} + \frac{622451000581x^3}{1761607680} + \frac{32317459357x^2}{335544320} + \frac{360310431231x}{18790481920} + \frac{8225778834909}{75161927680}\right) + \frac{606427533063\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}(x-\frac{1}{4})}{23}\right)}{8589934592}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**4,x)
```

output
```
sqrt(2*x**2 - x + 3)*(625*x**11/6 + 5125*x**10/24 + 33055*x**9/64 + 181892
5*x**8/2304 + 81213077*x**7/73728 + 778286825*x**6/688128 + 16491197869*x*
*5/16515072 + 31499817401*x**4/47185920 + 622451000581*x**3/1761607680 + 3
2317459357*x**2/335544320 + 360310431231*x/18790481920 + 8225778834909/751
61927680) + 606427533063*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/8589934592
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.89

$$\begin{aligned}\int (3-x+2x^2)^{3/2}(2+3x+5x^2)^4\,dx &= \frac{625}{24}(2x^2-x+3)^{\frac{5}{2}}x^7 \\ &+ \frac{7625}{96}(2x^2-x+3)^{\frac{5}{2}}x^6 + \frac{95165}{768}(2x^2-x+3)^{\frac{5}{2}}x^5 \\ &+ \frac{941905}{9216}(2x^2-x+3)^{\frac{5}{2}}x^4 + \frac{10444117}{294912}(2x^2-x+3)^{\frac{5}{2}}x^3 \\ &- \frac{56422489}{8257536}(2x^2-x+3)^{\frac{5}{2}}x^2 + \frac{48669967}{22020096}(2x^2-x+3)^{\frac{5}{2}}x \\ &+ \frac{2124689283}{146800640}(2x^2-x+3)^{\frac{5}{2}} + \frac{382121949}{33554432}(2x^2-x+3)^{\frac{3}{2}}x \\ &- \frac{382121949}{134217728}(2x^2-x+3)^{\frac{3}{2}} + \frac{26366414481}{536870912}\sqrt{2x^2-x+3}x \\ &+ \frac{606427533063}{8589934592}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{26366414481}{2147483648}\sqrt{2x^2-x+3}\end{aligned}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
625/24*(2*x^2 - x + 3)^(5/2)*x^7 + 7625/96*(2*x^2 - x + 3)^(5/2)*x^6 + 951
65/768*(2*x^2 - x + 3)^(5/2)*x^5 + 941905/9216*(2*x^2 - x + 3)^(5/2)*x^4 +
 10444117/294912*(2*x^2 - x + 3)^(5/2)*x^3 - 56422489/8257536*(2*x^2 - x +
 3)^(5/2)*x^2 + 48669967/22020096*(2*x^2 - x + 3)^(5/2)*x + 2124689283/146
800640*(2*x^2 - x + 3)^(5/2) + 382121949/33554432*(2*x^2 - x + 3)^(3/2)*x
- 382121949/134217728*(2*x^2 - x + 3)^(3/2) + 26366414481/536870912*sqrt(2
*x^2 - x + 3)*x + 606427533063/8589934592*sqrt(2)*arcsinh(1/23*sqrt(23)*(4
*x - 1)) - 26366414481/2147483648*sqrt(2*x^2 - x + 3)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.45

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^4 \, dx = \frac{1}{676457349120} \left(4 \left(8 \left(4 \left(16 \left(20 \left(8 \left(28 \left(160 \left(12 \left(200 \left(20\,x+41\right)x+19833\right)x+363785\right)x+81213\right.\right.\right.\right.\right.\right.\right.\right.\right. - \frac{606427533063}{8589934592}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1/676457349120*(4*(8*(4*(16*(20*(8*(28*(160*(12*(200*(20*x + 41)*x + 19833
)*x + 363785)*x + 81213077)*x + 2334860475)*x + 16491197869)*x + 220498721
807)*x + 1867353001743)*x + 2035999939491)*x + 3242793881079)*x + 74032009
514181)*sqrt(2*x^2 - x + 3) - 606427533063/8589934592*sqrt(2)*log(-2*sqrt(
2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^4 \, dx = \int \left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^2+3\,x+2\right)^4 \,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^4, x)
```

3.66 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 \, dx$

3.66.1 Optimal result

Integrand size = 27, antiderivative size = 189

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^3 \, dx = -\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432} - \frac{667795(1-4x)\,(3-x+2x^2)^{3/2}}{2097152} - \frac{4625907(3-x+2x^2)^{5/2}}{2293760} - \frac{81685x(3-x+2x^2)^{5/2}}{114688} + \frac{384739x^2(3-x+2x^2)^{5/2}}{43008} + \frac{27785x^3(3-x+2x^2)^{5/2}}{1536} + \frac{725}{48}x^4\left(3-x+2x^2\right)^{5/2} + \frac{25}{4}x^5\left(3-x+2x^2\right)^{5/2} - \frac{1059790665\,\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{67108864\sqrt{2}}$$

output
```
-667795/2097152*(1-4*x)*(2*x^2-x+3)^(3/2)-4625907/2293760*(2*x^2-x+3)^(5/2
)-81685/114688*x*(2*x^2-x+3)^(5/2)+384739/43008*x^2*(2*x^2-x+3)^(5/2)+2778
5/1536*x^3*(2*x^2-x+3)^(5/2)+725/48*x^4*(2*x^2-x+3)^(5/2)+25/4*x^5*(2*x^2-
x+3)^(5/2)-1059790665/134217728*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-460
77855/33554432*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.50

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \, dx = \frac{4\sqrt{3-x+2x^2}(-72152399943+53985432012x+199615064544x^2+389257196928x^3+4878$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-72152399943 + 53985432012*x + 199615064544*x^2 +
389257196928*x^3 + 487891884032*x^4 + 571298324480*x^5 + 430820229120*x^6
+ 328328806400*x^7 + 124780544000*x^8 + 88080384000*x^9) - 111278019825*Sq
rt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/14092861440
```

3.66.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{3/2} \left(5x^2+3x+2\right)^3 \, dx$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{20}\int \frac{5}{2}\left(2x^2-x+3\right)^{3/2}\left(2175x^5+1530x^4+1656x^3+912x^2+288x+64\right)dx+\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \ 27$$

$$\frac{1}{8}\int \left(2x^2-x+3\right)^{3/2}\left(2175x^5+1530x^4+1656x^3+912x^2+288x+64\right)dx+\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{8}\left(\frac{1}{18}\int \frac{3}{2}\left(2x^2-x+3\right)^{3/2}\left(27785x^4+2472x^3+10944x^2+3456x+768\right)dx+\frac{725}{6}\left(2x^2-x+3\right)^{5/2}x^4\right)+ \frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{12}\int \left(2x^2-x+3\right)^{3/2}\left(27785x^4+2472x^3+10944x^2+3456x+768\right)dx+\frac{725}{6}\left(2x^2-x+3\right)^{5/2}x^4\right)+ \frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{16}\int \frac{1}{2}\left(2x^2-x+3\right)^{3/2}\left(384739x^3-149922x^2+110592x+24576\right)dx+\frac{27785}{16}\left(2x^2-x+3\right)^{5/2}x^3\right)+ \right. \frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\int \left(2x^2-x+3\right)^{3/2}\left(384739x^3-149922x^2+110592x+24576\right)dx+\frac{27785}{16}\left(2x^2-x+3\right)^{5/2}x^3\right)+\frac{72}{6}\right. \frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{1}{14}\int \frac{3}{2}\left(-245055x^2-506764x+229376\right)\left(2x^2-x+3\right)^{3/2}dx+\frac{384739}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{2778}{16}\right.\right. \frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\int \left(-245055x^2-506764x+229376\right)\left(2x^2-x+3\right)^{3/2}dx+\frac{384739}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{27785}{16}\right.\right. \frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{12}\int \frac{3}{2}(2325118-4625907x)\left(2x^2-x+3\right)^{3/2}dx-\frac{81685}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{384739}{14}x^2(2x^2\right.\right.\right. \frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 27$$

3.66. $\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\, dx$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\int (2325118-4625907x)\left(2x^2-x+3\right)^{3/2}dx-\frac{81685}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{384739}{14}x^2\left(2x^2-\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{4625907}{10}\left(2x^2-x+3\right)^{5/2}\right)-\frac{81685}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{4625907}{10}\left(2x^2-x+3\right)^{5\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+\right.\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}\right(\right.\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 222$$

$$\frac{1}{8}\left(\frac{1}{12}\left(\frac{1}{32}\left(\frac{3}{28}\left(\frac{1}{8}\left(\frac{4674565}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)\right.\right.\right.\right.\right.\right.$$
$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

3.66. $\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx$

output
```
(25*x^5*(3 - x + 2*x^2)^(5/2))/4 + ((725*x^4*(3 - x + 2*x^2)^(5/2))/6 + ((
27785*x^3*(3 - x + 2*x^2)^(5/2))/16 + ((384739*x^2*(3 - x + 2*x^2)^(5/2))/
14 + (3*((-81685*x*(3 - x + 2*x^2)^(5/2))/4 + ((-4625907*(3 - x + 2*x^2)^(
5/2))/10 + (4674565*(-1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*(
(1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqr
t[2])))/32))/4)/8))/28)/32)/12)/8
```

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.66.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

method	result
risch	$\frac{(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+487891884032x^4+389257196928x^3+199}{3523215360}$
trager	$\left(25x^9+\frac{425}{12}x^8+\frac{35785}{384}x^7+\frac{438253}{3584}x^6+\frac{13947713}{86016}x^5+\frac{34032637}{245760}x^4+\frac{1013690617}{9175040}x^3+\frac{297046227}{5242880}x^2+\frac{449878}{293601}\right.$
default	$\frac{667795(-1+4x)(2x^2-x+3)^{\frac{3}{2}}}{2097152}+\frac{46077855(-1+4x)\sqrt{2x^2-x+3}}{33554432}+\frac{1059790665\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{134217728}-\frac{4625907(2x^2-x+}{2293760}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3523215360*(88080384000*x^9+124780544000*x^8+328328806400*x^7+4308202291
20*x^6+571298324480*x^5+487891884032*x^4+389257196928*x^3+199615064544*x^2
+53985432012*x-72152399943)*(2*x^2-x+3)^(1/2)+1059790665/134217728*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\int (3-x+2x^2)^{3/2}(2+3x+5x^2)^3\,dx=\frac{1}{3523215360}\left(88080384000\,x^9+124780544000\,x^8+328328806400\,x^7+430820229120\,x^6+57\right.$$
$$+\frac{1059790665}{268435456}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/3523215360*(88080384000*x^9 + 124780544000*x^8 + 328328806400*x^7 + 4308
20229120*x^6 + 571298324480*x^5 + 487891884032*x^4 + 389257196928*x^3 + 19
9615064544*x^2 + 53985432012*x - 72152399943)*sqrt(2*x^2 - x + 3) + 105979
0665/268435456*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.50

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx=\sqrt{2x^2-x+3}\cdot\left(25x^9+\frac{425x^8}{12}+\frac{35785x^7}{384}+\frac{438253x^6}{3584}+\frac{13947713x^5}{86016}+\frac{34032637x^4}{245760}+\frac{1013690617x^3}{9175040}+\frac{297046227x^2}{5242880}+\frac{4498786001x}{293601280}-\frac{24050799981}{1174405120}\right)+\frac{1059790665\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{134217728}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**3,x)
```

output
```
sqrt(2*x**2 - x + 3)*(25*x**9 + 425*x**8/12 + 35785*x**7/384 + 438253*x**6
/3584 + 13947713*x**5/86016 + 34032637*x**4/245760 + 1013690617*x**3/91750
40 + 297046227*x**2/5242880 + 4498786001*x/293601280 - 24050799981/1174405
120) + 1059790665*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/134217728
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx=\frac{25}{4}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^5+\frac{725}{48}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{27785}{1536}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{384739}{43008}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2-\frac{81685}{114688}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x-\frac{4625907}{2293760}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{667795}{524288}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x-\frac{667795}{2097152}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{46077855}{8388608}\sqrt{2\,x^2-x+3}x+\frac{1059790665}{134217728}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{46077855}{33554432}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
25/4*(2*x^2 - x + 3)^(5/2)*x^5 + 725/48*(2*x^2 - x + 3)^(5/2)*x^4 + 27785/
1536*(2*x^2 - x + 3)^(5/2)*x^3 + 384739/43008*(2*x^2 - x + 3)^(5/2)*x^2 -
81685/114688*(2*x^2 - x + 3)^(5/2)*x - 4625907/2293760*(2*x^2 - x + 3)^(5/
2) + 667795/524288*(2*x^2 - x + 3)^(3/2)*x - 667795/2097152*(2*x^2 - x + 3
)^(3/2) + 46077855/8388608*sqrt(2*x^2 - x + 3)*x + 1059790665/134217728*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46077855/33554432*sqrt(2*x^2 - x
+ 3)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.49

$$\int \left(3-x+2x^2\right)^{3/2} (2+3x+5x^2)^3 \, dx = \frac{1}{3523215360}\,(4\,(8\,(4\,(16\,(20\,(8\,(140\,(160\,(12\,x+17)x+7157)x+1314759)x+13947713)x+2\ldots - \frac{1059790665}{134217728}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1/3523215360*(4*(8*(4*(16*(20*(8*(140*(160*(12*x + 17)*x + 7157)*x + 13147
59)*x + 13947713)*x + 238228459)*x + 3041071851)*x + 6237970767)*x + 13496
358003)*x - 72152399943)*sqrt(2*x^2 - x + 3) - 1059790665/134217728*sqrt(2
)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^3 \, dx = \int \left(2\,x^2-x+3\right)^{3/2} \left(5\,x^2+3\,x+2\right)^3 dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3, x)
```

3.67 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 \, dx$

3.67.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^2 \, dx = \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576} + \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608} + \frac{73861(3-x+2x^2)^{5/2}}{215040} + \frac{24499x(3-x+2x^2)^{5/2}}{10752} + \frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2} + \frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2} + \frac{12850997\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

output
```
24293/196608*(1-4*x)*(2*x^2-x+3)^(3/2)+73861/215040*(2*x^2-x+3)^(5/2)+2449
9/10752*x*(2*x^2-x+3)^(5/2)+1235/448*x^2*(2*x^2-x+3)^(5/2)+25/16*x^3*(2*x^
2-x+3)^(5/2)+12850997/4194304*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+55873
9/1048576*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^2 \, dx = \frac{4\sqrt{3-x+2x^2}(439831323+1619403428x+1799647136x^2+2728413312x^3+2061273088x^4}{440}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(439831323 + 1619403428*x + 1799647136*x^2 + 272841
3312*x^3 + 2061273088*x^4 + 2025840640*x^5 + 525926400*x^6 + 688128000*x^7
) + 1349354685*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/440401920
```

3.67.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{3/2} \left(5x^2+3x+2\right)^2 \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16} \int \frac{1}{2} \left(2x^2-x+3\right)^{3/2} \left(1235x^3+478x^2+384x+128\right) dx + \frac{25}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32} \int \left(2x^2-x+3\right)^{3/2} \left(1235x^3+478x^2+384x+128\right) dx + \frac{25}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32} \left(\frac{1}{14} \int \frac{1}{2} \left(2x^2-x+3\right)^{3/2} \left(24499x^2-4068x+3584\right) dx + \frac{1235}{14} x^2 \left(2x^2-x+3\right)^{5/2}\right) + \frac{25}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32} \left(\frac{1}{28} \int \left(2x^2-x+3\right)^{3/2} \left(24499x^2-4068x+3584\right) dx + \frac{1235}{14} x^2 \left(2x^2-x+3\right)^{5/2}\right) + \frac{25}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32} \left(\frac{1}{28} \left(\frac{1}{12} \int -\frac{1}{2} (60978-73861x) \left(2x^2-x+3\right)^{3/2} dx + \frac{24499}{12} x \left(2x^2-x+3\right)^{5/2}\right) + \frac{1235}{14} x^2 \left(2x^2-x+3\right)^{5/2} \right. \\ \frac{25}{16} \left(2x^2-x+3\right)^{5/2} x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{24499}{12}x\left(2x^2-x+3\right)^{5/2}-\frac{1}{24}\int (60978-73861x)\left(2x^2-x+3\right)^{3/2}dx\right)+\frac{1235}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+ \frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\int \left(2x^2-x+3\right)^{3/2}dx\right)+\frac{24499}{12}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{1235}{14}x^2(2 \frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)\right)+\frac{2449}{12} \frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1 \frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2 \frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{73861}{10}\left(2x^2-x+3\right)^{5/2}-\frac{170051}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1- \frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

3.67. $\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2 dx$

output
```
(25*x^3*(3 - x + 2*x^2)^(5/2))/16 + ((1235*x^2*(3 - x + 2*x^2)^(5/2))/14 +
 ((24499*x*(3 - x + 2*x^2)^(5/2))/12 + ((73861*(3 - x + 2*x^2)^(5/2))/10 -
 (170051*(-1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*S
qrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32)
)/4)/24)/28)/32
```

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.67.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+1619403428x+439831323)\sqrt{2x^2-x+}}{110100480}$
trager	$\left(\frac{25}{4}x^7+\frac{535}{112}x^6+\frac{49459}{2688}x^5+\frac{143783}{7680}x^4+\frac{7105243}{286720}x^3+\frac{8034139}{491520}x^2+\frac{404850857}{27525120}x+\frac{146610441}{36700160}\right)\sqrt{2x^2-x+}$
default	$-\frac{24293(-1+4x)(2x^2-x+3)^{\frac{3}{2}}}{196608}-\frac{558739(-1+4x)\sqrt{2x^2-x+3}}{1048576}-\frac{12850997\sqrt{2}\,\text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{73861(2x^2-x+3)^{\frac{5}{2}}}{215040}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/110100480*(688128000*x^7+525926400*x^6+2025840640*x^5+2061273088*x^4+272
8413312*x^3+1799647136*x^2+1619403428*x+439831323)*(2*x^2-x+3)^(1/2)-12850
997/4194304*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int (3-x+2x^2)^{3/2}(2+3x+5x^2)^2\,dx=\frac{1}{110100480}\left(688128000\,x^7+525926400\,x^6+2025840640\,x^5+2061273088\,x^4+2728413312\,x^3\right.$$
$$+\frac{12850997}{8388608}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/110100480*(688128000*x^7 + 525926400*x^6 + 2025840640*x^5 + 2061273088*x
^4 + 2728413312*x^3 + 1799647136*x^2 + 1619403428*x + 439831323)*sqrt(2*x^
2 - x + 3) + 12850997/8388608*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4
*x - 1) - 32*x^2 + 16*x - 25)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int (3-x+2x^2)^{3/2}(2+3x+5x^2)^2\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{25x^7}{4}+\frac{535x^6}{112}+\frac{49459x^5}{2688}+\frac{143783x^4}{7680}+\frac{7105243x^3}{286720}+\frac{8034139x^2}{491520}+\frac{404850857x}{27525120}+\frac{146610441}{36700160}\right)-\frac{12850997\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}(x-\frac{1}{4})}{23}\right)}{4194304}$$

input

```
integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2,x)
```

output

```
sqrt(2*x**2 - x + 3)*(25*x**7/4 + 535*x**6/112 + 49459*x**5/2688 + 143783*
x**4/7680 + 7105243*x**3/286720 + 8034139*x**2/491520 + 404850857*x/275251
20 + 146610441/36700160) - 12850997*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)
/4194304
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int (3-x+2x^2)^{3/2}(2+3x+5x^2)^2\,dx = \frac{25}{16}(2x^2-x+3)^{\frac{5}{2}}x^3+\frac{1235}{448}(2x^2-x+3)^{\frac{5}{2}}x^2+\frac{24499}{10752}(2x^2-x+3)^{\frac{5}{2}}x+\frac{73861}{215040}(2x^2-x+3)^{\frac{5}{2}}-\frac{24293}{49152}(2x^2-x+3)^{\frac{3}{2}}x+\frac{24293}{196608}(2x^2-x+3)^{\frac{3}{2}}-\frac{558739}{262144}\sqrt{2x^2-x+3}\,x-\frac{12850997}{4194304}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{558739}{1048576}\sqrt{2x^2-x+3}$$

input

```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output

```
25/16*(2*x^2 - x + 3)^(5/2)*x^3 + 1235/448*(2*x^2 - x + 3)^(5/2)*x^2 + 244
99/10752*(2*x^2 - x + 3)^(5/2)*x + 73861/215040*(2*x^2 - x + 3)^(5/2) - 24
293/49152*(2*x^2 - x + 3)^(3/2)*x + 24293/196608*(2*x^2 - x + 3)^(3/2) - 5
58739/262144*sqrt(2*x^2 - x + 3)*x - 12850997/4194304*sqrt(2)*arcsinh(1/23
*sqrt(23)*(4*x - 1)) + 558739/1048576*sqrt(2*x^2 - x + 3)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int (3-x+2x^2)^{3/2} (2+3x +5x^2)^2 \, dx = \frac{1}{110100480} (4 (8 (4 (16 (20 (120 (140 x + 107)x + 49459)x + 1006481)x + 21315729)x + 56238 + \frac{12850997}{4194304} \sqrt{2} \log \left(-2 \sqrt{2}\left(\sqrt{2}x - \sqrt{2 x^2 - x + 3}\right) + 1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
1/110100480*(4*(8*(4*(16*(20*(120*(140*x + 107)*x + 49459)*x + 1006481)*x
+ 21315729)*x + 56238973)*x + 404850857)*x + 439831323)*sqrt(2*x^2 - x + 3
) + 12850997/4194304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3)) + 1)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2)^2 \, dx = \int (2 x^2 - x + 3)^{3/2} (5 x^2 + 3 x + 2)^2 \, dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2, x)
```

3.68 $\int (3-x+2x^2)^{3/2} (2+3x+5x^2) \, dx$

3.68.1 Optimal result

Integrand size = 25, antiderivative size = 105

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2) \, dx = -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192} - \frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536} + \frac{107}{240}\left(3-x+2x^2\right)^{5/2} + \frac{5}{12}x\left(3-x+2x^2\right)^{5/2} - \frac{94691\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

output
```
-179/1536*(1-4*x)*(2*x^2-x+3)^(3/2)+107/240*(2*x^2-x+3)^(5/2)+5/12*x*(2*x^
2-x+3)^(5/2)-94691/32768*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4117/8192*
(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int (3-x+2x^2)^{3/2} (2+3x+5x^2) \, dx = \frac{4\sqrt{3-x+2x^2}(388341+565276x+319072x^2+561024x^3+14336x^4+204800x^5) - 1420365}{491520}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(388341 + 565276*x + 319072*x^2 + 561024*x^3 + 1433
6*x^4 + 204800*x^5) - 1420365*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2
]])/491520
```

3.68.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{12}\int \frac{1}{2}(107x+18)\left(2x^2-x+3\right)^{3/2}dx+\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int (107x+18)\left(2x^2-x+3\right)^{3/2}dx+\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{24}\left(\frac{179}{4}\int \left(2x^2-x+3\right)^{3/2}dx+\frac{107}{10}\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{24}\left(\frac{179}{4}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{107}{10}\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{24}\left(\frac{179}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{107}{10}\left(2x^2-\right.\right.$$
$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{24}\left(\frac{179}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)\right.$$
$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

$\big\downarrow$ 222

$$\frac{1}{24}\left(\frac{179}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{107}{10}\left(2x^2-x+\right.\right.$$
$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

output
```
(5*x*(3 - x + 2*x^2)^(5/2))/12 + ((107*(3 - x + 2*x^2)^(5/2))/10 + (179*(-
1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x +
 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/4)/24
```

3.68.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.68.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result
risch	$\frac{\left(204800 x^{5}+14336 x^{4}+561024 x^{3}+319072 x^{2}+565276 x +388341\right) \sqrt{2 x^{2}-x +3}}{122880}+\frac{94691 \sqrt{2}\, \operatorname{arcsinh}\left(\frac{4 \sqrt{23}\, \left(x -\frac{1}{4}\right)}{23}\right)}{32768}$
trager	$\left(\frac{5}{3} x^{5}+\frac{7}{60} x^{4}+\frac{1461}{320} x^{3}+\frac{9971}{3840} x^{2}+\frac{141319}{30720} x +\frac{129447}{40960}\right) \sqrt{2 x^{2}-x +3}+\frac{94691 \operatorname{RootOf}\left(\_Z^{2}-2\right) \ln\left(4 \operatorname{RootOf}\left(\right.\right.}{}$
default	$\frac{179 \left(-1+4 x \right) \left(2 x^{2}-x +3\right)^{\frac{3}{2}}}{1536}+\frac{4117 \left(-1+4 x \right) \sqrt{2 x^{2}-x +3}}{8192}+\frac{94691 \sqrt{2}\, \operatorname{arcsinh}\left(\frac{4 \sqrt{23}\, \left(x -\frac{1}{4}\right)}{23}\right)}{32768}+\frac{107 \left(2 x^{2}-x +3\right)^{\frac{5}{2}}}{240}+\frac{5 x \left(2 x^{2}-x\right.}{12}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/122880*(204800*x^5+14336*x^4+561024*x^3+319072*x^2+565276*x+388341)*(2*x
^2-x+3)^(1/2)+94691/32768*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)\,dx=\frac{1}{122880}\left(204800\,x^5+14336\,x^4+561024\,x^3+319072\,x^2+565276\,x+388341\right)\sqrt{2\,x^2-x+3}+\frac{94691}{65536}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/122880*(204800*x^5 + 14336*x^4 + 561024*x^3 + 319072*x^2 + 565276*x + 38
8341)*sqrt(2*x^2 - x + 3) + 94691/65536*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2
- x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)\,dx=\sqrt{2x^2-x+3}\cdot\left(\frac{5x^5}{3}+\frac{7x^4}{60}+\frac{1461x^3}{320}+\frac{9971x^2}{3840}+\frac{141319x}{30720}+\frac{129447}{40960}\right)+\frac{94691\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**5/3 + 7*x**4/60 + 1461*x**3/320 + 9971*x**2/384
0 + 141319*x/30720 + 129447/40960) + 94691*sqrt(2)*asinh(4*sqrt(23)*(x - 1
/4)/23)/32768
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)\,dx=\frac{5}{12}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x+\frac{107}{240}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{179}{384}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x-\frac{179}{1536}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{4117}{2048}\sqrt{2\,x^2-x+3}x+\frac{94691}{32768}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{4117}{8192}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
5/12*(2*x^2 - x + 3)^(5/2)*x + 107/240*(2*x^2 - x + 3)^(5/2) + 179/384*(2*
x^2 - x + 3)^(3/2)*x - 179/1536*(2*x^2 - x + 3)^(3/2) + 4117/2048*sqrt(2*x
^2 - x + 3)*x + 94691/32768*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 411
7/8192*sqrt(2*x^2 - x + 3)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)\,dx=\frac{1}{122880}\left(4\left(8\left(4\left(16\left(100\,x+7\right)x+4383\right)x+9971\right)x+141319\right)x+388341\right)\sqrt{2\,x^2-x+3}-\frac{94691}{32768}\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
1/122880*(4*(8*(4*(16*(100*x + 7)*x + 4383)*x + 9971)*x + 141319)*x + 3883
41)*sqrt(2*x^2 - x + 3) - 94691/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3)) + 1)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int (3 - x + 2x^2)^{3/2} (2 + 3x + 5x^2) \, dx = \int (2\,x^2 - x + 3)^{3/2} (5\,x^2 + 3\,x + 2) \, dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2), x)
```

3.69 $\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx$

3.69.1 Optimal result

Integrand size = 27, antiderivative size = 197

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2}-\frac{2203\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(8+61\sqrt{2}+\left(130+69\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \frac{11}{125}\sqrt{\frac{11}{31}\left(-247+500\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(-247+500\sqrt{2}\right)}}\left(8-61\sqrt{2}+\left(130-69\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
-2203/2000*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/100*(49-20*x)*(2*x^2-x
+3)^(1/2)-11/3875*arctanh(1/62*(8+x*(130-69*2^(1/2))-61*2^(1/2))*682^(1/2)
/(-247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-84227+170500*2^(1/2))^(1/2)
+11/3875*arctan(1/62*(8+61*2^(1/2)+x*(130+69*2^(1/2)))*682^(1/2)/(247+500*
2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(84227+170500*2^(1/2))^(1/2)
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.16

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \frac{20(-49+20x)\sqrt{3-x+2x^2}-2203\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)+1936\text{Ro}}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

output
```
(20*(-49 + 20*x)*Sqrt[3 - x + 2*x^2] - 2203*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6
 - 2*x + 4*x^2]] + 1936*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*
#1^3 - 5*#1^4 & , (-36*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 6*Sq
rt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 13*Log[-(Sqrt[2]*x
) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2
- 10*#1^3) & ])/2000
```

3.69.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {1308, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}}{5x^2+3x+2}dx$$

$$\Big\downarrow \text{1308}$$

$$-\frac{1}{50}\int -\frac{2203x^2-1195x+1462}{4\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{100}\sqrt{2x^2-x+3}(49-20x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{200}\int \frac{2203x^2-1195x+1462}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \text{2143}$$

$$\frac{1}{200}\left(\frac{2203}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int \frac{968(3-13x)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{200}\left(\frac{2203}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{968}{5}\int \frac{3-13x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{200}\left(\frac{968}{5}\int \frac{3-13x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{2203\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{5\sqrt{46}}\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{200}\left(\frac{968}{5}\int \frac{3-13x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{2203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \; 1368$$

$$\frac{1}{200}\left(\frac{968}{5}\left(\frac{\int -\frac{11\left(\left(10+13\sqrt{2}\right)x-3\sqrt{2}+16\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(10-13\sqrt{2}\right)x+3\sqrt{2}+16\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{2203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{200}\left(\frac{968}{5}\left(\frac{\int \frac{\left(10-13\sqrt{2}\right)x+3\sqrt{2}+16}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(10+13\sqrt{2}\right)x-3\sqrt{2}+16}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{2203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}\right)-\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$$\Big\downarrow \; 1362$$

3.69. $\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx$

$$\frac{1}{200}\left(\frac{968}{5}\left(\sqrt{2}\left(247-500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8\right)^2}{2x^2-x+3}-62\left(247-500\sqrt{2}\right)}d\frac{\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8}{\sqrt{2x^2-x+3}}\right.\right.$$
$$\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$\downarrow$ 217

$$\frac{1}{200}\left(\frac{968}{5}\left(\sqrt{2}\left(247-500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8\right)^2}{2x^2-x+3}-62\left(247-500\sqrt{2}\right)}d\frac{\left(130-69\sqrt{2}\right)x-61\sqrt{2}+8}{\sqrt{2x^2-x+3}}\right.\right.$$
$$\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

$\downarrow$ 219

$$\frac{1}{200}\left(\frac{2203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}+\frac{968}{5}\left(\sqrt{\frac{1}{341}\left(247+500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right.\right.\right.$$
$$\frac{1}{100}(49-20x)\sqrt{2x^2-x+3}$$

input
```
Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

output
```
-1/100*((49 - 20*x)*Sqrt[3 - x + 2*x^2]) + ((2203*ArcSinh[(-1 + 4*x)/Sqrt[
23]])/(5*Sqrt[2]) + (968*(Sqrt[(247 + 500*Sqrt[2])/341]*ArcTan[(Sqrt[11/(6
2*(247 + 500*Sqrt[2]))]*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x))/Sqrt[3 -
x + 2*x^2]] + ((247 - 500*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-247 + 500*Sqrt[2
]))]*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[3
41*(-247 + 500*Sqrt[2])]))/5)/200
```

3.69. $\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx$

3.69.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1308
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Simp[1/(2*f^2*(p + q)*(2*p + 2*q + 1))   Int[(a + b
*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p
)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
 + c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p +
q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*
(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(
p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
 - 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2
*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

3.69. $\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx$

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.69.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.50 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.99

method	result
trager	Expression too large to display
risch	$\frac{(-49+20x)\sqrt{2x^2-x+3}}{100}+\frac{2203\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2000}+\frac{11\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(1535\sqrt{-775687}\right.}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-49/100+1/5*x)*(2*x^2-x+3)^(1/2)+1/100*RootOf(24025*_Z^4+163063472*_Z^2+2
267598080000)*ln((5429049375*x*RootOf(24025*_Z^4+163063472*_Z^2+2267598080
000)^5+40888264630400*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)^3*x+
13640929511440000*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)^2*(2*x^2
-x+3)^(1/2)+154372254960000*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000
)^3-592661349855657984*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)*x+5
2586627694873395200*(2*x^2-x+3)^(1/2)+2295791036224716800*RootOf(24025*_Z^
4+163063472*_Z^2+2267598080000))/(775*x*RootOf(24025*_Z^4+163063472*_Z^2+2
267598080000)^2+6431392*x+5068448))+1/15500*RootOf(_Z^2+24025*RootOf(24025
*_Z^4+163063472*_Z^2+2267598080000)^2+163063472)*ln(-(-8686479*RootOf(_Z^2
+24025*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)^2+163063472)*RootOf
(24025*_Z^4+163063472*_Z^2+2267598080000)^4*x-52493234464*RootOf(24025*_Z^
4+163063472*_Z^2+2267598080000)^2*RootOf(_Z^2+24025*RootOf(24025*_Z^4+1630
63472*_Z^2+2267598080000)^2+163063472)*x+3382950518837120*RootOf(24025*_Z^
4+163063472*_Z^2+2267598080000)^2*(2*x^2-x+3)^(1/2)+246995607936*RootOf(24
025*_Z^4+163063472*_Z^2+2267598080000)^2*RootOf(_Z^2+24025*RootOf(24025*_Z
^4+163063472*_Z^2+2267598080000)^2+163063472)+992130849952000*RootOf(_Z^2+
24025*RootOf(24025*_Z^4+163063472*_Z^2+2267598080000)^2+163063472)*x+99194
17776240896000*(2*x^2-x+3)^(1/2)-1996846869248000*RootOf(_Z^2+24025*RootOf
(24025*_Z^4+163063472*_Z^2+2267598080000)^2+163063472))/(775*x*RootOf(2...
```

3.69. $\int \frac{\left(3-x+2x^2\right)^{3/2}}{2+3x+5x^2}\,dx$

3.69.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.60

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \frac{1}{15500}\sqrt{31}\sqrt{316778i\sqrt{31}-657514}\log\left(-\frac{2\sqrt{2x^2-x+3}(2\sqrt{31}-i)\sqrt{316778i
$$
$$-\frac{1}{15500}\sqrt{31}\sqrt{316778i\sqrt{31}-657514}\log\left(\frac{2\sqrt{2x^2-x+3}(2\sqrt{31}-i)\sqrt{316778i\sqrt{31}-657514}-1375\sqrt{3}}{x}\right.$$
$$+\frac{1}{15500}\sqrt{31}\sqrt{-316778i\sqrt{31}-657514}\log\left(-\frac{2\sqrt{2x^2-x+3}(2\sqrt{31}+i)\sqrt{-316778i\sqrt{31}-657514}+13}{x}\right.$$
$$-\frac{1}{15500}\sqrt{31}\sqrt{-316778i\sqrt{31}-657514}\log\left(\frac{2\sqrt{2x^2-x+3}(2\sqrt{31}+i)\sqrt{-316778i\sqrt{31}-657514}-137}{x}\right.$$
$$+\frac{1}{100}\sqrt{2x^2-x+3}(20x-49)$$
$$+\frac{2203}{4000}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/15500*sqrt(31)*sqrt(316778*I*sqrt(31) - 657514)*log(-(2*sqrt(2*x^2 - x +
 3)*(2*sqrt(31) - I)*sqrt(316778*I*sqrt(31) - 657514) + 1375*sqrt(31)*(-I*
x + 6*I) - 26125*x + 30250)/x) - 1/15500*sqrt(31)*sqrt(316778*I*sqrt(31) -
 657514)*log((2*sqrt(2*x^2 - x + 3)*(2*sqrt(31) - I)*sqrt(316778*I*sqrt(31
) - 657514) - 1375*sqrt(31)*(-I*x + 6*I) + 26125*x - 30250)/x) + 1/15500*s
qrt(31)*sqrt(-316778*I*sqrt(31) - 657514)*log(-(2*sqrt(2*x^2 - x + 3)*(2*s
qrt(31) + I)*sqrt(-316778*I*sqrt(31) - 657514) + 1375*sqrt(31)*(I*x - 6*I)
 - 26125*x + 30250)/x) - 1/15500*sqrt(31)*sqrt(-316778*I*sqrt(31) - 657514
)*log((2*sqrt(2*x^2 - x + 3)*(2*sqrt(31) + I)*sqrt(-316778*I*sqrt(31) - 65
7514) - 1375*sqrt(31)*(I*x - 6*I) + 26125*x - 30250)/x) + 1/100*sqrt(2*x^2
 - x + 3)*(20*x - 49) + 2203/4000*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.69.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}}{5x^2+3x+2}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2), x)
```

3.69.7 Maxima [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{3}{2}}}{5\,x^2+3\,x+2}\,dx$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2), x)
```

3.69.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = \int \frac{{\left(2\,x^2-x+3\right)}^{3/2}}{5\,x^2+3\,x+2}\,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2), x)
```

3.70 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx$

3.70.1 Optimal result

Integrand size = 27, antiderivative size = 232

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} dx = \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)(3-x+2x^2)^{3/2}}{31(2+3x+5x^2)} - \frac{2}{25}\sqrt{2}\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{\sqrt{\frac{11}{31}(3169333+2265350\sqrt{2})}\arctan\left(\frac{\sqrt{\frac{11}{62(3169333+2265350\sqrt{2})}}\left(3514+2963\sqrt{2}+\left(9440+6477\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{1550} - \frac{\sqrt{\frac{11}{31}(-3169333+2265350\sqrt{2})}\text{arctanh}\left(\frac{\sqrt{\frac{11}{62(-3169333+2265350\sqrt{2})}}\left(3514-2963\sqrt{2}+\left(9440-6477\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{1550}$$

output
```
1/31*(3+10*x)*(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)-2/25*arcsinh(1/23*(1-4*x)*23
^(1/2))*2^(1/2)+4/155*(4-5*x)*(2*x^2-x+3)^(1/2)-1/48050*arctanh(1/62*(3514
+x*(9440-6477*2^(1/2))-2963*2^(1/2))*682^(1/2)/(-3169333+2265350*2^(1/2))^
(1/2)/(2*x^2-x+3)^(1/2))*(-1080742553+772484350*2^(1/2))^(1/2)+1/48050*arc
tan(1/62*(3514+2963*2^(1/2)+x*(9440+6477*2^(1/2)))*682^(1/2)/(3169333+2265
350*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(1080742553+772484350*2^(1/2))^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.79

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx = \frac{50\left(\frac{55(7+13x)\sqrt{3-x+2x^2}}{2+3x+5x^2} - 62\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)\right) + 682\text{RootSum}\Big[-\ldots}{\ldots}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
(50*((55*(7 + 13*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) - 62*Sqrt[2]*Lo
g[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]]) + 682*RootSum[-56 - 26*Sqrt[2]*#1 +
17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (999*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
 2*x^2] - #1] + 310*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#
1 + 100*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] +
17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 11*RootSum[-56 - 26*Sqrt[2]*#1 + 1
7*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-72888*Log[-(Sqrt[2]*x) + Sqrt[3 - x
 + 2*x^2] - #1] + 8230*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1
]*#1 + 2025*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2
] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/38750
```

3.70.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {1302, 27, 2138, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}}{(5x^2+3x+2)^2}\,dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)} - \frac{1}{31}\int -\frac{\left(-80x^2-26x+69\right)\sqrt{2x^2-x+3}}{2\left(5x^2+3x+2\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{62}\int \frac{\left(-80x^2-26x+69\right)\sqrt{2x^2-x+3}}{5x^2+3x+2}dx+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 2138}$$

$$\frac{1}{62}\left(\frac{8}{5}(4-5x)\sqrt{2x^2-x+3}-\frac{1}{100}\int -\frac{20\left(248x^2-575x+1307\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{62}\left(\frac{1}{5}\int \frac{248x^2-575x+1307}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{8}{5}\sqrt{2x^2-x+3}(4-5x)\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 2143}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{248}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int \frac{11(549-329x)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+\frac{8}{5}\sqrt{2x^2-x+3}(4-5x)\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{248}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{11}{5}\int \frac{549-329x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+\frac{8}{5}\sqrt{2x^2-x+3}(4-5x)\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\int \frac{549-329x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{124}{5}\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{8}{5}\sqrt{2x^2-x+3}(4\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\int \frac{549-329x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{124}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{8}{5}\sqrt{2x^2-x+3}(4-5x)\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

3.70. $\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx$

$\Big\downarrow$ 1368

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\left(\frac{\int -\frac{11\left(-\left(\left(220-329\sqrt{2}\right)x\right)-549\sqrt{2}+878\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(220+329\sqrt{2}\right)x\right)+549\sqrt{2}+878\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{124}{5}\sqrt{2}\text{arcsinh}\left(\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 27

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\left(\frac{\int \frac{-\left(\left(220+329\sqrt{2}\right)x\right)+549\sqrt{2}+878}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(220-329\sqrt{2}\right)x\right)-549\sqrt{2}+878}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{124}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 1362

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\left(\sqrt{2}\left(3169333-2265350\sqrt{2}\right)\int\frac{1}{-\frac{11\left(\left(9440-6477\sqrt{2}\right)x-2963\sqrt{2}+3514\right)^2}{2x^2-x+3}-62\left(3169333-2265350\sqrt{2}\right)}d\frac{(9\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 217

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{11}{5}\left(\sqrt{2}\left(3169333-2265350\sqrt{2}\right)\int\frac{1}{-\frac{11\left(\left(9440-6477\sqrt{2}\right)x-2963\sqrt{2}+3514\right)^2}{2x^2-x+3}-62\left(3169333-2265350\sqrt{2}\right)}d\frac{(9\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 219

3.70. $\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx$

$$\frac{1}{62}\left(\frac{1}{5}\left(\frac{124}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{11}{5}\left(\sqrt{\frac{1}{341}\left(3169333+2265350\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(3169333+2265350\sqrt{2}\right)}}((9440}{\sqrt{2x^2}}\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}$$

input
```
Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(31*(2 + 3*x + 5*x^2)) + ((8*(4 - 5*x)*
Sqrt[3 - x + 2*x^2])/5 + ((124*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/5 + (
11*(Sqrt[(3169333 + 2265350*Sqrt[2])/341]*ArcTan[(Sqrt[11/(62*(3169333 + 2
265350*Sqrt[2]))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3
- x + 2*x^2]] + ((3169333 - 2265350*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-316933
3 + 2265350*Sqrt[2]))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sqrt[2])*x))/Sq
rt[3 - x + 2*x^2]])/Sqrt[341*(-3169333 + 2265350*Sqrt[2])]))/5)/5)/62
```

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.70. $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} \, dx$

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.70.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.40 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.20

method	result
trager	Expression too large to display
risch	$\frac{11(7+13x)\sqrt{2x^2-x+3}}{155(5x^2+3x+2)}+\frac{2\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{25}+\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(126130\sqrt{-775687+54\right.}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
11/155*(7+13*x)/(5*x^2+3*x+2)*(2*x^2-x+3)^(1/2)-2/25*RootOf(_Z^2-2)*ln(-4*
RootOf(_Z^2-2)*x+4*(2*x^2-x+3)^(1/2)+RootOf(_Z^2-2))+2/155*RootOf(3075200*
_Z^4+8645940424*_Z^2+6209490853225)*ln((41069142240000*x*RootOf(3075200*_Z
^4+8645940424*_Z^2+6209490853225)^5+167323715982737600*RootOf(3075200*_Z^4
+8645940424*_Z^2+6209490853225)^3*x+26624410668240000*RootOf(3075200*_Z^4+
8645940424*_Z^2+6209490853225)^3+409907135729221900 0*RootOf(3075200*_Z^4+8
645940424*_Z^2+6209490853225)^2*(2*x^2-x+3)^(1/2)+138906735546068157756*Ro
otOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)*x+77561425919598618800*Ro
otOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)+58054041501434884274 05*(2
*x^2-x+3)^(1/2))/(6200*x*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225
)^2+9409004*x+924451))+1/48050*RootOf(_Z^2+384400*RootOf(3075200*_Z^4+8645
940424*_Z^2+6209490853225)^2+1080742553)*ln((65710627584*RootOf(_Z^2+38440
0*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2+1080742553)*RootOf(
3075200*_Z^4+8645940424*_Z^2+6209490853225)^4*x+101773581037216*RootOf(307
5200*_Z^4+8645940424*_Z^2+6209490853225)^2*RootOf(_Z^2+384400*RootOf(30752
00*_Z^4+8645940424*_Z^2+6209490853225)^2+1080742553)*x-4066278786433881248
*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2*(2*x^2-x+3)^(1/2)-42
599057069184*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2*RootOf(_
Z^2+384400*RootOf(3075200*_Z^4+8645940424*_Z^2+6209490853225)^2+1080742553
)-11025935123814325*RootOf(_Z^2+384400*RootOf(3075200*_Z^4+8645940424*_...
```

3.70.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.66

$$\int \frac{(3 - x + 2x^2)^{3/2}}{(2 + 3x + 5x^2)^2}\, dx = \frac{\sqrt{62}(5\,x^2 + 3\,x + 2)\sqrt{924451i\,\sqrt{31} - 34862663}\log\left(\frac{\sqrt{62}\sqrt{2\,x^2-x+3}\sqrt{924451i\,\sqrt{31}-34862\ldots}}{}\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/192200*(sqrt(62)*(5*x^2 + 3*x + 2)*sqrt(924451*I*sqrt(31) - 34862663)*lo
g((sqrt(62)*sqrt(2*x^2 - x + 3)*sqrt(924451*I*sqrt(31) - 34862663)*(1757*I
*sqrt(31) - 13609) - 35112925*sqrt(31)*(-I*x + 6*I) + 667145575*x - 772484
350)/x) - sqrt(62)*(5*x^2 + 3*x + 2)*sqrt(924451*I*sqrt(31) - 34862663)*lo
g((sqrt(62)*sqrt(2*x^2 - x + 3)*sqrt(924451*I*sqrt(31) - 34862663)*(-1757*
I*sqrt(31) + 13609) - 35112925*sqrt(31)*(-I*x + 6*I) + 667145575*x - 77248
4350)/x) - sqrt(62)*(5*x^2 + 3*x + 2)*sqrt(-924451*I*sqrt(31) - 34862663)*
log((sqrt(62)*sqrt(2*x^2 - x + 3)*(1757*I*sqrt(31) + 13609)*sqrt(-924451*I
*sqrt(31) - 34862663) - 35112925*sqrt(31)*(I*x - 6*I) + 667145575*x - 7724
84350)/x) + sqrt(62)*(5*x^2 + 3*x + 2)*sqrt(-924451*I*sqrt(31) - 34862663)
*log((sqrt(62)*sqrt(2*x^2 - x + 3)*(-1757*I*sqrt(31) - 13609)*sqrt(-924451
*I*sqrt(31) - 34862663) - 35112925*sqrt(31)*(I*x - 6*I) + 667145575*x - 77
2484350)/x) + 7688*sqrt(2)*(5*x^2 + 3*x + 2)*log(-4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 13640*sqrt(2*x^2 - x + 3)*(13*x +
7))/(5*x^2 + 3*x + 2)
```

3.70.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx=\int \frac{(2x^2-x+3)^{\frac{3}{2}}}{(5x^2+3x+2)^2}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**2, x)
```

3.70.7 Maxima [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx=\int \frac{(2\,x^2-x+3)^{\frac{3}{2}}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

3.70.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{174900625,[8]%%%}+%%%{%%{[-419761500,0]:[1,0,-2]%%},[7]%%%
}+%%%{-68
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2} \, dx = \int \frac{(2\,x^2-x+3)^{3/2}}{(5\,x^2+3\,x+2)^2} \, dx$$

input
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^2, x)
```

3.71 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx$

3.71.1 Optimal result

Integrand size = 27, antiderivative size = 223

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \frac{(3+10x)\,(3-x+2x^2)^{3/2}}{62\,(2+3x+5x^2)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\,(2+3x+5x^2)} + \frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(366990269+259509026\sqrt{2}\right)}}\left(29367+20575\sqrt{2}+\left(70517+49942\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{7688} - \frac{3\sqrt{\frac{1}{682}\left(-366990269+259509026\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-366990269+259509026\sqrt{2}\right)}}\left(29367-20575\sqrt{2}+\left(70517-49942\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{7688}$$

output
```
1/62*(3+10*x)*(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2+3/3844*(277+696*x)*(2*x^2-
x+3)^(1/2)/(5*x^2+3*x+2)-3/5243216*arctanh(1/31*(29367+x*(70517-49942*2^(1
/2))-20575*2^(1/2))*341^(1/2)/(-366990269+259509026*2^(1/2))^(1/2)/(2*x^2-
x+3)^(1/2))*(-250287363458+176985155732*2^(1/2))^(1/2)+3/5243216*arctan(1/
31*(29367+20575*2^(1/2)+x*(70517+49942*2^(1/2)))*341^(1/2)/(366990269+2595
09026*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(250287363458+176985155732*2^(1/2)
)^(1/2)
```

3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.83 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.57

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \frac{\frac{3306250\sqrt{3-x+2x^2}(2220+8343x+10171x^2+11680x^3)}{(2+3x+5x^2)^2} - 42578694225\text{RootSum}\left[-56-26\sqrt{2}\#\right.}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((3306250*Sqrt[3 - x + 2*x^2]*(2220 + 8343*x + 10171*x^2 + 11680*x^3))/(2
+ 3*x + 5*x^2)^2 - 42578694225*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*S
qrt[2]*#1^3 - 5*#1^4 & , Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]/(-13
*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 406695200*RootSum[-56 -
 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (93*Log[-(Sqrt[2]*x
) + Sqrt[3 - x + 2*x^2] - #1] + 10*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] + 1
4*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (492
6449381*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 26609914
65*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1
 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 186*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1
^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (155209944*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqr
t[3 - x + 2*x^2] - #1]*#1 - 248390285*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^
2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/12709
225000
```

3.71.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1302, 27, 1346, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}}{(5x^2+3x+2)^3}\,dx$$

$$\Big\downarrow \; 1302$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}-\frac{1}{62}\int -\frac{3(63-22x)\sqrt{2x^2-x+3}}{2\left(5x^2+3x+2\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\frac{3}{124}\int \frac{(63-22x)\sqrt{2x^2-x+3}}{\left(5x^2+3x+2\right)^2}dx+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 1346$$

$$\frac{3}{124}\left(\frac{(696x+277)\sqrt{2x^2-x+3}}{31\left(5x^2+3x+2\right)}-\frac{1}{31}\int -\frac{4453-1804x}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{124}\left(\frac{1}{62}\int \frac{4453-1804x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{\sqrt{2x^2-x+3}(696x+277)}{31\left(5x^2+3x+2\right)}\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 1368$$

$$\frac{3}{124}\left(\frac{1}{62}\left(\frac{\int -\frac{11\left(-\left(\left(2649-1804\sqrt{2}\right)x\right)-4453\sqrt{2}+6257\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(2649+1804\sqrt{2}\right)x\right)+4453\sqrt{2}+6257\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{\sqrt{2x^2-x\cdots}}{31\,(5x\cdots}\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{124}\left(\frac{1}{62}\left(\frac{\int \frac{-\left(\left(2649+1804\sqrt{2}\right)x\right)+4453\sqrt{2}+6257}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(2649-1804\sqrt{2}\right)x\right)-4453\sqrt{2}+6257}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(696x+\cdots}{31\left(5x^2+3x+2\right)}\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 1362$$

3.71. $\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx$

$$\frac{3}{124}\left(\frac{1}{62}\left(\frac{\left(366990269-259509026\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(70517-49942\sqrt{2}\right)x-20575\sqrt{2}+29367\right)^2}{2x^2-x+3}-31\left(366990269-259509026\sqrt{2}\right)}d\frac{\left(70517-4994\right.}{}}{\sqrt{2}}\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 217

$$\frac{3}{124}\left(\frac{1}{62}\left(\frac{\left(366990269-259509026\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(70517-49942\sqrt{2}\right)x-20575\sqrt{2}+29367\right)^2}{2x^2-x+3}-31\left(366990269-259509026\sqrt{2}\right)}d\frac{\left(70517-4994\right.}{}}{\sqrt{2}}\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 219

$$\frac{3}{124}\left(\frac{1}{62}\left(\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(366990269+259509026\sqrt{2}\right)}}\left(\left(70517+49942\sqrt{2}\right)x+205\right.}{\sqrt{2x^2-x+3}}\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}$$

input
```
Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(((277 +
696*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(366990269 + 25
9509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))
]*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]
] + ((366990269 - 259509026*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-366990269 + 25
9509026*Sqrt[2]))]*(29367 - 20575*Sqrt[2] + (70517 - 49942*Sqrt[2])*x))/Sq
rt[3 - x + 2*x^2]])/Sqrt[682*(-366990269 + 259509026*Sqrt[2])])/62))/124
```

3.71. $\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx$

3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1346
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(g*b - 2*a*h - (b*h - 2*g*
c)*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1)
)), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d
+ e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
 (2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*
(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

3.71. $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx$

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.71.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.81 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.17

method	result
trager	Expression too large to display
risch	$\frac{(11680x^3+10171x^2+8343x+2220)\sqrt{2x^2-x+3}}{3844(5x^2+3x+2)^2}+\frac{3\sqrt{\frac{8(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}}\sqrt{2}\left(1915561\sqrt{-775687+549362\sqrt{\right.}}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3844*(11680*x^3+10171*x^2+8343*x+2220)/(5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2)
+3/5243216*RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+8009195630656*_Z^2+1
6836233643867169)^2+250287363458)*ln((36607893262336*RootOf(_Z^2+29767936*
RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^2+250287363458
)*RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^4*x+32626452
3201744512*RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^2*R
ootOf(_Z^2+29767936*RootOf(952573952*_Z^4+8009195630656*_Z^2+1683623364386
7169)^2+250287363458)*x+3707589189779200*RootOf(952573952*_Z^4+80091956306
56*_Z^2+16836233643867169)^2*RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+80
09195630656*_Z^2+16836233643867169)^2+250287363458)+1225107842671457930662
4*(2*x^2-x+3)^(1/2)*RootOf(952573952*_Z^4+8009195630656*_Z^2+1683623364386
7169)^2+72386220273374938520l*RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+8
009195630656*_Z^2+16836233643867169)^2+250287363458)*x+1759868749834835570
0*RootOf(_Z^2+29767936*RootOf(952573952*_Z^4+8009195630656*_Z^2+1683623364
3867169)^2+250287363458)+5152337237574050505771805 4*(2*x^2-x+3)^(1/2))/(21
824*x*RootOf(952573952*_Z^4+8009195630656*_Z^2+16836233643867169)^2+921288
44*x+508369))+3/961*RootOf(952573952*_Z^4+8009195630656*_Z^2+1683623364386
7169)*ln(-(-585726292197376*x*RootOf(952573952*_Z^4+8009195630656*_Z^2+168
36233643867169)^5-462928419465770086 4*RootOf(952573952*_Z^4+8009195630656*
_Z^2+16836233643867169)^3*x+3592691620737413286 4*(2*x^2-x+3)^(1/2)*Root...
```

3.71.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.78

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx =$$

$$-\frac{\sqrt{341}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\sqrt{4575321i\,\sqrt{31}-3302912421}\,\log\left(\frac{\sqrt{341}\sqrt{2\,x^2-x+3}\sqrt{4575321i\,\sqrt{31}-3302\cdots}}{\cdots}\right.}{\cdots}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
-1/10486432*(sqrt(341)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(4575321*
I*sqrt(31) - 3302912421)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(4575321*I
*sqrt(31) - 3302912421)*(29367*I*sqrt(31) + 193967) - 12067169709*sqrt(31)
*(I*x - 6*I) + 229276224471*x - 265477733598)/x) - sqrt(341)*(25*x^4 + 30*
x^3 + 29*x^2 + 12*x + 4)*sqrt(4575321*I*sqrt(31) - 3302912421)*log((sqrt(3
41)*sqrt(2*x^2 - x + 3)*sqrt(4575321*I*sqrt(31) - 3302912421)*(-29367*I*sq
rt(31) - 193967) - 12067169709*sqrt(31)*(I*x - 6*I) + 229276224471*x - 265
477733598)/x) - sqrt(341)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(-4575
321*I*sqrt(31) - 3302912421)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(29367*I*s
qrt(31) - 193967)*sqrt(-4575321*I*sqrt(31) - 3302912421) - 12067169709*sqr
t(31)*(-I*x + 6*I) + 229276224471*x - 265477733598)/x) + sqrt(341)*(25*x^4
 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(-4575321*I*sqrt(31) - 3302912421)*log(
(sqrt(341)*sqrt(2*x^2 - x + 3)*(-29367*I*sqrt(31) + 193967)*sqrt(-4575321*
I*sqrt(31) - 3302912421) - 12067169709*sqrt(31)*(-I*x + 6*I) + 22927622447
1*x - 265477733598)/x) - 2728*(11680*x^3 + 10171*x^2 + 8343*x + 2220)*sqrt
(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

3.71.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}}{(5x^2+3x+2)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**3, x)
```

3.71.7 Maxima [F]

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{3}{2}}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

3.71.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(3/2)/(3*x + 5*x^2 + 2)^3, x)
```

3.72 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 \, dx$

3.72.1 Optimal result

Integrand size = 27, antiderivative size = 254

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 \, dx = -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368} - \frac{9226119881(1-4x)(3-x+2x^2)^{3/2}}{2147483648} - \frac{401135647(1-4x)(3-x+2x^2)^{5/2}}{335544320} + \frac{25250178739(3-x+2x^2)^{7/2}}{5725224960} + \frac{112244125x(3-x+2x^2)^{7/2}}{122683392} + \frac{122595067x^2(3-x+2x^2)^{7/2}}{19169280} + \frac{23460839x^3(3-x+2x^2)^{7/2}}{532480} + \frac{3684995x^4(3-x+2x^2)^{7/2}}{39936} + \frac{1046225x^5(3-x+2x^2)^{7/2}}{9984} + \frac{13875}{208}x^6(3-x+2x^2)^{7/2} + \frac{625}{28}x^7(3-x+2x^2)^{7/2} - \frac{14641852251147\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{68719476736\sqrt{2}}$$

output
```
-9226119881/2147483648*(1-4*x)*(2*x^2-x+3)^(3/2)-401135647/335544320*(1-4*
x)*(2*x^2-x+3)^(5/2)+25250178739/5725224960*(2*x^2-x+3)^(7/2)+112244125/12
2683392*x*(2*x^2-x+3)^(7/2)+122595067/19169280*x^2*(2*x^2-x+3)^(7/2)+23460
839/532480*x^3*(2*x^2-x+3)^(7/2)+3684995/39936*x^4*(2*x^2-x+3)^(7/2)+10462
25/9984*x^5*(2*x^2-x+3)^(7/2)+13875/208*x^6*(2*x^2-x+3)^(7/2)+625/28*x^7*(
2*x^2-x+3)^(7/2)-14641852251147/137438953472*arcsinh(1/23*(1-4*x)*23^(1/2)
)*2^(1/2)-636602271789/34359738368*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.45

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx = \frac{4\sqrt{3-x+2x^2}(10820567498568669+12071614275862524x+50064174038215008x^2+14249$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 50064174
038215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 40546838
2284161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 4390645
58846345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 13723
3466130432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13) - 59
958384968446965*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/5628125144
67840
```

3.72.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.20, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.741, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{5/2} \left(5x^2+3x+2\right)^4 dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{28}\int \frac{7}{2}\left(2x^2-x+3\right)^{5/2}\left(13875x^7+15050x^6+18720x^5+14088x^4+7488x^3+3008x^2+768x+128\right)dx+\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int \left(2x^2-x+3\right)^{5/2}\left(13875x^7+15050x^6+18720x^5+14088x^4+7488x^3+3008x^2+768x+128\right)dx+\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{26}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(1046225x^6+473940x^5+732576x^4+389376x^3+156416x^2+39936x+6656\right)dx+\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\int \left(2x^2-x+3\right)^{5/2}\left(1046225x^6+473940x^5+732576x^4+389376x^3+156416x^2+39936x+6656\right)dx+\frac{1}{}\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{24}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(40534945x^5+3776898x^4+18690048x^3+7507968x^2+1916928x+319488\right)dx\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\int \left(2x^2-x+3\right)^{5/2}\left(40534945x^5+3776898x^4+18690048x^3+7507968x^2+1916928x+319488\right)dx+\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{1}{22}\int \frac{33}{2}\left(2x^2-x+3\right)^{5/2}\left(23460839x^4-4559896x^3+10010624x^2+2555904x+425984\right)dx+\frac{3684}{2}\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\int \left(2x^2-x+3\right)^{5/2}\left(23460839x^4-4559896x^3+10010624x^2+2555904x+425984\right)dx+\frac{3684995}{2}\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\big\downarrow$ 2192

3.72. $\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^4\,dx$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{20}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(122595067x^3-21870142x^2+102236160x+17039360\right)dx+\frac{23460839}{20}\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\int \left(2x^2-x+3\right)^{5/2}\left(122595067x^3-21870142x^2+102236160x+17039360\right)dx+\frac{23460839}{20}x^3\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{18}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(561220625x^2+2209360956x+613416960\right)dx+\frac{122595067}{18}x^2\left(2x^2\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\int \left(2x^2-x+3\right)^{5/2}\left(561220625x^2+2209360956x+613416960\right)dx+\frac{122595067}{18}x^2\left(2x^2-\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\downarrow$ 2192

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{1}{16}\int \frac{3}{2}(25250178739x+5420672990)\left(2x^2-x+3\right)^{5/2}dx+\frac{561220625}{16}x\left(2x^2-x+3\right)^{7}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\int (25250178739x+5420672990)\left(2x^2-x+3\right)^{5/2}dx+\frac{561220625}{16}x\left(2x^2-x+3\right)^{7/2}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\downarrow$ 1160

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\int \left(2x^2-x+3\right)^{5/2}dx+\frac{25250178739}{14}\left(2x^2-x+3\right)^{7/2}\right)+\frac{56122062}{16}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\downarrow$ 1087

3.72. $\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^4\,dx$

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)+\frac{25250\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\Big\downarrow$ 1087

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}(\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\Big\downarrow$ 1087

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\Big\downarrow$ 1090

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

$\Big\downarrow$ 222

$$\frac{1}{8}\left(\frac{1}{52}\left(\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{40}\left(\frac{1}{36}\left(\frac{3}{32}\left(\frac{46932870699}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7$$

input
```
Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

3.72. $\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^4\,dx$

output
```
(625*x^7*(3 - x + 2*x^2)^(7/2))/28 + ((13875*x^6*(3 - x + 2*x^2)^(7/2))/26
 + ((1046225*x^5*(3 - x + 2*x^2)^(7/2))/24 + ((3684995*x^4*(3 - x + 2*x^2)
^(7/2))/2 + (3*((23460839*x^3*(3 - x + 2*x^2)^(7/2))/20 + ((122595067*x^2*
(3 - x + 2*x^2)^(7/2))/18 + ((561220625*x*(3 - x + 2*x^2)^(7/2))/16 + (3*(
(25250178739*(3 - x + 2*x^2)^(7/2))/14 + (46932870699*(-1/24*((1 - 4*x)*(3
 - x + 2*x^2)^(5/2)) + (115*(-1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69
*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(16*Sqrt[2])))/32))/48))/4))/32)/36)/40))/4)/48)/52)/8
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.72.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.37

method	result
risch	$\frac{(25125558681600000x^{13}+37398427729920000x^{12}+137233466130432000x^{11}+204932411660697600x^{10}+363646430503501824x^{9}+43}{}$
trager	$\left(\frac{1250}{7}x^{13}+\frac{48375}{182}x^{12}+\frac{1217225}{1248}x^{11}+\frac{50895515}{34944}x^{10}+\frac{172023939}{66560}x^{9}+\frac{52340574127}{16773120}x^{8}+\frac{2023708176167}{536739840}x^{7}+\frac{246}{7}\right.$
default	$\frac{401135647(-1+4x)(2x^2-x+3)^{\frac{5}{2}}}{335544320}+\frac{9226119881(-1+4x)(2x^2-x+3)^{\frac{3}{2}}}{2147483648}+\frac{636602271789(-1+4x)\sqrt{2x^2-x+3}}{34359738368}+\frac{146418522511}{}$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/140703128616960*(25125558681600000*x^13+37398427729920000*x^12+137233466
130432000*x^11+204932411660697600*x^10+363646430503501824*x^9+439064558846
345216*x^8+530502956133122048*x^7+485091164642279424*x^6+40546838228416102
4*x^5+257786732552566784*x^4+142490931553577856*x^3+50064174038215008*x^2+
12071614275862524*x+10820567498568669)*(2*x^2-x+3)^(1/2)+14641852251147/13
7438953472*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.46

$$\int (3-x+2x^2)^{5/2}(2+3x+5x^2)^4\,dx = \frac{1}{140703128616960}\left(25125558681600000\,x^{13}+37398427729920000\,x^{12}+137233466130432000\right.$$
$$+\frac{14641852251147}{274877906944}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

output
```
1/140703128616960*(25125558681600000*x^13 + 37398427729920000*x^12 + 13723
3466130432000*x^11 + 204932411660697600*x^10 + 363646430503501824*x^9 + 43
9064558846345216*x^8 + 530502956133122048*x^7 + 485091164642279424*x^6 + 4
05468382284161024*x^5 + 257786732552566784*x^4 + 142490931553577856*x^3 +
50064174038215008*x^2 + 12071614275862524*x + 10820567498568669)*sqrt(2*x^
2 - x + 3) + 14641852251147/274877906944*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2
 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.49

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^4 \, dx = \sqrt{2x^2-x+3} \cdot \left(\frac{1250x^{13}}{7} + \frac{48375x^{12}}{182} + \frac{1217225x^{11}}{1248} + \frac{50895515x^{10}}{34944} + \frac{172023939x^9}{66560} + \frac{52340574127x^8}{16773120} + \frac{2023708176167x^7}{536739840} + \frac{2467301252453x^6}{715653120} + \frac{49495652134297x^5}{17175674880} + \frac{17981775429169x^4}{9814671360} + \frac{371070134254109x^3}{366414397440} + \frac{24833419661813x^2}{69793218560} + \frac{335322618773959x}{3908420239360} + \frac{1202285277618741}{15633680957440}\right) + \frac{14641852251147\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}(x-\frac{1}{4})}{23}\right)}{137438953472}$$

input
```
integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**4,x)
```

output
```
sqrt(2*x**2 - x + 3)*(1250*x**13/7 + 48375*x**12/182 + 1217225*x**11/1248
+ 50895515*x**10/34944 + 172023939*x**9/66560 + 52340574127*x**8/16773120
+ 2023708176167*x**7/536739840 + 2467301252453*x**6/715653120 + 4949565213
4297*x**5/17175674880 + 17981775429169*x**4/9814671360 + 371070134254109*x
**3/366414397440 + 24833419661813*x**2/69793218560 + 335322618773959*x/390
8420239360 + 1202285277618741/15633680957440) + 14641852251147*sqrt(2)*asi
nh(4*sqrt(23)*(x - 1/4)/23)/137438953472
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.93

$$\int (3-x+2x^2)^{5/2}\,(2+3x+5x^2)^4\,dx = \frac{625}{28}\,(2\,x^2-x+3)^{\frac{7}{2}}x^7 + \frac{13875}{208}\,(2\,x^2-x+3)^{\frac{7}{2}}x^6 + \frac{1046225}{9984}\,(2\,x^2-x+3)^{\frac{7}{2}}x^5 + \frac{3684995}{39936}\,(2\,x^2-x+3)^{\frac{7}{2}}x^4 + \frac{23460839}{532480}\,(2\,x^2-x+3)^{\frac{7}{2}}x^3 + \frac{122595067}{19169280}\,(2\,x^2-x+3)^{\frac{7}{2}}x^2 + \frac{112244125}{122683392}\,(2\,x^2-x+3)^{\frac{7}{2}}x + \frac{25250178739}{5725224960}\,(2\,x^2-x+3)^{\frac{7}{2}} + \frac{401135647}{83886080}\,(2\,x^2-x+3)^{\frac{5}{2}}x - \frac{401135647}{335544320}\,(2\,x^2-x+3)^{\frac{5}{2}} + \frac{9226119881}{536870912}\,(2\,x^2-x+3)^{\frac{3}{2}}x - \frac{9226119881}{2147483648}\,(2\,x^2-x+3)^{\frac{3}{2}} + \frac{636602271789}{8589934592}\,\sqrt{2\,x^2-x+3}x + \frac{14641852251147}{137438953472}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{636602271789}{34359738368}\,\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

output
```
625/28*(2*x^2 - x + 3)^(7/2)*x^7 + 13875/208*(2*x^2 - x + 3)^(7/2)*x^6 + 1
046225/9984*(2*x^2 - x + 3)^(7/2)*x^5 + 3684995/39936*(2*x^2 - x + 3)^(7/2
)*x^4 + 23460839/532480*(2*x^2 - x + 3)^(7/2)*x^3 + 122595067/19169280*(2*
x^2 - x + 3)^(7/2)*x^2 + 112244125/122683392*(2*x^2 - x + 3)^(7/2)*x + 252
50178739/5725224960*(2*x^2 - x + 3)^(7/2) + 401135647/83886080*(2*x^2 - x
+ 3)^(5/2)*x - 401135647/335544320*(2*x^2 - x + 3)^(5/2) + 9226119881/5368
70912*(2*x^2 - x + 3)^(3/2)*x - 9226119881/2147483648*(2*x^2 - x + 3)^(3/2
) + 636602271789/8589934592*sqrt(2*x^2 - x + 3)*x + 14641852251147/1374389
53472*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 636602271789/34359738368*
sqrt(2*x^2 - x + 3)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.44

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx = \frac{1}{140703128616960} \left(4 \left(8 \left(4 \left(16 \left(4 \left(8 \left(4 \left(32 \left(12 \left(200 \left(20 \left(240 \left(260 \, x+387\right)x+340823\right)x+10179\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$-\frac{14641852251147}{137438953472} \sqrt{2} \log \left(-2 \sqrt{2}\left(\sqrt{2}x-\sqrt{2 \, x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

output
```
1/140703128616960*(4*(8*(4*(16*(4*(8*(4*(32*(12*(200*(20*(240*(260*x + 387
)*x + 340823)*x + 10179103)*x + 3612502719)*x + 52340574127)*x + 202370817
6167)*x + 7401903757359)*x + 49495652134297)*x + 125872428004183)*x + 1113
210402762327)*x + 1564505438694219)*x + 3017903568965631)*x + 108205674985
68669)*sqrt(2*x^2 - x + 3) - 14641852251147/137438953472*sqrt(2)*log(-2*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx = \int \left(2\,x^2-x+3\right)^{5/2} \left(5\,x^2+3\,x+2\right)^4 \, dx$$

input
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^4, x)
```

3.73 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 \, dx$

3.73.1 Optimal result

Integrand size = 27, antiderivative size = 212

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 \, dx = -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x)(3-x+2x^2)^{3/2}}{67108864} - \frac{57915(1-4x)(3-x+2x^2)^{5/2}}{2097152} - \frac{1696165(3-x+2x^2)^{7/2}}{2752512} + \frac{509257x(3-x+2x^2)^{7/2}}{294912} + \frac{80483x^2(3-x+2x^2)^{7/2}}{9216} + \frac{3823}{256}x^3(3-x+2x^2)^{7/2} + \frac{1175}{96}x^4(3-x+2x^2)^{7/2} + \frac{125}{24}x^5(3-x+2x^2)^{7/2} - \frac{10569777075\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2147483648\sqrt{2}}$$

output
```
-6660225/67108864*(1-4*x)*(2*x^2-x+3)^(3/2)-57915/2097152*(1-4*x)*(2*x^2-x
+3)^(5/2)-1696165/2752512*(2*x^2-x+3)^(7/2)+509257/294912*x*(2*x^2-x+3)^(7
/2)+80483/9216*x^2*(2*x^2-x+3)^(7/2)+3823/256*x^3*(2*x^2-x+3)^(7/2)+1175/9
6*x^4*(2*x^2-x+3)^(7/2)+125/24*x^5*(2*x^2-x+3)^(7/2)-10569777075/429496729
6*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-459555525/1073741824*(1-4*x)*(2*x
^2-x+3)^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.50

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 \, dx = \frac{4\sqrt{3-x+2x^2}(-1191399152715+4560943728924x+10060731582048x^2+20384824684416}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-1191399152715 + 4560943728924*x + 10060731582048*
x^2 + 20384824684416*x^3 + 26186527209472*x^4 + 34378613923840*x^5 + 28347
538538496*x^6 + 27835561148416*x^7 + 14341894045696*x^8 + 12943588589568*x
^9 + 2395786444800*x^10 + 2818572288000*x^11) - 665895955725*Sqrt[2]*Log[1
 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/270582939648
```

3.73.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.593$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{5/2} \left(5x^2+3x+2\right)^3 \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{24}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2} \left(12925x^5+9930x^4+9936x^3+5472x^2+1728x+384\right)dx+\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\int \left(2x^2-x+3\right)^{5/2} \left(12925x^5+9930x^4+9936x^3+5472x^2+1728x+384\right)dx+\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{48}\left(\frac{1}{22}\int \frac{33}{2}\left(2x^2-x+3\right)^{5/2}\left(19115x^4+3848x^3+7296x^2+2304x+512\right)dx+\frac{1175}{2}\left(2x^2-x+3\right)^{7/2}x^4\right)+$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{48}\left(\frac{3}{4}\int \left(2x^2-x+3\right)^{5/2}\left(19115x^4+3848x^3+7296x^2+2304x+512\right)dx+\frac{1175}{2}\left(2x^2-x+3\right)^{7/2}x^4\right)+$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{20}\int \frac{5}{2}\left(2x^2-x+3\right)^{5/2}\left(80483x^3-10446x^2+18432x+4096\right)dx+\frac{3823}{4}x^3\left(2x^2-x+3\right)^{7/2}\right)+\frac{1175}{2}(2$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\int \left(2x^2-x+3\right)^{5/2}\left(80483x^3-10446x^2+18432x+4096\right)dx+\frac{3823}{4}x^3\left(2x^2-x+3\right)^{7/2}\right)+\frac{1175}{2}(2x^2$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{18}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(509257x^2-302244x+147456\right)dx+\frac{80483}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{3823}{4}x^3($$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\int \left(2x^2-x+3\right)^{5/2}\left(509257x^2-302244x+147456\right)dx+\frac{80483}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{3823}{4}x^3(2x$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{1}{16}\int \frac{15}{2}(110870-339233x)\left(2x^2-x+3\right)^{5/2}dx+\frac{509257}{16}x\left(2x^2-x+3\right)^{7/2}\right)+\frac{80483}{18}x^2(2x^2-$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \; 27$$

3.73. $\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3\,dx$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\int(110870-339233x)\left(2x^2-x+3\right)^{5/2}dx+\frac{509257}{16}x\left(2x^2-x+3\right)^{7/2}\right)+\frac{80483}{18}x^2\left(2x^2-x+\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\int\left(2x^2-x+3\right)^{5/2}dx-\frac{339233}{14}\left(2x^2-x+3\right)^{7/2}\right)+\frac{509257}{16}x\left(2x^2-x+3\right)^{7/2}\right)+\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\int\left(2x^2-x+3\right)^{3/2}dx-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)-\frac{339233}{14}\left(2x^2-x+3\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\left(\frac{69}{32}\int\sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}(1-4x)\left(2x^2-\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

$$\Big\downarrow \text{222}$$

3.73. $\int\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3\,dx$

$$\frac{1}{48}\left(\frac{3}{4}\left(\frac{1}{8}\left(\frac{1}{36}\left(\frac{15}{32}\left(\frac{104247}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5$$

input
```
Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

output
```
(125*x^5*(3 - x + 2*x^2)^(7/2))/24 + ((1175*x^4*(3 - x + 2*x^2)^(7/2))/2 +
 (3*((3823*x^3*(3 - x + 2*x^2)^(7/2))/4 + ((80483*x^2*(3 - x + 2*x^2)^(7/2
))/18 + ((509257*x*(3 - x + 2*x^2)^(7/2))/16 + (15*((-339233*(3 - x + 2*x^
2)^(7/2))/14 + (104247*(-1/24*((1 - 4*x)*(3 - x + 2*x^2)^(5/2)) + (115*(-1
/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x +
2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/48))/4))/3
2)/36)/8))/4)/48
```

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.73.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.40

method	result
risch	$\frac{(2818572288000x^{11}+2395786444800x^{10}+12943588589568x^{9}+14341894045696x^{8}+27835561148416x^{7}+28347538538496x^{6}+343786\cdots}{676457349\cdots}$
trager	$\left(\frac{125}{3}x^{11}+\frac{425}{12}x^{10}+\frac{6123}{32}x^{9}+\frac{244241}{1152}x^{8}+\frac{15169177}{36864}x^{7}+\frac{144183037}{344064}x^{6}+\frac{4196608145}{8257536}x^{5}+\frac{1826627177}{4718592}x^{4}+\frac{530\cdots}{17\cdots}\right.$
default	$\frac{57915(-1+4x)(2x^{2}-x+3)^{\frac{5}{2}}}{2097152}+\frac{6660225(-1+4x)(2x^{2}-x+3)^{\frac{3}{2}}}{67108864}+\frac{459555525(-1+4x)\sqrt{2x^{2}-x+3}}{1073741824}+\frac{10569777075\sqrt{2}\,\mathrm{arcsinh}\left(\cdots\right)}{4294967296\cdots}$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/67645734912*(2818572288000*x^11+2395786444800*x^10+12943588589568*x^9+14
341894045696*x^8+27835561148416*x^7+28347538538496*x^6+34378613923840*x^5+
26186527209472*x^4+20384824684416*x^3+10060731582048*x^2+4560943728924*x-1
191399152715)*(2*x^2-x+3)^(1/2)+10569777075/4294967296*2^(1/2)*arcsinh(4/2
3*23^(1/2)*(x-1/4))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.51

$$\int \left(3-x+2x^2\right)^{5/2} (2+3x+5x^2)^3\, dx = \frac{1}{67645734912} \left(2818572288000\, x^{11} + 2395786444800\, x^{10} + 12943588589568\, x^9 + 143418940456\right.$$
$$+ \frac{10569777075}{8589934592} \sqrt{2} \log \left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/67645734912*(2818572288000*x^11 + 2395786444800*x^10 + 12943588589568*x^
9 + 14341894045696*x^8 + 27835561148416*x^7 + 28347538538496*x^6 + 3437861
3923840*x^5 + 26186527209472*x^4 + 20384824684416*x^3 + 10060731582048*x^2
 + 4560943728924*x - 1191399152715)*sqrt(2*x^2 - x + 3) + 10569777075/8589
934592*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*
x - 25)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3\, dx = \sqrt{2x^2-x+3} \cdot \left(\frac{125x^{11}}{3} + \frac{425x^{10}}{12} + \frac{6123x^9}{32} + \frac{244241x^8}{1152} + \frac{15169177x^7}{36864} + \frac{144183037x^6}{344064} + \frac{4196608145x^5}{8257536} + \frac{1826627177x^4}{4718592} + \frac{53085480949x^3}{176160768} + \frac{4990442253x^2}{33554432} + \frac{126692881359x}{1879048192} - \frac{132377683635}{7516192768}\right) + \frac{10569777075\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4294967296}$$

input
```
integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)
```

output
```
sqrt(2*x**2 - x + 3)*(125*x**11/3 + 425*x**10/12 + 6123*x**9/32 + 244241*x
**8/1152 + 15169177*x**7/36864 + 144183037*x**6/344064 + 4196608145*x**5/8
257536 + 1826627177*x**4/4718592 + 53085480949*x**3/176160768 + 4990442253
*x**2/33554432 + 126692881359*x/1879048192 - 132377683635/7516192768) + 10
569777075*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4294967296
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.95

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 \, dx = \frac{125}{24} (2x^2-x+3)^{\frac{7}{2}} x^5 + \frac{1175}{96} (2x^2-x+3)^{\frac{7}{2}} x^4 + \frac{3823}{256} (2x^2-x+3)^{\frac{7}{2}} x^3 + \frac{80483}{9216} (2x^2-x+3)^{\frac{7}{2}} x^2 + \frac{509257}{294912} (2x^2-x+3)^{\frac{7}{2}} x - \frac{1696165}{2752512} (2x^2-x+3)^{\frac{7}{2}} + \frac{57915}{524288} (2x^2-x+3)^{\frac{5}{2}} x - \frac{57915}{2097152} (2x^2-x+3)^{\frac{5}{2}} + \frac{6660225}{16777216} (2x^2-x+3)^{\frac{3}{2}} x - \frac{6660225}{67108864} (2x^2-x+3)^{\frac{3}{2}} + \frac{459555525}{268435456} \sqrt{2x^2-x+3}x + \frac{10569777075}{4294967296} \sqrt{2} \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{459555525}{1073741824} \sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
125/24*(2*x^2 - x + 3)^(7/2)*x^5 + 1175/96*(2*x^2 - x + 3)^(7/2)*x^4 + 382
3/256*(2*x^2 - x + 3)^(7/2)*x^3 + 80483/9216*(2*x^2 - x + 3)^(7/2)*x^2 + 5
09257/294912*(2*x^2 - x + 3)^(7/2)*x - 1696165/2752512*(2*x^2 - x + 3)^(7/
2) + 57915/524288*(2*x^2 - x + 3)^(5/2)*x - 57915/2097152*(2*x^2 - x + 3)^
(5/2) + 6660225/16777216*(2*x^2 - x + 3)^(3/2)*x - 6660225/67108864*(2*x^2
 - x + 3)^(3/2) + 459555525/268435456*sqrt(2*x^2 - x + 3)*x + 10569777075/
4294967296*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 459555525/1073741824
*sqrt(2*x^2 - x + 3)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 \, dx = \frac{1}{67645734912} (4 (8 (4 (16 (4 (8 (28 (32 (12 (200 (20x+17)x+18369)x+244241)x+15169177) - \frac{10569777075}{4294967296} \sqrt{2} \log\left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1/67645734912*(4*(8*(4*(16*(4*(8*(28*(32*(12*(200*(20*x + 17)*x + 18369)*x
 + 244241)*x + 15169177)*x + 432549111)*x + 4196608145)*x + 12786390239)*x
 + 159256442847)*x + 314397861939)*x + 1140235932231)*x - 1191399152715)*s
qrt(2*x^2 - x + 3) - 10569777075/4294967296*sqrt(2)*log(-2*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^3 \, dx = \int (2\,x^2-x+3)^{5/2} (5\,x^2+3\,x+2)^3 \, dx$$

input
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3, x)
```

3.74 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 \, dx$

3.74.1 Optimal result

Integrand size = 27, antiderivative size = 170

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 \, dx = -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216} - \frac{177905(1-4x)\,(3-x+2x^2)^{3/2}}{3145728} - \frac{1547(1-4x)\,(3-x+2x^2)^{5/2}}{98304} + \frac{23225(3-x+2x^2)^{7/2}}{43008} + \frac{8467x(3-x+2x^2)^{7/2}}{4608} + \frac{305}{144}x^2(3-x+2x^2)^{7/2} + \frac{5}{4}x^3(3-x+2x^2)^{7/2} - \frac{94111745\,\mathrm{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{33554432\sqrt{2}}$$

output
```
-177905/3145728*(1-4*x)*(2*x^2-x+3)^(3/2)-1547/98304*(1-4*x)*(2*x^2-x+3)^(
5/2)+23225/43008*(2*x^2-x+3)^(7/2)+8467/4608*x*(2*x^2-x+3)^(7/2)+305/144*x
^2*(2*x^2-x+3)^(7/2)+5/4*x^3*(2*x^2-x+3)^(7/2)-94111745/67108864*arcsinh(1
/23*(1-4*x)*23^(1/2))*2^(1/2)-4091815/16777216*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^2 \, dx = \frac{4\sqrt{3-x+2x^2}(14824182519+39533249652x+42992644128x^2+77872272000x^3+5714746}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(14824182519 + 39533249652*x + 42992644128*x^2 + 77
872272000*x^3 + 57147467776*x^4 + 75389820928*x^5 + 26401898496*x^6 + 4416
3137536*x^7 + 2055208960*x^8 + 10569646080*x^9) - 5929039935*Sqrt[2]*Log[1
 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/4227858432
```

3.74.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{5/2} \left(5x^2+3x+2\right)^2 dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{20}\int \frac{5}{2}\left(2x^2-x+3\right)^{5/2}\left(305x^3+142x^2+96x+32\right)dx+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \left(2x^2-x+3\right)^{5/2}\left(305x^3+142x^2+96x+32\right)dx+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{18}\int \frac{1}{2}\left(2x^2-x+3\right)^{5/2}\left(8467x^2-204x+1152\right)dx+\frac{305}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{36}\int \left(2x^2-x+3\right)^{5/2}\left(8467x^2-204x+1152\right)dx+\frac{305}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{1}{16}\int -\frac{3}{2}(4646-23225x)\left(2x^2-x+3\right)^{5/2}dx+\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}\right)+\frac{305}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\int (4646-23225x)\left(2x^2-x+3\right)^{5/2}dx\right)+\frac{305}{18}x^2\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\int \left(2x^2-x+3\right)^{5/2}dx-\frac{23225}{14}\left(2x^2-x+3\right)^{7/2}\right)\right)+\frac{305}{18}x^2\left(2x^2 -\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)-\frac{232}{1}\right.\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{2}\right.\right.\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{}{1}\right.\right.\right.\right.\right.$$
$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 1090$$

3.74. $\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2\,dx$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\right.\right.\right.\right.\right.\right.$$

$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{8}\left(\frac{1}{36}\left(\frac{8467}{16}x\left(2x^2-x+3\right)^{7/2}-\frac{3}{32}\left(-\frac{4641}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}\right.\right.\right.\right.\right.$$

$$\frac{5}{4}x^3\left(2x^2-x+3\right)^{7/2}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

output
```
(5*x^3*(3 - x + 2*x^2)^(7/2))/4 + ((305*x^2*(3 - x + 2*x^2)^(7/2))/18 + ((
8467*x*(3 - x + 2*x^2)^(7/2))/16 - (3*((-23225*(3 - x + 2*x^2)^(7/2))/14 -
 (4641*(-1/24*((1 - 4*x)*(3 - x + 2*x^2)^(5/2)) + (115*(-1/16*((1 - 4*x)*(
3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*Ar
cSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/48))/4))/32)/36)/8
```

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.74.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467776x^4+77872272000x^3+4299264412}{1056964608}$
trager	$\left(10x^9+\frac{35}{18}x^8+\frac{24067}{576}x^7+\frac{134287}{5376}x^6+\frac{9202859}{129024}x^5+\frac{3986291}{73728}x^4+\frac{202792375}{2752512}x^3+\frac{63977149}{1572864}x^2+\frac{3294437471}{88080384}x\right.$
default	$\frac{1547(-1+4x)(2x^2-x+3)^{\frac{5}{2}}}{98304}+\frac{177905(-1+4x)(2x^2-x+3)^{\frac{3}{2}}}{3145728}+\frac{4091815(-1+4x)\sqrt{2x^2-x+3}}{16777216}+\frac{94111745\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,(x-}{23}\right.}{67108864}$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1056964608*(10569646080*x^9+2055208960*x^8+44163137536*x^7+26401898496*x
^6+75389820928*x^5+57147467776*x^4+77872272000*x^3+42992644128*x^2+3953324
9652*x+14824182519)*(2*x^2-x+3)^(1/2)+94111745/67108864*2^(1/2)*arcsinh(4/
23*23^(1/2)*(x-1/4))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^2 \, dx = \frac{1}{1056964608} \left(10569646080\, x^9 + 2055208960\, x^8 + 44163137536\, x^7 + 26401898496\, x^6 + 7538982\right.$$
$$+ \frac{94111745}{134217728} \sqrt{2} \log \left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/1056964608*(10569646080*x^9 + 2055208960*x^8 + 44163137536*x^7 + 2640189
8496*x^6 + 75389820928*x^5 + 57147467776*x^4 + 77872272000*x^3 + 429926441
28*x^2 + 39533249652*x + 14824182519)*sqrt(2*x^2 - x + 3) + 94111745/13421
7728*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x
- 25)
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^2 \, dx = \sqrt{2x^2-x+3} \cdot \left(10x^9 + \frac{35x^8}{18} + \frac{24067x^7}{576} + \frac{134287x^6}{5376} + \frac{9202859x^5}{129024} + \frac{3986291x^4}{73728} + \frac{202792375x^3}{2752512} + \frac{63977149x^2}{1572864} + \frac{3294437471x}{88080384} + \frac{1647131391}{117440512}\right) + \frac{94111745\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{67108864}$$

input
```
integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**2,x)
```

output
```
sqrt(2*x**2 - x + 3)*(10*x**9 + 35*x**8/18 + 24067*x**7/576 + 134287*x**6/
5376 + 9202859*x**5/129024 + 3986291*x**4/73728 + 202792375*x**3/2752512 +
 63977149*x**2/1572864 + 3294437471*x/88080384 + 1647131391/117440512) + 9
4111745*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/67108864
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2\,dx=\frac{5}{4}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^3+\frac{305}{144}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^2+\frac{8467}{4608}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x+\frac{23225}{43008}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{1547}{24576}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x-\frac{1547}{98304}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{177905}{786432}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x-\frac{177905}{3145728}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{4091815}{4194304}\sqrt{2\,x^2-x+3x}+\frac{94111745}{67108864}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{4091815}{16777216}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
5/4*(2*x^2 - x + 3)^(7/2)*x^3 + 305/144*(2*x^2 - x + 3)^(7/2)*x^2 + 8467/4
608*(2*x^2 - x + 3)^(7/2)*x + 23225/43008*(2*x^2 - x + 3)^(7/2) + 1547/245
76*(2*x^2 - x + 3)^(5/2)*x - 1547/98304*(2*x^2 - x + 3)^(5/2) + 177905/786
432*(2*x^2 - x + 3)^(3/2)*x - 177905/3145728*(2*x^2 - x + 3)^(3/2) + 40918
15/4194304*sqrt(2*x^2 - x + 3)*x + 94111745/67108864*sqrt(2)*arcsinh(1/23*
sqrt(23)*(4*x - 1)) - 4091815/16777216*sqrt(2*x^2 - x + 3)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.55

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2\,dx=\frac{1}{1056964608}\left(4\left(8\left(4\left(16\left(4\left(8\left(28\left(160\left(36\,x+7\right)x+24067\right)x+402861\right)x+9202859\right)x+27904\right.\right.\right.\right.\right.\right.\right. -\frac{94111745}{67108864}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
1/1056964608*(4*(8*(4*(16*(4*(8*(28*(160*(36*x + 7)*x + 24067)*x + 402861)
*x + 9202859)*x + 27904037)*x + 608377125)*x + 1343520129)*x + 9883312413)
*x + 14824182519)*sqrt(2*x^2 - x + 3) - 94111745/67108864*sqrt(2)*log(-2*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^2 \, dx = \int \left(2\,x^2-x+3\right)^{5/2} \left(5\,x^2+3\,x+2\right)^2 dx$$

input
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2, x)
```

3.75 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2) \, dx$

3.75.1 Optimal result

Integrand size = 25, antiderivative size = 128

$$\int (3-x+2x^2)^{5/2} (2+3x+5x^2) \, dx = -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288} - \frac{31855(1-4x)(3-x+2x^2)^{3/2}}{98304} - \frac{277(1-4x)(3-x+2x^2)^{5/2}}{3072} + \frac{141}{448}(3-x+2x^2)^{7/2} + \frac{5}{16}x(3-x+2x^2)^{7/2} - \frac{16851295\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1048576\sqrt{2}}$$

output
```
-31855/98304*(1-4*x)*(2*x^2-x+3)^(3/2)-277/3072*(1-4*x)*(2*x^2-x+3)^(5/2)+
141/448*(2*x^2-x+3)^(7/2)+5/16*x*(2*x^2-x+3)^(7/2)-16851295/2097152*arcsin
h(1/23*(1-4*x)*23^(1/2))*2^(1/2)-732665/524288*(1-4*x)*(2*x^2-x+3)^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int (3-x+2x^2)^{5/2} (2+3x +5x^2) \, dx = \frac{4\sqrt{3-x+2x^2}(58536675+148957444x+67272352x^2+172684416x^3-1619968x^4+1188085}{44040192}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(58536675 + 148957444*x + 67272352*x^2 + 172684416*
x^3 - 1619968*x^4 + 118808576*x^5 - 13565952*x^6 + 27525120*x^7) - 3538771
95*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/44040192
```

3.75.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2192, 27, 1160, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2x^2-x+3\right)^{5/2}\left(5x^2+3x+2\right)dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\int \frac{1}{2}(141x+34)\left(2x^2-x+3\right)^{5/2}dx+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\int (141x+34)\left(2x^2-x+3\right)^{5/2}dx+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{32}\left(\frac{277}{4}\int \left(2x^2-x+3\right)^{5/2}dx+\frac{141}{14}\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\int \left(2x^2-x+3\right)^{3/2}dx-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)+\frac{141}{14}\left(2x^2-x+3\right)^{7/2}\right)+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}(1-4x)\left(2x^2-x+3\right)^{5/2}\right)+\frac{141}{14}(\ldots\right)+\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}(1$$
$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$\Big\downarrow$ 1090

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x\right.\right.\right.\right.$$
$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

$\Big\downarrow$ 222

$$\frac{1}{32}\left(\frac{277}{4}\left(\frac{115}{48}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{24}(1-\right.\right.$$
$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

output
```
(5*x*(3 - x + 2*x^2)^(7/2))/16 + ((141*(3 - x + 2*x^2)^(7/2))/14 + (277*(-
1/24*((1 - 4*x)*(3 - x + 2*x^2)^(5/2)) + (115*(-1/16*((1 - 4*x)*(3 - x + 2
*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1
 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/48))/4)/32
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.75.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

method	result
risch	$\frac{\left(27525120 x^{7}-13565952 x^{6}+118808576 x^{5}-1619968 x^{4}+172684416 x^{3}+67272352 x^{2}+148957444 x +58536675\right) \sqrt{2 x^{2}-x +3}}{11010048}+\frac{168512}{}$
trager	$\left(\frac{5}{2} x^{7}-\frac{69}{56} x^{6}+\frac{14503}{1344} x^{5}-\frac{113}{768} x^{4}+\frac{449699}{28672} x^{3}+\frac{300323}{49152} x^{2}+\frac{37239361}{2752512} x +\frac{19512225}{3670016}\right) \sqrt{2 x^{2}-x +3}+\frac{1685}{}$
default	$\frac{277 \left(-1+4 x \right) \left(2 x^{2}-x +3\right)^{\frac{5}{2}}}{3072}+\frac{31855 \left(-1+4 x \right) \left(2 x^{2}-x +3\right)^{\frac{3}{2}}}{98304}+\frac{732665 \left(-1+4 x \right) \sqrt{2 x^{2}-x +3}}{524288}+\frac{16851295 \sqrt{2}\, \operatorname{arcsinh}\left(\frac{4 \sqrt{23}\, \left(x -\frac{1}{4}\right)}{23}\right.}{2097152}$

input
```
int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/11010048*(27525120*x^7-13565952*x^6+118808576*x^5-1619968*x^4+172684416*
x^3+67272352*x^2+148957444*x+58536675)*(2*x^2-x+3)^(1/2)+16851295/2097152*
2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

$$\int \left(3-x+2x^2\right)^{5/2} (2+3x+5x^2)\, dx = \frac{1}{11010048}\left(27525120\,x^7 - 13565952\,x^6 + 118808576\,x^5 - 1619968\,x^4 + 172684416\,x^3 + 6727235\right. + \frac{16851295}{4194304}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/11010048*(27525120*x^7 - 13565952*x^6 + 118808576*x^5 - 1619968*x^4 + 17
2684416*x^3 + 67272352*x^2 + 148957444*x + 58536675)*sqrt(2*x^2 - x + 3) +
 16851295/4194304*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 3
2*x^2 + 16*x - 25)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \left(3-x+2x^2\right)^{5/2} (2+3x+5x^2)\, dx = \sqrt{2x^2-x+3} \cdot \left(\frac{5x^7}{2} - \frac{69x^6}{56} + \frac{14503x^5}{1344} - \frac{113x^4}{768} + \frac{449699x^3}{28672} + \frac{300323x^2}{49152} + \frac{37239361x}{2752512} + \frac{19512225}{3670016}\right) + \frac{16851295\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2097152}$$

input
```
integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**7/2 - 69*x**6/56 + 14503*x**5/1344 - 113*x**4/7
68 + 449699*x**3/28672 + 300323*x**2/49152 + 37239361*x/2752512 + 19512225
/3670016) + 16851295*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/2097152
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)\,dx = \frac{5}{16}\left(2x^2-x+3\right)^{\frac{7}{2}}x + \frac{141}{448}\left(2x^2-x+3\right)^{\frac{7}{2}} + \frac{277}{768}\left(2x^2-x+3\right)^{\frac{5}{2}}x - \frac{277}{3072}\left(2x^2-x+3\right)^{\frac{5}{2}} + \frac{31855}{24576}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{31855}{98304}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{732665}{131072}\sqrt{2x^2-x+3}\,x + \frac{16851295}{2097152}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{732665}{524288}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
5/16*(2*x^2 - x + 3)^(7/2)*x + 141/448*(2*x^2 - x + 3)^(7/2) + 277/768*(2*
x^2 - x + 3)^(5/2)*x - 277/3072*(2*x^2 - x + 3)^(5/2) + 31855/24576*(2*x^2
 - x + 3)^(3/2)*x - 31855/98304*(2*x^2 - x + 3)^(3/2) + 732665/131072*sqrt
(2*x^2 - x + 3)*x + 16851295/2097152*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x -
1)) - 732665/524288*sqrt(2*x^2 - x + 3)
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)\,dx = \frac{1}{11010048}\left(4\left(8\left(4\left(16\left(4\left(24\left(140x-69\right)x+14503\right)x-791\right)x+1349097\right)x+2102261\right)x+3723\right. - \frac{16851295}{2097152}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
1/11010048*(4*(8*(4*(16*(4*(24*(140*x - 69)*x + 14503)*x - 791)*x + 134909
7)*x + 2102261)*x + 37239361)*x + 58536675)*sqrt(2*x^2 - x + 3) - 16851295
/2097152*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)\,dx=\int \left(2\,x^2-x+3\right)^{5/2}\left(5\,x^2+3\,x+2\right)\,dx$$

input
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2), x)
```

3.76 $\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx$

3.76.1 Optimal result

Integrand size = 27, antiderivative size = 222

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{7216203\,\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}} - \frac{121\sqrt{\frac{11}{31}\left(-15457+25000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(-15457+25000\sqrt{2}\right)}}\left(196-443\sqrt{2}-\left(690+247\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{3125} + \frac{121\sqrt{\frac{11}{31}\left(15457+25000\sqrt{2}\right)}\,\text{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(15457+25000\sqrt{2}\right)}}\left(196+443\sqrt{2}-\left(690-247\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{3125}$$

output
```
-1/600*(103-60*x)*(2*x^2-x+3)^(3/2)-7216203/1600000*arcsinh(1/23*(1-4*x)*2
3^(1/2))*2^(1/2)-1/80000*(226249-99620*x)*(2*x^2-x+3)^(1/2)-121/96875*arct
an(1/62*(196-443*2^(1/2)-x*(690+247*2^(1/2)))*682^(1/2)/(-15457+25000*2^(1
/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-5270837+8525000*2^(1/2))^(1/2)+121/96875*a
rctanh(1/62*(196-x*(690-247*2^(1/2))+443*2^(1/2))*682^(1/2)/(15457+25000*2
^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(5270837+8525000*2^(1/2))^(1/2)
```

3.76.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \frac{20\sqrt{3-x+2x^2}(-802347+412060x-106400x^2+48000x^3)-21648609\sqrt{2}\log(}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

output
```
(20*Sqrt[3 - x + 2*x^2]*(-802347 + 412060*x - 106400*x^2 + 48000*x^3) - 21
648609*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]] - 2044416*RootSum[-5
6 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (368*Log[-(Sqrt[
2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 22*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 -
 x + 2*x^2] - #1]*#1 - 119*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1
^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/4800000
```

3.76.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {1308, 27, 2138, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{5/2}}{5x^2+3x+2}dx$$

$$\Big\downarrow \ 1308$$

$$-\frac{1}{300}\int -\frac{3\sqrt{2x^2-x+3}(4981x^2-2045x+3154)}{4(5x^2+3x+2)}dx-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{400}\int \frac{\sqrt{2x^2-x+3}(4981x^2-2045x+3154)}{5x^2+3x+2}dx-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \ 2138$$

$$\frac{1}{400}\left(-\frac{1}{100}\int -\frac{7216203x^2-3779795x+2136862}{4\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{200}\sqrt{2x^2-x+3}(226249-99620x)\right)-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{400}\left(\frac{1}{400}\int \frac{7216203x^2-3779795x+2136862}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx-\frac{1}{200}(226249-99620x)\sqrt{2x^2-x+3}\right)-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \; 2143$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int -\frac{340736(119x+11)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{200}(226249-99620x)\sqrt{2x\right.$$
$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{340736}{5}\int \frac{119x+11}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{200}(226249-99620x)\right.$$
$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{5\sqrt{46}}-\frac{340736}{5}\int \frac{119x+11}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{200}(226249-99\right.$$
$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\int \frac{119x+11}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)-\frac{1}{200}(226249-99620x)\sqrt{2x\right.$$
$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$$\Big\downarrow \; 1368$$

3.76. $\int \frac{\left(3-x+2x^2\right)^{5/2}}{2+3x+5x^2}\,dx$

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(\frac{\int\frac{11\left(\left(130+119\sqrt{2}\right)x+11\sqrt{2}+108\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int\frac{11\left(\left(130-119\sqrt{2}\right)x-11\sqrt{2}+108\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}d\right.\right.\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$\downarrow$ 27

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(\frac{\int\frac{\left(130+119\sqrt{2}\right)x+11\sqrt{2}+108}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int\frac{\left(130-119\sqrt{2}\right)x-11\sqrt{2}+108}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)\right)-\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$\downarrow$ 1362

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(\sqrt{2}\left(15457+25000\sqrt{2}\right)\int\frac{1}{62\left(15457+25000\sqrt{2}\right)-\frac{11\left(-\left(\left(690-24\right.\right.\right.}{}}\right.\right.\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$\downarrow$ 217

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(\sqrt{2}\left(15457+25000\sqrt{2}\right)\int\frac{1}{62\left(15457+25000\sqrt{2}\right)-\frac{11\left(-\left(\left(690-24\right.\right.\right.}{}}\right.\right.\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

$\downarrow$ 219

$$\frac{1}{400}\left(\frac{1}{400}\left(\frac{7216203\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}-\frac{340736}{5}\left(-\frac{\left(15457-25000\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{11}{62\left(25000\sqrt{2}-15457\right)}}\left(-\left(\left(690+247\sqrt{2}\right)x\right.\right.}{\sqrt{2x^2-x+3}}\right.}{\sqrt{341\left(25000\sqrt{2}-15457\right)}}\right.\right.\right.$$

$$\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}$$

3.76. $\int\frac{\left(3-x+2x^2\right)^{5/2}}{2+3x+5x^2}\,dx$

input
```
Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

output
```
-1/600*((103 - 60*x)*(3 - x + 2*x^2)^(3/2)) + (-1/200*((226249 - 99620*x)*
Sqrt[3 - x + 2*x^2]) + ((7216203*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(5*Sqrt[2])
 - (340736*(-(((15457 - 25000*Sqrt[2])*ArcTan[(Sqrt[11/(62*(-15457 + 25000
*Sqrt[2]))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x))/Sqrt[3 - x + 2*x^
2]])/Sqrt[341*(-15457 + 25000*Sqrt[2])]) - Sqrt[(15457 + 25000*Sqrt[2])/34
1]*ArcTanh[(Sqrt[11/(62*(15457 + 25000*Sqrt[2]))]*(196 + 443*Sqrt[2] - (69
0 - 247*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]))/5)/400)/400
```

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1308
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Simp[1/(2*f^2*(p + q)*(2*p + 2*q + 1))   Int[(a + b
*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p
)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
 + c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p +
q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*
(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(
p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
 - 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2
*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.76. $\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx$

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.76.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.64 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.70

method	result
trager	Expression too large to display
risch	$\frac{(48000x^3-106400x^2+412060x-802347)\sqrt{2x^2-x+3}}{240000}+\frac{7216203\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{1600000}+\frac{121\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)}{\left(\sqrt{2}+1-x\right)^2}}}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
(1/5*x^3-133/300*x^2+20603/12000*x-267449/80000)*(2*x^2-x+3)^(1/2)+1/12400
000*RootOf(_Z^2+24025*RootOf(24025*_Z^4-1264358596886528*_Z^2+870315550029
3816320000000)^2-1264358596886528)*ln((43476601*RootOf(_Z^2+24025*RootOf(
24025*_Z^4-1264358596886528*_Z^2+8703155500293816320000000)^2-12643585968
86528)*RootOf(24025*_Z^4-1264358596886528*_Z^2+8703155500293816320000000)
^4*x-7853588332187811184*RootOf(24025*_Z^4-1264358596886528*_Z^2+8703155500
293816320000000)^2*RootOf(_Z^2+24025*RootOf(24025*_Z^4-1264358596886528*_
Z^2+8703155500293816320000000)^2-1264358596886528)*x-7351087056682614784*
RootOf(24025*_Z^4-1264358596886528*_Z^2+8703155500293816320000000)^2*Root
Of(_Z^2+24025*RootOf(24025*_Z^4-1264358596886528*_Z^2+870315550029381632000
000000)^2-1264358596886528)-17702265231141688416665600*RootOf(24025*_Z^4-
1264358596886528*_Z^2+8703155500293816320000000)^2*(2*x^2-x+3)^(1/2)-9065
0044207406572476825600000*RootOf(_Z^2+24025*RootOf(24025*_Z^4-12643585968
86528*_Z^2+8703155500293816320000000)^2-1264358596886528)*x+40856458713110
3036728934400000*RootOf(_Z^2+24025*RootOf(24025*_Z^4-1264358596886528*_Z^2
+8703155500293816320000000)^2-1264358596886528)+514379782078184624892371 5
547955200000*(2*x^2-x+3)^(1/2))/(775*x*RootOf(24025*_Z^4-1264358596886528*
_Z^2+8703155500293816320000000)^2-4299695792128 0*x-30138769768448))-1/800
00*RootOf(24025*_Z^4-1264358596886528*_Z^2+8703155500293816320000000)*ln(
(-27172875625*x*RootOf(24025*_Z^4-1264358596886528*_Z^2+87031555002938 1...
```

3.76.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.45

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \frac{1}{387500}\sqrt{31}\sqrt{1839524522i\sqrt{31}+4978730614}\log\left(-\frac{\sqrt{2x^2-x+3}(27\sqrt{31}+49\right.$$
$$-\frac{1}{387500}\sqrt{31}\sqrt{1839524522i\sqrt{31}+4978730614}\log\left(\frac{\sqrt{2x^2-x+3}(27\sqrt{31}+49i)\sqrt{1839524522i\sqrt{31}+\right.$$
$$+\frac{1}{387500}\sqrt{31}\sqrt{-1839524522i\sqrt{31}+4978730614}\log\left(-\frac{\sqrt{2x^2-x+3}(27\sqrt{31}-49i)\sqrt{-1839524522i\sqrt{3}}\right.$$
$$-\frac{1}{387500}\sqrt{31}\sqrt{-1839524522i\sqrt{31}+4978730614}\log\left(\frac{\sqrt{2x^2-x+3}(27\sqrt{31}-49i)\sqrt{-1839524522i\sqrt{31}}\right.$$
$$+\frac{1}{240000}(48000x^3-106400x^2+412060x-802347)\sqrt{2x^2-x+3}$$
$$+\frac{7216203}{3200000}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

input

```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output

```
1/387500*sqrt(31)*sqrt(1839524522*I*sqrt(31) + 4978730614)*log(-(sqrt(2*x^
2 - x + 3)*(27*sqrt(31) + 49*I)*sqrt(1839524522*I*sqrt(31) + 4978730614) +
 756250*sqrt(31)*(-I*x + 6*I) - 14368750*x + 16637500)/x) - 1/387500*sqrt(
31)*sqrt(1839524522*I*sqrt(31) + 4978730614)*log((sqrt(2*x^2 - x + 3)*(27*
sqrt(31) + 49*I)*sqrt(1839524522*I*sqrt(31) + 4978730614) - 756250*sqrt(31
)*(-I*x + 6*I) + 14368750*x - 16637500)/x) + 1/387500*sqrt(31)*sqrt(-18395
24522*I*sqrt(31) + 4978730614)*log(-(sqrt(2*x^2 - x + 3)*(27*sqrt(31) - 49
*I)*sqrt(-1839524522*I*sqrt(31) + 4978730614) + 756250*sqrt(31)*(I*x - 6*I
) - 14368750*x + 16637500)/x) - 1/387500*sqrt(31)*sqrt(-1839524522*I*sqrt(
31) + 4978730614)*log((sqrt(2*x^2 - x + 3)*(27*sqrt(31) - 49*I)*sqrt(-1839
524522*I*sqrt(31) + 4978730614) - 756250*sqrt(31)*(I*x - 6*I) + 14368750*x
 - 16637500)/x) + 1/240000*(48000*x^3 - 106400*x^2 + 412060*x - 802347)*sq
rt(2*x^2 - x + 3) + 7216203/3200000*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.76. $\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx$

3.76.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{5}{2}}}{5x^2+3x+2}\,dx$$

input
```
integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
```

output
```
Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2), x)
```

3.76.7 Maxima [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{5}{2}}}{5\,x^2+3\,x+2}\,dx$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2), x)
```

3.76.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = \int \frac{(2\,x^2-x+3)^{5/2}}{5\,x^2+3\,x+2}\,dx$$

input
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2), x)
```

3.77 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx$

3.77.1 Optimal result

Integrand size = 27, antiderivative size = 255

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)\left(3-x+2x^2\right)^{3/2} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{31\left(2+3x+5x^2\right)} - \frac{4799\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2500\sqrt{2}} + \frac{11\sqrt{\frac{11}{31}\left(224510383+194487500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(224510383+194487500\sqrt{2}\right)}}\left(21136+33287\sqrt{2}+\left(87710+54423\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{38750} - \frac{11\sqrt{\frac{11}{31}\left(-224510383+194487500\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{62\left(-224510383+194487500\sqrt{2}\right)}}\left(21136-33287\sqrt{2}+\left(87710-54423\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{38750}$$

output
```
4/155*(4-5*x)*(2*x^2-x+3)^(3/2)+1/31*(3+10*x)*(2*x^2-x+3)^(5/2)/(5*x^2+3*x
+2)-4799/5000*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1/7750*(1277+2240*x)*
(2*x^2-x+3)^(1/2)-11/1201250*arctanh(1/62*(21136+x*(87710-54423*2^(1/2))-3
3287*2^(1/2))*682^(1/2)/(-224510383+194487500*2^(1/2))^(1/2)/(2*x^2-x+3)^(
1/2))*(-76558040603+66320237500*2^(1/2))^(1/2)+11/1201250*arctan(1/62*(211
36+33287*2^(1/2)+x*(87710+54423*2^(1/2)))*682^(1/2)/(224510383+194487500*2
^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(76558040603+66320237500*2^(1/2))^(1/2)
```

3.77.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.70

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \frac{\frac{500\sqrt{3-x+2x^2}(8996+9289x-12555x^2+3100x^3)}{2+3x+5x^2} - 3719225\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((500*Sqrt[3 - x + 2*x^2]*(8996 + 9289*x - 12555*x^2 + 3100*x^3))/(2 + 3*x
 + 5*x^2) - 3719225*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]] + 30008
*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (5237
*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2880*Sqrt[2]*Log[-(Sqrt[2]
*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 2225*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] -
242*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^
4 & , (639994*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 22980
*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1175*Sqrt[2]*Log[-(Sqrt
[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*
#1^2 - 10*#1^3) & ])/3875000
```

3.77.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1302, 27, 2138, 27, 2138, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{5/2}}{(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)} - \frac{1}{31}\int -\frac{5(-32x^2-6x+15)(2x^2-x+3)^{3/2}}{2(5x^2+3x+2)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{62}\int \frac{\left(-32x^2-6x+15\right)\left(2x^2-x+3\right)^{3/2}}{5x^2+3x+2}dx+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2138}$$

$$\frac{5}{62}\left(\frac{8}{25}(4-5x)\left(2x^2-x+3\right)^{3/2}-\frac{1}{600}\int -\frac{24\left(-896x^2-905x+1461\right)\sqrt{2x^2-x+3}}{5x^2+3x+2}dx\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{62}\left(\frac{1}{25}\int \frac{\left(-896x^2-905x+1461\right)\sqrt{2x^2-x+3}}{5x^2+3x+2}dx+\frac{8}{25}(4-5x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2138}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(-\frac{1}{100}\int -\frac{2\left(148769x^2-175535x+243476\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx-\frac{1}{25}\sqrt{2x^2-x+3}(2240x+1277)\right)+\frac{8}{25}(4-5x)\left(2x^2\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\int \frac{148769x^2-175535x+243476}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx-\frac{1}{25}(2240x+1277)\sqrt{2x^2-x+3}\right)+\frac{8}{25}(4-5x)\left(2x^2-x+3\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2143}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{148769}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int \frac{242(3801-5471x)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)-\frac{1}{25}(2240x+1277)\sqrt{2x^2-x\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{148769}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{242}{5}\int \frac{3801-5471x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)-\frac{1}{25}(2240x+1277)\sqrt{2x^2-\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

3.77. $\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx$

$\Big\downarrow$ 1090

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\int \frac{3801-5471x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{148769\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{5\sqrt{46}}\right)-\frac{1}{25}(2240x+1277)\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 222

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\int \frac{3801-5471x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{148769\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}\right)-\frac{1}{25}(2240x+1277)\sqrt{2x^2-x\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 1368

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\left(\frac{\int -\frac{11\left(\left(1670+5471\sqrt{2}\right)x-3801\sqrt{2}+9272\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(1670-5471\sqrt{2}\right)x+3801\sqrt{2}+9272\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{148769a}{}\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 27

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\left(\frac{\int \frac{\left(1670-5471\sqrt{2}\right)x+3801\sqrt{2}+9272}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(1670+5471\sqrt{2}\right)x-3801\sqrt{2}+9272}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{148769\text{arcsinh}\left(\frac{4x-}{\sqrt{23}}\right.}{5\sqrt{2}}\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 1362

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\left(\sqrt{2}\left(224510383-194487500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(87710-54423\sqrt{2}\right)x-33287\sqrt{2}+21136\right)^2}{2x^2-x+3}-62\,(224510383}\right.\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$\Big\downarrow$ 217

3.77. $\displaystyle\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^2}\,dx$

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{242}{5}\left(\sqrt{2}\left(224510383-194487500\sqrt{2}\right)\int\frac{1}{-\frac{11\left(\left(87710-54423\sqrt{2}\right)x-33287\sqrt{2}+21136\right)^2}{2x^2-x+3}-62\left(224510383\right.}\right.\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

$\downarrow$ 219

$$\frac{5}{62}\left(\frac{1}{25}\left(\frac{1}{50}\left(\frac{148769\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{5\sqrt{2}}+\frac{242}{5}\left(\sqrt{\frac{1}{341}\left(224510383+194487500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{62\left(224510383+194487\right.}}}{}\right.\right.\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{31\left(5x^2+3x+2\right)}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

output
```
((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) + (5*((8*(4 - 5*
x)*(3 - x + 2*x^2)^(3/2))/25 + (-1/25*((1277 + 2240*x)*Sqrt[3 - x + 2*x^2]
) + ((148769*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(5*Sqrt[2]) + (242*(Sqrt[(22451
0383 + 194487500*Sqrt[2])/341]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*
Sqrt[2]))]*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x))/Sqrt[3 - x
 + 2*x^2]] + ((224510383 - 194487500*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-22451
0383 + 194487500*Sqrt[2]))]*(21136 - 33287*Sqrt[2] + (87710 - 54423*Sqrt[2
])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[341*(-224510383 + 194487500*Sqrt[2])]))/
5)/50)/25))/62
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.77. $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} \, dx$

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.77.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.10 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.05

method	result
trager	Expression too large to display
risch	$\frac{\left(3100x^3-12555x^2+9289x+8996\right)\sqrt{2x^2-x+3}}{38750x^2+23250x+15500}+\frac{4799\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{5000}+\frac{11\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}}{}$
default	Expression too large to display

input
```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7750*(3100*x^3-12555*x^2+9289*x+8996)/(5*x^2+3*x+2)*(2*x^2-x+3)^(1/2)-47
99/5000*RootOf(_Z^2-2)*ln(-4*RootOf(_Z^2-2)*x+4*(2*x^2-x+3)^(1/2)+RootOf(_
Z^2-2))+1/1201250*RootOf(_Z^2+96100*RootOf(96100*_Z^4+9263522912963*_Z^2+3
35049907908469531250)^2+9263522912963)*ln(-(-50635674288*RootOf(_Z^2+96100
*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^2+92635229129
63)*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^4*x-248394
3074332490408*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^
2*RootOf(_Z^2+96100*RootOf(96100*_Z^4+9263522912963*_Z^2+33504990790846953
1250)^2+9263522912963)*x+1972262224934562786833300 0*RootOf(96100*_Z^4+9263
522912963*_Z^2+335049907908469531250)^2*(2*x^2-x+3)^(1/2)+5394974354905351
392*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^2*RootOf(_
Z^2+96100*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^2+92
63522912963)+1762315286807623678840531 25*RootOf(_Z^2+96100*RootOf(96100*_Z
^4+9263522912963*_Z^2+335049907908469531250)^2+9263522912963)*x+9164762297
3463476291055296999375 0*(2*x^2-x+3)^(1/2)-2123614134501225303247750 00*Root
Of(_Z^2+96100*RootOf(96100*_Z^4+9263522912963*_Z^2+335049907908469531250)^
2+9263522912963))/(3100*x*RootOf(96100*_Z^4+9263522912963*_Z^2+33504990790
8469531250)^2+92436758755*x-75966534842))-1/3875*RootOf(96100*_Z^4+9263522
912963*_Z^2+335049907908469531250)*ln((-7911824107500*x*RootOf(96100*_Z^4+
9263522912963*_Z^2+335049907908469531250)^5-113719843996664697220 0*Root...
```

3.77.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.58

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx =$$

$$-\frac{2\sqrt{62}(5x^2+3x+2)\sqrt{37983267421i\sqrt{31}-298823319773}\log\left(-\frac{2\sqrt{62}\sqrt{2x^2-x+3}\sqrt{37983267421i\sqrt{31}-2988233197}}{}\right.}{}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
-1/9610000*(2*sqrt(62)*(5*x^2 + 3*x + 2)*sqrt(37983267421*I*sqrt(31) - 298
823319773)*log(-(2*sqrt(62)*sqrt(2*x^2 - x + 3)*sqrt(37983267421*I*sqrt(31
) - 298823319773)*(2642*I*sqrt(31) - 35929) + 16580059375*sqrt(31)*(-I*x +
 6*I) - 315021128125*x + 364761306250)/x) - 2*sqrt(62)*(5*x^2 + 3*x + 2)*s
qrt(37983267421*I*sqrt(31) - 298823319773)*log(-(2*sqrt(62)*sqrt(2*x^2 - x
 + 3)*sqrt(37983267421*I*sqrt(31) - 298823319773)*(-2642*I*sqrt(31) + 3592
9) + 16580059375*sqrt(31)*(-I*x + 6*I) - 315021128125*x + 364761306250)/x)
 - 2*sqrt(62)*(5*x^2 + 3*x + 2)*sqrt(-37983267421*I*sqrt(31) - 29882331977
3)*log(-(2*sqrt(62)*sqrt(2*x^2 - x + 3)*(2642*I*sqrt(31) + 35929)*sqrt(-37
983267421*I*sqrt(31) - 298823319773) + 16580059375*sqrt(31)*(I*x - 6*I) -
315021128125*x + 364761306250)/x) + 2*sqrt(62)*(5*x^2 + 3*x + 2)*sqrt(-379
83267421*I*sqrt(31) - 298823319773)*log(-(2*sqrt(62)*sqrt(2*x^2 - x + 3)*(
-2642*I*sqrt(31) - 35929)*sqrt(-37983267421*I*sqrt(31) - 298823319773) + 1
6580059375*sqrt(31)*(I*x - 6*I) - 315021128125*x + 364761306250)/x) - 4611
839*sqrt(2)*(5*x^2 + 3*x + 2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1)
 - 32*x^2 + 16*x - 25) - 1240*(3100*x^3 - 12555*x^2 + 9289*x + 8996)*sqrt(
2*x^2 - x + 3))/(5*x^2 + 3*x + 2)
```

3.77.6 Sympy [F]

$$\int \frac{(3 - x + 2x^2)^{5/2}}{(2 + 3x + 5x^2)^2}\, dx = \int \frac{(2x^2 - x + 3)^{\frac{5}{2}}}{(5x^2 + 3x + 2)^2}\, dx$$

input
```
integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**2, x)
```

3.77.7 Maxima [F]

$$\int \frac{(3 - x + 2x^2)^{5/2}}{(2 + 3x + 5x^2)^2}\, dx = \int \frac{(2\,x^2 - x + 3)^{\frac{5}{2}}}{(5\,x^2 + 3\,x + 2)^2}\, dx$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^2, x)
```

3.77.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{15625,[8]%%%}+%%%{%%{[-37500,0]:[1,0,-2]%%},[7]%%%}+%%%{-6
1250,[6]%
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \int \frac{(2\,x^2-x+3)^{5/2}}{(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^2, x)
```

3.78 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} \, dx$

3.78.1 Optimal result

Integrand size = 27, antiderivative size = 281

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} \, dx = \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{62(2+3x+5x^2)^2} + \frac{(769+2336x)(3-x+2x^2)^{3/2}}{3844(2+3x+5x^2)} - \frac{4}{125}\sqrt{2}\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{\sqrt{11(1+4\sqrt{2})}(2937349+1978861\sqrt{2})\arctan\left(\sqrt{\frac{11}{62(3531015707557+2498852071250\sqrt{2})}}\right.}{29791000}$$

output
```
1/62*(3+10*x)*(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2+1/3844*(769+2336*x)*(2*x^2
-x+3)^(3/2)/(5*x^2+3*x+2)-4/125*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/4
8050*(11359-12920*x)*(2*x^2-x+3)^(1/2)-1/29791000*arctanh(1/62*(3957722+x*
(9832420-6895071*2^(1/2))-2937349*2^(1/2))*682^(1/2)/(-3531015707557+24988
52071250*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(2937349-1978861*2^(1/2))*(-11+
44*2^(1/2))^(1/2)+1/29791000*arctan(1/62*(3957722+2937349*2^(1/2)+x*(98324
20+6895071*2^(1/2)))*682^(1/2)/(3531015707557+2498852071250*2^(1/2))^(1/2)
/(2*x^2-x+3)^(1/2))*(2937349+1978861*2^(1/2))*(11+44*2^(1/2))^(1/2)
```

3.78.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.19

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \frac{\frac{15812500\sqrt{3-x+2x^2}(22552+69621x+93872x^2+97155x^3)}{(2+3x+5x^2)^2} - 4420600000\sqrt{2}\log\left(1-4x+2\sqrt{6-\right.}{}$$

input
```
Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

output
```
((15812500*Sqrt[3 - x + 2*x^2]*(22552 + 69621*x + 93872*x^2 + 97155*x^3))/
(2 + 3*x + 5*x^2)^2 - 4420600000*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*
x^2]] + 972532000*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 -
 5*#1^4 & , (3781*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 630*Sqrt[
2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 150*Log[-(Sqrt[2]*x)
+ Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 -
10*#1^3) & ] + 682*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[
2]*#1^3 - 5*#1^4 & , (4978708507*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2
*x^2] - #1] - 165870920*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 +
1110955025*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13
*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 11*Sqrt[2]*RootSum[-56
- 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (492740319684*Sqrt
[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] - 128644699540*Log[-(Sqrt
[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 55365920925*Sqrt[2]*Log[-(Sqrt[2]*
x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2
 - 10*#1^3) & ])/138143750000
```

3.78.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1302, 27, 2132, 27, 2138, 27, 2143, 27, 1090, 222, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{5/2}}{(5x^2+3x+2)^3}\,dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}-\frac{1}{62}\int -\frac{5\left(-16x^2-14x+39\right)\left(2x^2-x+3\right)^{3/2}}{2\left(5x^2+3x+2\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{124}\int \frac{\left(-16x^2-14x+39\right)\left(2x^2-x+3\right)^{3/2}}{\left(5x^2+3x+2\right)^2}dx+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2132}$$

$$\frac{5}{124}\left(\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{155\left(5x^2+3x+2\right)}-\frac{1}{155}\int -\frac{\left(-20672x^2-5900x+13347\right)\sqrt{2x^2-x+3}}{2\left(5x^2+3x+2\right)}dx\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{124}\left(\frac{1}{310}\int \frac{\left(-20672x^2-5900x+13347\right)\sqrt{2x^2-x+3}}{5x^2+3x+2}dx+\frac{(2336x+769)\left(2x^2-x+3\right)^{3/2}}{155\left(5x^2+3x+2\right)}\right)+\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2138}$$

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{4}{25}(11359-12920x)\sqrt{2x^2-x+3}-\frac{1}{100}\int -\frac{4\left(61504x^2-579685x+1356541\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx\right)+\frac{(2336x+769\ldots}{155\left(5x\ldots\right.}\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\int \frac{61504x^2-579685x+1356541}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{4}{25}\sqrt{2x^2-x+3}(11359-12920x)\right)+\frac{(2336x+769)\left(2x^2\ldots\right.}{155\left(5x^2+3x\ldots\right.}\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \text{2143}$$

3.78. $\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{61504}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{5}\int \frac{11(605427-280267x)}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)+\frac{4}{25}\sqrt{2x^2-x+3}(11359-\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 27

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{61504}{5}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{11}{5}\int \frac{605427-280267x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx\right)+\frac{4}{25}\sqrt{2x^2-x+3}(11359-\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 1090

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\int \frac{605427-280267x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{30752}{5}\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{4}{25}\sqrt{2x\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 222

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\int \frac{605427-280267x}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{30752}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{4}{25}\sqrt{2x^2-x+3}(11359\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 1368

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\left(\frac{\int -\frac{11\left(-\left(\left(325160-280267\sqrt{2}\right)x\right)-605427\sqrt{2}+885694\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(325160+280267\sqrt{2}\right)x\right)+605427\sqrt{2}+\right.}{\sqrt{2x^2-x+3}(5x^2+3x+2)}}{22\sqrt{2}}\right.\right.\right.\right.$$
$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 27

3.78. $\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\left(\frac{\int \frac{-\left(\left(325160+280267\sqrt{2}\right)x\right)+605427\sqrt{2}+885694}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(325160-280267\sqrt{2}\right)x\right)-605427\sqrt{2}+885694}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 1362

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\left(\sqrt{2}\left(3531015707557-2498852071250\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(9832420-6895071\sqrt{2}\right)x-2937349\sqrt{2}+3957722\right)^2}{2x^2-x+3}}\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 217

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{11}{5}\left(\sqrt{2}\left(3531015707557-2498852071250\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(9832420-6895071\sqrt{2}\right)x-2937349\sqrt{2}+3957722\right)^2}{2x^2-x+3}}\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 219

$$\frac{5}{124}\left(\frac{1}{310}\left(\frac{1}{25}\left(\frac{30752}{5}\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{11}{5}\left(\sqrt{\frac{1}{341}\left(3531015707557+2498852071250\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\ldots}}{62(\ldots}\right.\right.\right.\right.\right.$$

$$\frac{(10x+3)\left(2x^2-x+3\right)^{5/2}}{62\left(5x^2+3x+2\right)^2}$$

input
```
Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

3.78. $\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$

output
```
((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + (5*(((769 +
2336*x)*(3 - x + 2*x^2)^(3/2))/(155*(2 + 3*x + 5*x^2)) + ((4*(11359 - 1292
0*x)*Sqrt[3 - x + 2*x^2])/25 + ((30752*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]
])/5 + (11*(Sqrt[(3531015707557 + 2498852071250*Sqrt[2])/341]*ArcTan[(Sqrt
[11/(62*(3531015707557 + 2498852071250*Sqrt[2]))]*(3957722 + 2937349*Sqrt[
2] + (9832420 + 6895071*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((353101570755
7 - 2498852071250*Sqrt[2])*ArcTanh[(Sqrt[11/(62*(-3531015707557 + 24988520
71250*Sqrt[2]))]*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*Sqrt[2])*
x))/Sqrt[3 - x + 2*x^2]])/Sqrt[341*(-3531015707557 + 2498852071250*Sqrt[2]
)]))/5)/25)/310))/124
```

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

3.78. $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} \, dx$

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.78.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.00 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.18

method	result
trager	Expression too large to display
risch	$\frac{11\left(97155x^3+93872x^2+69621x+22552\right)\sqrt{2x^2-x+3}}{96100\left(5x^2+3x+2\right)^2}+\frac{4\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{125}+\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}\cdots}}{\cdots}$
default	Expression too large to display

3.78. $\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx$

input
```
int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
11/96100*(97155*x^3+93872*x^2+69621*x+22552)/(5*x^2+3*x+2)^2*(2*x^2-x+3)^(
1/2)+1/24025*RootOf(49203200*_Z^4+38530443400861984*_Z^2+75555566255284022
42640625)*ln((36815059399680000*x*RootOf(49203200*_Z^4+38530443400861984*_
Z^2+7555556625528402242640625)^5+35093779815505808148083200*RootOf(4920320
0*_Z^4+38530443400861984*_Z^2+7555556625528402242640625)^3*x+2403325288603
23273780028700000*(2*x^2-x+3)^(1/2)*RootOf(49203200*_Z^4+38530443400861984
*_Z^2+7555556625528402242640625)^2+1823557958071135735680000*RootOf(492032
00*_Z^4+38530443400861984*_Z^2+7555556625528402242640625)^3+79667570132996
79497362622070599592*RootOf(49203200*_Z^4+38530443400861984*_Z^2+755555662
5528402242640625)*x+94293965883068184162712639837646059625*(2*x^2-x+3)^(1/
2)+1058392736159831951768700463021600*RootOf(49203200*_Z^4+385304434008619
84*_Z^2+7555556625528402242640625))/(24800*x*RootOf(49203200*_Z^4+38530443
400861984*_Z^2+7555556625528402242640625)^2+9922195093316*x+282535863379))
+1/29791000*RootOf(_Z^2+1537600*RootOf(49203200*_Z^4+38530443400861984*_Z^
2+7555556625528402242640625)^2+1204076356276937)*ln((29452047519744*RootOf
(_Z^2+1537600*RootOf(49203200*_Z^4+38530443400861984*_Z^2+7555556625528402
242640625)^2+1204076356276937)*RootOf(49203200*_Z^4+38530443400861984*_Z^2
+7555556625528402242640625)^4*x+18052075604500342109056*RootOf(49203200*_Z
^4+38530443400861984*_Z^2+7555556625528402242640625)^2*RootOf(_Z^2+1537600
*RootOf(49203200*_Z^4+38530443400861984*_Z^2+7555556625528402242640625)...
```

3.78.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.62

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \frac{\sqrt{62}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\sqrt{282535863379i\,\sqrt{31}-38841172783127}\,\mathrm{lo}}{}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/119164000*(sqrt(62)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(282535863
379*I*sqrt(31) - 38841172783127)*log((sqrt(62)*sqrt(2*x^2 - x + 3)*sqrt(28
2535863379*I*sqrt(31) - 38841172783127)*(1978861*I*sqrt(31) - 13728257) -
38732207104375*sqrt(31)*(-I*x + 6*I) + 735911934983125*x - 852108556296250
)/x) - sqrt(62)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(282535863379*I*
sqrt(31) - 38841172783127)*log((sqrt(62)*sqrt(2*x^2 - x + 3)*sqrt(28253586
3379*I*sqrt(31) - 38841172783127)*(-1978861*I*sqrt(31) + 13728257) - 38732
207104375*sqrt(31)*(-I*x + 6*I) + 735911934983125*x - 852108556296250)/x)
- sqrt(62)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(-282535863379*I*sqrt
(31) - 38841172783127)*log((sqrt(62)*sqrt(2*x^2 - x + 3)*(1978861*I*sqrt(3
1) + 13728257)*sqrt(-282535863379*I*sqrt(31) - 38841172783127) - 387322071
04375*sqrt(31)*(I*x - 6*I) + 735911934983125*x - 852108556296250)/x) + sqr
t(62)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(-282535863379*I*sqrt(31)
- 38841172783127)*log((sqrt(62)*sqrt(2*x^2 - x + 3)*(-1978861*I*sqrt(31) -
 13728257)*sqrt(-282535863379*I*sqrt(31) - 38841172783127) - 3873220710437
5*sqrt(31)*(I*x - 6*I) + 735911934983125*x - 852108556296250)/x) + 1906624
*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 13640*(97155*x^3 + 93872*x^2 + 6
9621*x + 22552)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

3.78.6 Sympy [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2x^2-x+3)^{\frac{5}{2}}}{(5x^2+3x+2)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**3, x)
```

3.78.7 Maxima [F]

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3}\,dx = \int \frac{(2\,x^2-x+3)^{\frac{5}{2}}}{(5\,x^2+3\,x+2)^3}\,dx$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^3, x)
```

3.78.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} \, dx = \int \frac{(2\,x^2-x+3)^{5/2}}{(5\,x^2+3\,x+2)^3} \, dx$$

input
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3,x)
```

output
```
int((2*x^2 - x + 3)^(5/2)/(3*x + 5*x^2 + 2)^3, x)
```

3.79 $\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$

3.79.1 Optimal result

Integrand size = 27, antiderivative size = 185

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{16493087661\sqrt{3-x+2x^2}}{29360128} + \frac{1572007407x\sqrt{3-x+2x^2}}{7340032} - \frac{15428243x^2\sqrt{3-x+2x^2}}{131072} - \frac{19750457x^3\sqrt{3-x+2x^2}}{229376} + \frac{686531x^4\sqrt{3-x+2x^2}}{6144} + \frac{2116475x^5\sqrt{3-x+2x^2}}{10752} + \frac{57375}{448}x^6\sqrt{3-x+2x^2} + \frac{625}{16}x^7\sqrt{3-x+2x^2} + \frac{2899366573\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8388608\sqrt{2}}$$

output
```
2899366573/16777216*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+16493087661/293
60128*(2*x^2-x+3)^(1/2)+1572007407/7340032*x*(2*x^2-x+3)^(1/2)-15428243/13
1072*x^2*(2*x^2-x+3)^(1/2)-19750457/229376*x^3*(2*x^2-x+3)^(1/2)+686531/61
44*x^4*(2*x^2-x+3)^(1/2)+2116475/10752*x^5*(2*x^2-x+3)^(1/2)+57375/448*x^6
*(2*x^2-x+3)^(1/2)+625/16*x^7*(2*x^2-x+3)^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(49479262983+18864088884x-10367779296x^2-7584175488x^3+9842108416x^4+173}{3523215}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(49479262983 + 18864088884*x - 10367779296*x^2 - 75
84175488*x^3 + 9842108416*x^4 + 17338163200*x^5 + 11280384000*x^6 + 344064
0000*x^7) + 60886698033*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/35
2321536
```

3.79.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.630$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{\sqrt{2x^2-x+3}}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\int \frac{57375x^7+48950x^6+74880x^5+56352x^4+29952x^3+12032x^2+3072x+512}{2\sqrt{2x^2-x+3}}dx+\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\int \frac{57375x^7+48950x^6+74880x^5+56352x^4+29952x^3+12032x^2+3072x+512}{\sqrt{2x^2-x+3}}dx+\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(\frac{1}{14}\int \frac{2116475x^6+31140x^5+1577856x^4+838656x^3+336896x^2+86016x+14336}{2\sqrt{2x^2-x+3}}dx+\frac{57375}{14}\sqrt{2x^2-x+\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\int \frac{2116475x^6+31140x^5+1577856x^4+838656x^3+336896x^2+86016x+14336}{\sqrt{2x^2-x+3}}dx+\frac{57375}{14}\sqrt{2x^2-x+\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{12}\int \frac{24028585x^5-25625706x^4+20127744x^3+8085504x^2+2064384x+344064}{2\sqrt{2x^2-x+3}}dx+\frac{2116475}{12}\sqrt{2x^2\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\int \frac{24028585x^5-25625706x^4+20127744x^3+8085504x^2+2064384x+344064}{\sqrt{2x^2-x+3}}dx+\frac{2116475}{12}\sqrt{2x^2\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{1}{10}\int \frac{15\left(-19750457x^4-11608744x^3+10780672x^2+2752512x+458752\right)}{2\sqrt{2x^2-x+3}}dx+\frac{4805717}{2}\sqrt{2x^2-\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\int \frac{-19750457x^4-11608744x^3+10780672x^2+2752512x+458752}{\sqrt{2x^2-x+3}}dx+\frac{4805717}{2}\sqrt{2x^2-x+3x\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{8}\int \frac{-323993103x^3+527998978x^2+44040192x+7340032}{2\sqrt{2x^2-x+3}}dx-\frac{19750457}{8}x^3\sqrt{2x^2-x+3}\right)+\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

3.79. $\int \frac{\left(2+3x+5x^2\right)^4}{\sqrt{3-x+2x^2}}\,dx$

$$\downarrow 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\int \frac{-323993103x^3+527998978x^2+44040192x+7340032}{\sqrt{2x^2-x+3}}dx-\frac{19750457}{8}x^3\sqrt{2x^2-x+3}\right)\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\downarrow 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{6}\int \frac{3(1572007407x^2+1472133180x+29360128)}{2\sqrt{2x^2-x+3}}dx-\frac{107997701}{2}x^2\sqrt{2x^2-x+3}\right)-\frac{19}{}\right.\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\downarrow 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\int \frac{1572007407x^2+1472133180x+29360128}{\sqrt{2x^2-x+3}}dx-\frac{107997701}{2}x^2\sqrt{2x^2-x+3}\right)-\frac{19750}{8}\right.\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\downarrow 2192$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{4}\int -\frac{9197163418-16493087661x}{2\sqrt{2x^2-x+3}}dx+\frac{1572007407}{4}\sqrt{2x^2-x+3}x\right)-\frac{107997701}{2}x^2\right.\right.\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\downarrow 27$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1572007407}{4}x\sqrt{2x^2-x+3}-\frac{1}{8}\int \frac{9197163418-16493087661x}{\sqrt{2x^2-x+3}}dx\right)-\frac{107997701}{2}x^2\sqrt{2}\right.\right.\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\downarrow 1160$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{16493087661}{2}\sqrt{2x^2-x+3}-\frac{20295566011}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1572007407}{4}\sqrt{2x\right.\right.\right.\right.\right.\right.$$
$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\downarrow 1090$$

3.79. $\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{16493087661}{2}\sqrt{2x^2-x+3}-\frac{20295566011\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}\right)+\frac{1572007}{4}\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

$$\Big\downarrow \; 222$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{16493087661}{2}\sqrt{2x^2-x+3}-\frac{20295566011\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}\right)+\frac{1572007407}{4}\sqrt{2x^2}\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7$$

input
```
Int[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

output
```
(625*x^7*Sqrt[3 - x + 2*x^2])/16 + ((57375*x^6*Sqrt[3 - x + 2*x^2])/14 + (
(2116475*x^5*Sqrt[3 - x + 2*x^2])/12 + ((4805717*x^4*Sqrt[3 - x + 2*x^2])/
2 + (3*((-19750457*x^3*Sqrt[3 - x + 2*x^2])/8 + ((-107997701*x^2*Sqrt[3 -
x + 2*x^2])/2 + ((1572007407*x*Sqrt[3 - x + 2*x^2])/4 + ((16493087661*Sqrt
[3 - x + 2*x^2])/2 - (20295566011*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]
))/8)/4)/16))/4)/24)/28)/32
```

3.79.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.79.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

method	result
risch	$\frac{(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x^2+18864088884x+49479262983)}{88080384}$
trager	$\left(\frac{625}{16}x^7+\frac{57375}{448}x^6+\frac{2116475}{10752}x^5+\frac{686531}{6144}x^4-\frac{19750457}{229376}x^3-\frac{15428243}{131072}x^2+\frac{1572007407}{7340032}x+\frac{16493087661}{29360128}\right)\sqrt{2x}$
default	$-\frac{2899366573\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16777216}+\frac{16493087661\sqrt{2x^2-x+3}}{29360128}+\frac{625x^7\sqrt{2x^2-x+3}}{16}+\frac{57375x^6\sqrt{2x^2-x+3}}{448}+\frac{2116475}{}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/88080384*(3440640000*x^7+11280384000*x^6+17338163200*x^5+9842108416*x^4-
7584175488*x^3-10367779296*x^2+18864088884*x+49479262983)*(2*x^2-x+3)^(1/2
)-2899366573/16777216*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{1}{88080384}\left(3440640000\,x^7 + 11280384000\,x^6 + 17338163200\,x^5 + 9842108416\,x^4 - 7584175488\,x^3 - 1036\right.$$
$$+ \frac{2899366573}{33554432}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/88080384*(3440640000*x^7 + 11280384000*x^6 + 17338163200*x^5 + 984210841
6*x^4 - 7584175488*x^3 - 10367779296*x^2 + 18864088884*x + 49479262983)*sq
rt(2*x^2 - x + 3) + 2899366573/33554432*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{625x^7}{16} + \frac{57375x^6}{448} + \frac{2116475x^5}{10752} + \frac{686531x^4}{6144} - \frac{19750457x^3}{229376} - \frac{15428243x^2}{131072} + \frac{1572007407x}{7340032} + \frac{16493087661}{29360128}\right) - \frac{2899366573\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16777216}$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(625*x**7/16 + 57375*x**6/448 + 2116475*x**5/10752 +
686531*x**4/6144 - 19750457*x**3/229376 - 15428243*x**2/131072 + 157200740
7*x/7340032 + 16493087661/29360128) - 2899366573*sqrt(2)*asinh(4*sqrt(23)*
(x - 1/4)/23)/16777216
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{625}{16}\sqrt{2\,x^2-x+3}x^7 + \frac{57375}{448}\sqrt{2\,x^2-x+3}x^6 + \frac{2116475}{10752}\sqrt{2\,x^2-x+3}x^5 + \frac{686531}{6144}\sqrt{2\,x^2-x+3}x^4 - \frac{19750457}{229376}\sqrt{2\,x^2-x+3}x^3 - \frac{15428243}{131072}\sqrt{2\,x^2-x+3}x^2 + \frac{1572007407}{7340032}\sqrt{2\,x^2-x+3}x - \frac{2899366573}{16777216}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{16493087661}{29360128}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
625/16*sqrt(2*x^2 - x + 3)*x^7 + 57375/448*sqrt(2*x^2 - x + 3)*x^6 + 21164
75/10752*sqrt(2*x^2 - x + 3)*x^5 + 686531/6144*sqrt(2*x^2 - x + 3)*x^4 - 1
9750457/229376*sqrt(2*x^2 - x + 3)*x^3 - 15428243/131072*sqrt(2*x^2 - x +
3)*x^2 + 1572007407/7340032*sqrt(2*x^2 - x + 3)*x - 2899366573/16777216*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16493087661/29360128*sqrt(2*x^2 -
 x + 3)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{1}{88080384}(4(8(4(16(100(120(140\,x+459)x+84659)x+4805717)x-59251371)x-323993103)x+4$$
$$+ \frac{2899366573}{16777216}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/88080384*(4*(8*(4*(16*(100*(120*(140*x + 459)*x + 84659)*x + 4805717)*x
- 59251371)*x - 323993103)*x + 4716022221)*x + 49479262983)*sqrt(2*x^2 - x
 + 3) + 2899366573/16777216*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
 - x + 3)) + 1)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^4}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(1/2), x)
```

3.80 $\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx$

3.80.1 Optimal result

Integrand size = 27, antiderivative size = 143

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = -\frac{203373\sqrt{3-x+2x^2}}{32768} - \frac{372783x\sqrt{3-x+2x^2}}{8192} - \frac{3387x^2\sqrt{3-x+2x^2}}{1024} + \frac{8185}{256}x^3\sqrt{3-x+2x^2} + \frac{1355}{48}x^4\sqrt{3-x+2x^2} + \frac{125}{12}x^5\sqrt{3-x+2x^2} - \frac{9267707\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}$$

output
```
-9267707/131072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-203373/32768*(2*x^2
-x+3)^(1/2)-372783/8192*x*(2*x^2-x+3)^(1/2)-3387/1024*x^2*(2*x^2-x+3)^(1/2
)+8185/256*x^3*(2*x^2-x+3)^(1/2)+1355/48*x^4*(2*x^2-x+3)^(1/2)+125/12*x^5*
(2*x^2-x+3)^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(-610119-4473396x-325152x^2+3143040x^3+2775040x^4+1024000x^5)-27803121\sqrt{}}{393216}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-610119 - 4473396*x - 325152*x^2 + 3143040*x^3 + 2
775040*x^4 + 1024000*x^5) - 27803121*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x
+ 4*x^2]])/393216
```

3.80.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{\sqrt{2x^2-x+3}}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{12}\int \frac{6775x^5+3090x^4+4968x^3+2736x^2+864x+192}{2\sqrt{2x^2-x+3}}dx+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{6775x^5+3090x^4+4968x^3+2736x^2+864x+192}{\sqrt{2x^2-x+3}}dx+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{24}\left(\frac{1}{10}\int \frac{15(8185x^4-4216x^3+3648x^2+1152x+256)}{2\sqrt{2x^2-x+3}}dx+\frac{1355}{2}\sqrt{2x^2-x+3}x^4\right)+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{24}\left(\frac{3}{4}\int \frac{8185x^4-4216x^3+3648x^2+1152x+256}{\sqrt{2x^2-x+3}}dx+\frac{1355}{2}\sqrt{2x^2-x+3}x^4\right)+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{8}\int \frac{-10161x^3-88962x^2+18432x+4096}{2\sqrt{2x^2-x+3}}dx+\frac{8185}{8}\sqrt{2x^2-x+3}x^3\right)+\frac{1355}{2}\sqrt{2x^2-x+3}x^4\right)+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\int \frac{-10161x^3-88962x^2+18432x+4096}{\sqrt{2x^2-x+3}}dx+\frac{8185}{8}\sqrt{2x^2-x+3}x^3\right)+\frac{1355}{2}\sqrt{2x^2-x+3}x^4\right)+\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{6}\int \frac{3\left(-372783x^2+114372x+16384\right)}{2\sqrt{2x^2-x+3}}dx-\frac{3387}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{8185}{8}\sqrt{2x^2-x+3}x^3\right)+\frac{13}{}\right.\ \frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\int \frac{-372783x^2+114372x+16384}{\sqrt{2x^2-x+3}}dx-\frac{3387}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{8185}{8}\sqrt{2x^2-x+3}x^3\right)+\frac{1355}{2}\right.\ \frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{4}\int \frac{2367770-203373x}{2\sqrt{2x^2-x+3}}dx-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\frac{3387}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{8185}{8}\sqrt{2x^2-}\right.\right.\ \frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\int \frac{2367770-203373x}{\sqrt{2x^2-x+3}}dx-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\frac{3387}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{8185}{8}\sqrt{2x^2-}\right.\right.\ \frac{125}{12}\sqrt{2x^2-x+3}x^5$$

3.80. $\int \frac{\left(2+3x+5x^2\right)^3}{\sqrt{3-x+2x^2}}\,dx$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{9267707}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{203373}{2}\sqrt{2x^2-x+3}\right)-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\frac{3387}{2}x^2\sqrt{}\right.\right.\right.\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{9267707\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{203373}{2}\sqrt{2x^2-x+3}\right)-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\right.\right.\right.\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{24}\left(\frac{3}{4}\left(\frac{1}{16}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{9267707\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{203373}{2}\sqrt{2x^2-x+3}\right)-\frac{372783}{4}x\sqrt{2x^2-x+3}\right)-\frac{3387}{2}x^2\sqrt{2}\right.\right.\right.\right.$$
$$\frac{125}{12}\sqrt{2x^2-x+3}x^5$$

input
```
Int[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

output
```
(125*x^5*Sqrt[3 - x + 2*x^2])/12 + ((1355*x^4*Sqrt[3 - x + 2*x^2])/2 + (3*
((8185*x^3*Sqrt[3 - x + 2*x^2])/8 + ((-3387*x^2*Sqrt[3 - x + 2*x^2])/2 + (
(-372783*x*Sqrt[3 - x + 2*x^2])/4 + ((-203373*Sqrt[3 - x + 2*x^2])/2 + (92
67707*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/8)/4)/16))/4)/24
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.80. $\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx$

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.80.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\left(1024000 x^{5}+2775040 x^{4}+3143040 x^{3}-325152 x^{2}-4473396 x -610119\right) \sqrt{2 x^{2}-x +3}}{98304}+\frac{9267707 \sqrt{2}\, \operatorname{arcsinh}\left(\frac{4 \sqrt{23}\, \left(x -\frac{1}{4}\right)}{23}\right)}{131072}$
trager	$\left(\frac{125}{12} x^{5}+\frac{1355}{48} x^{4}+\frac{8185}{256} x^{3}-\frac{3387}{1024} x^{2}-\frac{372783}{8192} x -\frac{203373}{32768}\right) \sqrt{2 x^{2}-x +3}-\frac{9267707 \operatorname{RootOf}\left(\_Z^{2}-2\right) \ln\left(-4\right.}{}$
default	$\frac{9267707 \sqrt{2}\, \operatorname{arcsinh}\left(\frac{4 \sqrt{23}\, \left(x -\frac{1}{4}\right)}{23}\right)}{131072}-\frac{203373 \sqrt{2 x^{2}-x +3}}{32768}+\frac{125 x^{5} \sqrt{2 x^{2}-x +3}}{12}+\frac{1355 x^{4} \sqrt{2 x^{2}-x +3}}{48}+\frac{8185 x^{3} \sqrt{2 x^{2}-x +3}}{256}-$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/98304*(1024000*x^5+2775040*x^4+3143040*x^3-325152*x^2-4473396*x-610119)*
(2*x^2-x+3)^(1/2)+9267707/131072*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{1}{98304}\left(1024000\,x^5+2775040\,x^4+3143040\,x^3-325152\,x^2-4473396\,x-610119\right)\sqrt{2\,x^2-x+3} + \frac{9267707}{262144}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/98304*(1024000*x^5 + 2775040*x^4 + 3143040*x^3 - 325152*x^2 - 4473396*x
- 610119)*sqrt(2*x^2 - x + 3) + 9267707/262144*sqrt(2)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.49

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3} \cdot \left(\frac{125x^5}{12}+\frac{1355x^4}{48}+\frac{8185x^3}{256}-\frac{3387x^2}{1024}-\frac{372783x}{8192}-\frac{203373}{32768}\right) + \frac{9267707\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(125*x**5/12 + 1355*x**4/48 + 8185*x**3/256 - 3387*x*
*2/1024 - 372783*x/8192 - 203373/32768) + 9267707*sqrt(2)*asinh(4*sqrt(23)
*(x - 1/4)/23)/131072
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{125}{12}\sqrt{2\,x^2-x+3}x^5 + \frac{1355}{48}\sqrt{2\,x^2-x+3}x^4 + \frac{8185}{256}\sqrt{2\,x^2-x+3}x^3 - \frac{3387}{1024}\sqrt{2\,x^2-x+3}x^2 - \frac{372783}{8192}\sqrt{2\,x^2-x+3}x + \frac{9267707}{131072}\sqrt{2}\,\text{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{203373}{32768}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
125/12*sqrt(2*x^2 - x + 3)*x^5 + 1355/48*sqrt(2*x^2 - x + 3)*x^4 + 8185/25
6*sqrt(2*x^2 - x + 3)*x^3 - 3387/1024*sqrt(2*x^2 - x + 3)*x^2 - 372783/819
2*sqrt(2*x^2 - x + 3)*x + 9267707/131072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*
x - 1)) - 203373/32768*sqrt(2*x^2 - x + 3)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.51

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{98304}(4\,(8\,(20\,(16\,(100\,x+271)x+4911)x-10161)x-1118349)x-610119)\sqrt{2\,x^2-x+3} - \frac{9267707}{131072}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/98304*(4*(8*(20*(16*(100*x + 271)*x + 4911)*x - 10161)*x - 1118349)*x -
610119)*sqrt(2*x^2 - x + 3) - 9267707/131072*sqrt(2)*log(-2*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}}\,dx = \int \frac{\left(5\,x^2+3\,x+2\right)^3}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(1/2), x)
```

3.81 $\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx$

3.81.1 Optimal result

Integrand size = 27, antiderivative size = 101

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = -\frac{11373\sqrt{3-x+2x^2}}{1024}+\frac{3443}{768}x\sqrt{3-x+2x^2}+\frac{655}{96}x^2\sqrt{3-x+2x^2}+\frac{25}{8}x^3\sqrt{3-x+2x^2}+\frac{30725\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

output
```
30725/4096*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-11373/1024*(2*x^2-x+3)^(
1/2)+3443/768*x*(2*x^2-x+3)^(1/2)+655/96*x^2*(2*x^2-x+3)^(1/2)+25/8*x^3*(2
*x^2-x+3)^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(-34119+13772x+20960x^2+9600x^3)+92175\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{12288}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-34119 + 13772*x + 20960*x^2 + 9600*x^3) + 92175*S
qrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/12288
```

3.81.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(5x^2+3x+2\right)^2}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\int \frac{655x^3+14x^2+192x+64}{2\sqrt{2x^2-x+3}}dx+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{655x^3+14x^2+192x+64}{\sqrt{2x^2-x+3}}dx+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\left(\frac{1}{6}\int \frac{3443x^2-5556x+768}{2\sqrt{2x^2-x+3}}dx+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{12}\int \frac{3443x^2-5556x+768}{\sqrt{2x^2-x+3}}dx+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{1}{4}\int -\frac{3(11373x+4838)}{2\sqrt{2x^2-x+3}}dx+\frac{3443}{4}\sqrt{2x^2-x+3}x\right)+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3443}{4}x\sqrt{2x^2-x+3}-\frac{3}{8}\int \frac{11373x+4838}{\sqrt{2x^2-x+3}}dx\right)+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 1160$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3443}{4}x\sqrt{2x^2-x+3}-\frac{3}{8}\left(\frac{30725}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx+\frac{11373}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3443}{4}x\sqrt{2x^2-x+3}-\frac{3}{8}\left(\frac{30725\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{11373}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{655}{6}\sqrt{2x^2-x}\right.$$
$$\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 222$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3443}{4}x\sqrt{2x^2-x+3}-\frac{3}{8}\left(\frac{30725\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{11373}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{655}{6}\sqrt{2x^2-x+3}x^2\right)+$$
$$\frac{25}{8}\sqrt{2x^2-x+3}x^3$$

input
```
Int[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

output
```
(25*x^3*Sqrt[3 - x + 2*x^2])/8 + ((655*x^2*Sqrt[3 - x + 2*x^2])/6 + ((3443
*x*Sqrt[3 - x + 2*x^2])/4 - (3*((11373*Sqrt[3 - x + 2*x^2])/2 + (30725*Arc
Sinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])))/8)/12)/16
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.81.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result
risch	$\frac{(9600x^3+20960x^2+13772x-34119)\sqrt{2x^2-x+3}}{3072}-\frac{30725\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$
trager	$\left(\frac{25}{8}x^3+\frac{655}{96}x^2+\frac{3443}{768}x-\frac{11373}{1024}\right)\sqrt{2x^2-x+3}-\frac{30725\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}\right.}{4096}$
default	$-\frac{30725\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{25x^3\sqrt{2x^2-x+3}}{8}+\frac{655x^2\sqrt{2x^2-x+3}}{96}+\frac{3443x\sqrt{2x^2-x+3}}{768}$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3072*(9600*x^3+20960*x^2+13772*x-34119)*(2*x^2-x+3)^(1/2)-30725/4096*2^(
1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\left(9600\,x^3+20960\,x^2+13772\,x-34119\right)\sqrt{2\,x^2-x+3} + \frac{30725}{8192}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3072*(9600*x^3 + 20960*x^2 + 13772*x - 34119)*sqrt(2*x^2 - x + 3) + 3072
5/8192*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x
 - 25)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{25x^3}{8}+\frac{655x^2}{96}+\frac{3443x}{768}-\frac{11373}{1024}\right) - \frac{30725\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(25*x**3/8 + 655*x**2/96 + 3443*x/768 - 11373/1024) -
 30725*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4096
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{25}{8}\sqrt{2\,x^2-x+3}x^3+\frac{655}{96}\sqrt{2\,x^2-x+3}x^2+\frac{3443}{768}\sqrt{2\,x^2-x+3}x - \frac{30725}{4096}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{11373}{1024}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
25/8*sqrt(2*x^2 - x + 3)*x^3 + 655/96*sqrt(2*x^2 - x + 3)*x^2 + 3443/768*s
qrt(2*x^2 - x + 3)*x - 30725/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
 - 11373/1024*sqrt(2*x^2 - x + 3)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}(4(40(60x+131)x+3443)x-34119)\sqrt{2x^2-x+3} + \frac{30725}{4096}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/3072*(4*(40*(60*x + 131)*x + 3443)*x - 34119)*sqrt(2*x^2 - x + 3) + 3072
5/4096*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx = \int \frac{(5x^2+3x+2)^2}{\sqrt{2x^2-x+3}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(1/2), x)
```

3.82 $\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx$

3.82.1 Optimal result

Integrand size = 25, antiderivative size = 59

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx = \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{17\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

output
```
17/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+39/16*(2*x^2-x+3)^(1/2)+5/4*x
*(2*x^2-x+3)^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}} dx = \frac{1}{64}\left(4(39+20x)\sqrt{3-x+2x^2} + 17\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)\right)$$

input
```
Integrate[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*(39 + 20*x)*Sqrt[3 - x + 2*x^2] + 17*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2
*x + 4*x^2]])/64
```

3.82.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{4}\int -\frac{14-39x}{2\sqrt{2x^2-x+3}}dx+\frac{5}{4}\sqrt{2x^2-x+3}x$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{4}x\sqrt{2x^2-x+3}-\frac{1}{8}\int \frac{14-39x}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{1}{8}\left(\frac{39}{2}\sqrt{2x^2-x+3}-\frac{17}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{5}{4}\sqrt{2x^2-x+3}x$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{8}\left(\frac{39}{2}\sqrt{2x^2-x+3}-\frac{17\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{8}\left(\frac{39}{2}\sqrt{2x^2-x+3}-\frac{17\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x$$

input
```
Int[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

output
```
(5*x*Sqrt[3 - x + 2*x^2])/4 + ((39*Sqrt[3 - x + 2*x^2])/2 - (17*ArcSinh[(-
1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/8
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.82.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{(20x+39)\sqrt{2x^2-x+3}}{16}-\frac{17\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{64}$	35
default	$-\frac{17\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{64}+\frac{39\sqrt{2x^2-x+3}}{16}+\frac{5x\sqrt{2x^2-x+3}}{4}$	45
trager	$\left(\frac{5x}{4}+\frac{39}{16}\right)\sqrt{2x^2-x+3}+\frac{17\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}+\mathrm{RootOf}\left(\_Z^2-2\right)\right)}{64}$	59

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(20*x+39)*(2*x^2-x+3)^(1/2)-17/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/
4))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{16}\,\sqrt{2\,x^2-x+3}(20\,x+39) + \frac{17}{128}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/128*sqrt(2)*log(4*sqrt(2)*sqrt(2
*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \left(\frac{5x}{4}+\frac{39}{16}\right)\sqrt{2x^2-x+3} - \frac{17\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

output
```
(5*x/4 + 39/16)*sqrt(2*x**2 - x + 3) - 17*sqrt(2)*asinh(4*sqrt(23)*(x - 1/
4)/23)/64
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{4}\sqrt{2\,x^2-x+3}x - \frac{17}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{39}{16}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
5/4*sqrt(2*x^2 - x + 3)*x - 17/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
 + 39/16*sqrt(2*x^2 - x + 3)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{16}\sqrt{2\,x^2-x+3}(20\,x+39) + \frac{17}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^2+3\,x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

3.83 $\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx$

3.83.1 Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(7+3\sqrt{2}+\left(13+10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \sqrt{\frac{1}{682}\left(-13+10\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-13+10\sqrt{2}\right)}}\left(7-3\sqrt{2}+\left(13-10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
-1/682*arctanh(1/31*(7+x*(13-10*2^(1/2))-3*2^(1/2))*341^(1/2)/(-13+10*2^(1
/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-8866+6820*2^(1/2))^(1/2)+1/682*arctan(1/31
*(7+3*2^(1/2)+x*(13+10*2^(1/2)))*341^(1/2)/(13+10*2^(1/2))^(1/2)/(2*x^2-x+
3)^(1/2))*(8866+6820*2^(1/2))^(1/2)
```

3.83.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \text{RootSum}\Big[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3 -5\#1^4\&, \frac{\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+2\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}\&\Big]$$

input
```
Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

output
```
RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (Log[-
(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 2*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqr
t[3 - x + 2*x^2] - #1]*#1)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3
) & ]
```

3.83.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1317, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}\,dx$$

$$\Big\downarrow \text{1317}$$

$$\frac{\int \frac{11\left(-x+\sqrt{2}+1\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}} - \frac{\int \frac{11\left(-x-\sqrt{2}+1\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-x+\sqrt{2}+1}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}} - \frac{\int \frac{-x-\sqrt{2}+1}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}$$

$$\Big\downarrow \text{1362}$$

$$\frac{\left(13-10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7\right)^2}{2x^2-x+3}-31\left(13-10\sqrt{2}\right)}d\frac{\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{\left(13+10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)^2}{2x^2-x+3}-31\left(13+10\sqrt{2}\right)}d\frac{\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7}{\sqrt{2x^2-x+3}}}{\sqrt{2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(13-10\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7\right)^2}{2x^2-x+3}-31\left(13-10\sqrt{2}\right)}d\frac{\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(13-10\sqrt{2}\right)\text{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(10\sqrt{2}-13\right)}}\left(\left(13-10\sqrt{2}\right)x-3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)}{\sqrt{682\left(10\sqrt{2}-13\right)}}$$

input
```
Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

output
```
Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))]*(7 + 3
*Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((13 - 10*Sqrt[2])
*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))]*(7 - 3*Sqrt[2] + (13 - 10*Sqrt[
2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-13 + 10*Sqrt[2])]
```

3.83. $\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx$

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

3.83.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.14 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.98

method	result
trager	$\mathrm{RootOf}\left(232562\_Z^4+4433\_Z^2+25\right)\ln\left(-\frac{74884964\,\mathrm{RootOf}\left(232562\_Z^4+4433\_Z^2+25\right)^5 x+3976060\,\mathrm{RootOf}}{}\right.$
default	$\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(369\sqrt{-775687+549362\sqrt{2}}\,\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\arctan\left(\frac{\sqrt{-775687+549362\sqrt{2}}\,\sqrt{-}}{}\right.\right.}{}$

input
```
int(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
RootOf(232562*_Z^4+4433*_Z^2+25)*ln(-(74884964*RootOf(232562*_Z^4+4433*_Z^
2+25)^5*x+3976060*RootOf(232562*_Z^4+4433*_Z^2+25)^3*x-954800*RootOf(23256
2*_Z^4+4433*_Z^2+25)^3+391468*RootOf(232562*_Z^4+4433*_Z^2+25)^2*(2*x^2-x+
3)^(1/2)+41625*RootOf(232562*_Z^4+4433*_Z^2+25)*x-37000*RootOf(232562*_Z^4
+4433*_Z^2+25)+3650*(2*x^2-x+3)^(1/2))/(682*RootOf(232562*_Z^4+4433*_Z^2+2
5)^2*x+5*x-2))-1/682*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)^2
+8866)*ln((18721241*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)^2+
8866)*RootOf(232562*_Z^4+4433*_Z^2+25)^4*x-280302*RootOf(232562*_Z^4+4433*
_Z^2+25)^2*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)^2+8866)*x+2
38700*RootOf(232562*_Z^4+4433*_Z^2+25)^2*RootOf(_Z^2+465124*RootOf(232562*
_Z^4+4433*_Z^2+25)^2+8866)+66745294*RootOf(232562*_Z^4+4433*_Z^2+25)^2*(2*
x^2-x+3)^(1/2)-1739*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)^2+
8866)*x-4700*RootOf(_Z^2+465124*RootOf(232562*_Z^4+4433*_Z^2+25)^2+8866)+6
49946*(2*x^2-x+3)^(1/2))/(341*RootOf(232562*_Z^4+4433*_Z^2+25)^2*x+4*x+1))
```

3.83. $\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx$

3.83.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx =$$
$$-\frac{1}{1364}\sqrt{341}\sqrt{i\sqrt{31}-13}\log\left(\frac{\sqrt{341}\sqrt{2x^2-x+3}(7i\sqrt{31}+31)\sqrt{i\sqrt{31}-13}-155\sqrt{31}(ix-6i)+29}{x}\right.$$
$$+\frac{1}{1364}\sqrt{341}\sqrt{-i\sqrt{31}-13}\log\left(\frac{\sqrt{341}\sqrt{2x^2-x+3}(7i\sqrt{31}-31)\sqrt{-i\sqrt{31}-13}-155\sqrt{31}(-ix+6i}{x}\right.$$
$$-\frac{1}{1364}\sqrt{341}\sqrt{-i\sqrt{31}-13}\log\left(\frac{\sqrt{341}\sqrt{2x^2-x+3}\sqrt{-i\sqrt{31}-13}(-7i\sqrt{31}+31)-155\sqrt{31}(-ix+}{x}\right.$$
$$+\frac{1}{1364}\sqrt{341}\sqrt{i\sqrt{31}-13}\log\left(\frac{\sqrt{341}\sqrt{2x^2-x+3}\sqrt{i\sqrt{31}-13}(-7i\sqrt{31}-31)-155\sqrt{31}(ix-6i)+}{x}\right.$$

input
```
integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/1364*sqrt(341)*sqrt(I*sqrt(31) - 13)*log((sqrt(341)*sqrt(2*x^2 - x + 3)
*(7*I*sqrt(31) + 31)*sqrt(I*sqrt(31) - 13) - 155*sqrt(31)*(I*x - 6*I) + 29
45*x - 3410)/x) + 1/1364*sqrt(341)*sqrt(-I*sqrt(31) - 13)*log((sqrt(341)*s
qrt(2*x^2 - x + 3)*(7*I*sqrt(31) - 31)*sqrt(-I*sqrt(31) - 13) - 155*sqrt(3
1)*(-I*x + 6*I) + 2945*x - 3410)/x) - 1/1364*sqrt(341)*sqrt(-I*sqrt(31) -
13)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(-I*sqrt(31) - 13)*(-7*I*sqrt(3
1) + 31) - 155*sqrt(31)*(-I*x + 6*I) + 2945*x - 3410)/x) + 1/1364*sqrt(341
)*sqrt(I*sqrt(31) - 13)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(I*sqrt(31)
 - 13)*(-7*I*sqrt(31) - 31) - 155*sqrt(31)*(I*x - 6*I) + 2945*x - 3410)/x)
```

3.83.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \int \frac{1}{\sqrt{2x^2-x+3}\cdot(5x^2+3x+2)}\,dx$$

input
```
integrate(1/(5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)), x)
```

3.83.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

3.83.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx = \int \frac{1}{\sqrt{2\,x^2-x+3}\,(5\,x^2+3\,x+2)}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)),x)
```

output
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)), x)
```

3.84 $\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} dx$

3.84.1 Optimal result

Integrand size = 27, antiderivative size = 188

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx=\frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(2119+1816\sqrt{2}+\left(5751+3935\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{1364}-\frac{\sqrt{\frac{1}{682}\left(-2343727+1678700\sqrt{2}\right)}\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-2343727+1678700\sqrt{2}\right)}}\left(2119-1816\sqrt{2}+\left(5751-3935\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{1364}$$

output
```
1/682*(4+65*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-1/930248*arctanh(1/31*(2119
+x*(5751-3935*2^(1/2))-1816*2^(1/2))*341^(1/2)/(-2343727+1678700*2^(1/2))^
(1/2)/(2*x^2-x+3)^(1/2))*(-1598421814+1144873400*2^(1/2))^(1/2)+1/930248*a
rctan(1/31*(2119+1816*2^(1/2)+x*(5751+3935*2^(1/2)))*341^(1/2)/(2343727+16
78700*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(1598421814+1144873400*2^(1/2))^(1
/2)
```

3.84.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{682\,(2+3x+5x^2)} + \frac{\text{RootSum}\left[-10580-2024\sqrt{2}\#1+68\#1^2+44\sqrt{2}\#1^3-5\#1^4\&, \frac{-9430\sqrt{2}\log\left(\sqrt{2}(-1+4x)-4\sqrt{3-x+2x^2}+\#1\right)+}{}\right.}{682\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

output
```
((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + RootSum[-10580
- 2024*Sqrt[2]*#1 + 68*#1^2 + 44*Sqrt[2]*#1^3 - 5*#1^4 & , (-9430*Sqrt[2]*
Log[Sqrt[2]*(-1 + 4*x) - 4*Sqrt[3 - x + 2*x^2] + #1] + 4492*Log[Sqrt[2]*(-
1 + 4*x) - 4*Sqrt[3 - x + 2*x^2] + #1]*#1 + 205*Sqrt[2]*Log[Sqrt[2]*(-1 +
4*x) - 4*Sqrt[3 - x + 2*x^2] + #1]*#1^2)/(-506*Sqrt[2] + 34*#1 + 33*Sqrt[2
]*#1^2 - 5*#1^3) & ]/(682*Sqrt[2])
```

3.84.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1305, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{(65x+4)\sqrt{2x^2-x+3}}{682\,(5x^2+3x+2)} - \frac{\int -\frac{11(332-205x)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{7502}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{332-205x}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{1364} + \frac{\sqrt{2x^2-x+3}(65x+4)}{682\,(5x^2+3x+2)}$$

$$\Big\downarrow \text{1368}$$

$$\frac{\frac{\int -\frac{11\left(-\left(\left(127-205\sqrt{2}\right)x\right)-332\sqrt{2}+537\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(127+205\sqrt{2}\right)x\right)+332\sqrt{2}+537\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}}{1364}+\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-\left(\left(127+205\sqrt{2}\right)x\right)+332\sqrt{2}+537}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(127-205\sqrt{2}\right)x\right)-332\sqrt{2}+537}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}}{1364}+\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1362}$$

$$\frac{\frac{\left(2343727-1678700\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119\right)^2}{2x^2-x+3}-31\left(2343727-1678700\sqrt{2}\right)}d\frac{\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{\left(2343727+1678700\ldots\right.}{\ldots}}{1364}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\left(2343727-1678700\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119\right)^2}{2x^2-x+3}-31\left(2343727-1678700\sqrt{2}\right)}d\frac{\left(5751-3935\sqrt{2}\right)x-1816\sqrt{2}+2119}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}\left(2343727\ldots\right.}}{1364}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(2343727-1678700\sqrt{2}\right)\ldots}{\ldots}}{1364}$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}$$

input
```
Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

3.84. $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx$

output
```
((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727
+ 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))]*
(2119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((
2343727 - 1678700*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-2343727 + 1678700*Sqrt[2
]))]*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
)/Sqrt[682*(-2343727 + 1678700*Sqrt[2])])/1364
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.84.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.82 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.52

method	result
trager	$\frac{(4+65x)\sqrt{2x^2-x+3}}{3410x^2+2046x+1364} - \frac{3\,\mathrm{RootOf}\left(1205601408\_Z^4+28771592652\_Z^2+176127105625\right)\ln\left(\frac{-394636742489088x\,\mathrm{RootOf}\left(120560140\right.}{}\right.}{}$
risch	$\frac{(4+65x)\sqrt{2x^2-x+3}}{3410x^2+2046x+1364} + \frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(153463\sqrt{-775687+549362\sqrt{2}}\,\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\mathrm{arc}\right.}{}$
default	Expression too large to display

input
```
int(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/682*(4+65*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-3/341*RootOf(1205601408*_Z^
4+28771592652*_Z^2+176127105625)*ln((-394636742489088*x*RootOf(1205601408*
_Z^4+28771592652*_Z^2+176127105625)^5-13779789867949248*RootOf(1205601408*
_Z^4+28771592652*_Z^2+176127105625)^3*x+32137043846114592*(2*x^2-x+3)^(1/2
)*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^2-2373323185094400
*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^3-96300551793960525
*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)*x+3866105782086668
75*(2*x^2-x+3)^(1/2)-59939356016807400*RootOf(1205601408*_Z^4+28771592652*_
Z^2+176127105625))/(98208*x*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127
105625)^2+1273745*x+135842))+1/930248*RootOf(_Z^2+66977856*RootOf(12056014
08*_Z^4+28771592652*_Z^2+176127105625)^2+1598421814)*ln(-(-2055399700464*x
*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^4*RootOf(_Z^2+66977
856*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^2+1598421814)-26
334199149588*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^2*RootO
f(_Z^2+66977856*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^2+15
98421814)*x+1369841493940634484*(2*x^2-x+3)^(1/2)*RootOf(1205601408*_Z^4+2
8771592652*_Z^2+176127105625)^2+12361058255700*RootOf(1205601408*_Z^4+2877
1592652*_Z^2+176127105625)^2*RootOf(_Z^2+66977856*RootOf(1205601408*_Z^4+2
8771592652*_Z^2+176127105625)^2+1598421814)+40592603336832*RootOf(_Z^2+669
77856*RootOf(1205601408*_Z^4+28771592652*_Z^2+176127105625)^2+159842181...
```

3.84.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \frac{\sqrt{341}(5\,x^2+3\,x+2)\sqrt{67921i\,\sqrt{31}-2343727}\,\log\left(\frac{\sqrt{341}\sqrt{2\,x^2-x+3}\sqrt{67921i\,\sqrt{31}-2343727}\left(2119i\,\sqrt{31}-16647\right)-26019850}{x}\right.}{}$$

input
```
integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/1860496*(sqrt(341)*(5*x^2 + 3*x + 2)*sqrt(67921*I*sqrt(31) - 2343727)*lo
g((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(67921*I*sqrt(31) - 2343727)*(2119*I*
sqrt(31) - 16647) - 26019850*sqrt(31)*(-I*x + 6*I) + 494377150*x - 5724367
00)/x) - sqrt(341)*(5*x^2 + 3*x + 2)*sqrt(67921*I*sqrt(31) - 2343727)*log(
(sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(67921*I*sqrt(31) - 2343727)*(-2119*I*s
qrt(31) + 16647) - 26019850*sqrt(31)*(-I*x + 6*I) + 494377150*x - 57243670
0)/x) - sqrt(341)*(5*x^2 + 3*x + 2)*sqrt(-67921*I*sqrt(31) - 2343727)*log(
(sqrt(341)*sqrt(2*x^2 - x + 3)*(2119*I*sqrt(31) + 16647)*sqrt(-67921*I*sqr
t(31) - 2343727) - 26019850*sqrt(31)*(I*x - 6*I) + 494377150*x - 572436700
)/x) + sqrt(341)*(5*x^2 + 3*x + 2)*sqrt(-67921*I*sqrt(31) - 2343727)*log((
sqrt(341)*sqrt(2*x^2 - x + 3)*(-2119*I*sqrt(31) - 16647)*sqrt(-67921*I*sqr
t(31) - 2343727) - 26019850*sqrt(31)*(I*x - 6*I) + 494377150*x - 572436700
)/x) + 2728*sqrt(2*x^2 - x + 3)*(65*x + 4))/(5*x^2 + 3*x + 2)
```

3.84.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)^2}\,dx$$

input
```
integrate(1/(5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2), x)
```

3.84.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^2\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x^2 - x + 3)), x)
```

3.84.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{\sqrt{2\,x^2-x+3}\,(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2),x)
```

output
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^2), x)
```

3.85 $\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx$

3.85.1 Optimal result

Integrand size = 27, antiderivative size = 223

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\,(2+3x+5x^2)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\,(2+3x+5x^2)}$$

$$+ \frac{25\sqrt{\frac{1}{682}\,(6414867847+4536374600\sqrt{2})}\ \arctan\left(\frac{\sqrt{\frac{11}{31(6414867847+4536374600\sqrt{2})}}\left(123161+85754\sqrt{2}+\left(294669+208915\sqrt{2}\cdots\right.\right.}{\sqrt{3-x+2x^2}}\cdots\right.}{3720992 \cdots}$$

$$- \frac{25\sqrt{\frac{1}{682}\,(-6414867847+4536374600\sqrt{2})}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31(-6414867847+4536374600\sqrt{2})}}\left(123161-85754\sqrt{2}+\left(294669-2089\cdots\right.\right.}{\sqrt{3-x+2x^2}}\cdots\right.}{3720992 \cdots}$$

output
```
1/1364*(4+65*x)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2+1/1860496*(26794+86265*x
)*(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)-25/2537716544*arctanh(1/31*(123161+x*(29
4669-208915*2^(1/2))-85754*2^(1/2))*341^(1/2)/(-6414867847+4536374600*2^(1
/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-4374939871654+3093807477200*2^(1/2))^(1/2)
+25/2537716544*arctan(1/31*(123161+85754*2^(1/2)+x*(294669+208915*2^(1/2))
)*341^(1/2)/(6414867847+4536374600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(4374
939871654+3093807477200*2^(1/2))^(1/2)
```

3.85.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.78

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx$$
$$= \frac{\sqrt{3-x+2x^2}(59044+341572x+392765x^2+431325x^3)}{1860496\,(2+3x+5x^2)^2}$$
$$+ \frac{3\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{-42330420383\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+11629301}{-}\right.}{49210119200}$$
$$- \frac{16\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{-720397\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+129160\sqrt{2}\log}{-13\sqrt{2}-}\right.}{4509725}$$

input
```
Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

output
```
(Sqrt[3 - x + 2*x^2]*(59044 + 341572*x + 392765*x^2 + 431325*x^3))/(186049
6*(2 + 3*x + 5*x^2)^2) + (3*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt
[2]*#1^3 - 5*#1^4 & , (-42330420383*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
 - #1] + 11629301740*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*
#1 - 2992879225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sq
rt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/49210119200 - (16*RootSum[-
56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-720397*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 129160*Sqrt[2]*Log[-(Sqrt[2]*x) +
 Sqrt[3 - x + 2*x^2] - #1]*#1 - 65525*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^
2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/45097
25
```

3.85.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1305, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)^3}\,dx$$

$$\Big\downarrow \text{ 1305}$$

$$\frac{(65x+4)\sqrt{2x^2-x+3}}{1364\,(5x^2+3x+2)^2}-\frac{\int -\frac{11(520x^2-589x+1050)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)^2}dx}{15004}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{520x^2-589x+1050}{\sqrt{2x^2-x+3}(5x^2+3x+2)^2}dx}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 2135}$$

$$\frac{\frac{\int \frac{275(18658-7445x)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{7502}+\frac{\sqrt{2x^2-x+3}(86265x+26794)}{682(5x^2+3x+2)}}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{25\int \frac{18658-7445x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{1364}+\frac{\sqrt{2x^2-x+3}(86265x+26794)}{682(5x^2+3x+2)}}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 1368}$$

$$\frac{\frac{25\left(\frac{\int -\frac{11\left(-\left(\left(11213-7445\sqrt{2}\right)x\right)-18658\sqrt{2}+26103\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(11213+7445\sqrt{2}\right)x\right)+18658\sqrt{2}+26103\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)}{1364}+\frac{\sqrt{2x^2-x+3}(86265x+26794)}{682(5x^2+3x+2)}}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{25\left(\frac{\int \frac{-\left(\left(11213+7445\sqrt{2}\right)x\right)+18658\sqrt{2}+26103}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(11213-7445\sqrt{2}\right)x\right)-18658\sqrt{2}+26103}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)}{1364}+\frac{\sqrt{2x^2-x+3}(86265x+26794)}{682(5x^2+3x+2)}}{2728}+\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\,(5x^2+3x+2)^2}$$

$$\Big\downarrow \text{ 1362}$$

3.85. $\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^3}\,dx$

$$25\left(\frac{\left(6414867847-4536374600\sqrt{2}\right)\int\frac{1}{-\frac{11\left(\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+123161\right)^2}{2x^2-x+3}-31\left(6414867847-4536374600\sqrt{2}\right)}d\frac{\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+1231}{\sqrt{2x^2-x+3}}}{\sqrt{2}}\right.$$

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 217

$$25\left(\frac{\left(6414867847-4536374600\sqrt{2}\right)\int\frac{1}{-\frac{11\left(\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+123161\right)^2}{2x^2-x+3}-31\left(6414867847-4536374600\sqrt{2}\right)}d\frac{\left(294669-208915\sqrt{2}\right)x-85754\sqrt{2}+1231}{\sqrt{2x^2-x+3}}}{\sqrt{2}}\right.$$

1364

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 219

$$25\left(\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(6414867847+4536374600\sqrt{2}\right)}}\left(\left(294669+208915\sqrt{2}\right)x+85754\sqrt{2}+123161\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(6414867847-453637\right.}{}\right.$$

1364

2728

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

3.85. $\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^3}\,dx$

output
```
((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + (((26794 + 8
6265*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (25*(Sqrt[(64148678
47 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600
*Sqrt[2]))]*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x))/Sqrt[3
 - x + 2*x^2]] + ((6414867847 - 4536374600*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(
-6414867847 + 4536374600*Sqrt[2]))]*(123161 - 85754*Sqrt[2] + (294669 - 20
8915*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-6414867847 + 4536374600
*Sqrt[2])]))/1364)/2728
```

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.85. $\int \frac{1}{\sqrt{3-x+2x^2}(2+3x+5x^2)^3} \, dx$

3.85.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.12 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.17

method	result
trager	Expression too large to display
risch	$\frac{(431325x^3+392765x^2+341572x+59044)\sqrt{2x^2-x+3}}{1860496(5x^2+3x+2)^2}+\frac{25\sqrt{\frac{8(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}}\sqrt{2}\left(11325170\sqrt{-775687+5\right.}{}$
default	Expression too large to display

input
```
int(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1860496*(431325*x^3+392765*x^2+341572*x+59044)/(5*x^2+3*x+2)^2*(2*x^2-x+
3)^(1/2)-25/2537716544*RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+787489
17689772*_Z^2+321542101742580625)^2+4374939871654)*ln((-7411434655680*x*Ro
otOf(4822405632*_Z^4+78748917689772*_Z^2+321542101742580625)^4*RootOf(_Z^2
+267911424*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542101742580625)^
2+4374939871654)-133779516386184108*RootOf(4822405632*_Z^4+78748917689772*
_Z^2+321542101742580625)^2*RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+78
748917689772*_Z^2+321542101742580625)^2+4374939871654)*x+35248490028539818
757496*(2*x^2-x+3)^(1/2)*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542
101742580625)^2-2440416054631500*RootOf(4822405632*_Z^4+78748917689772*_Z^
2+321542101742580625)^2*RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+78748
917689772*_Z^2+321542101742580625)^2+4374939871654)-5965090431215412614l3*
RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542101
742580625)^2+4374939871654)*x+2879887428875757890l6260666*(2*x^2-x+3)^(1/2
)-24428133718051268025*RootOf(_Z^2+267911424*RootOf(4822405632*_Z^4+787489
17689772*_Z^2+321542101742580625)^2+4374939871654))/(196416*x*RootOf(48224
05632*_Z^4+78748917689772*_Z^2+321542101742580625)^2+1614873451*x+14875319
))-75/465124*RootOf(4822405632*_Z^4+78748917689772*_Z^2+321542101742580625
)*ln(-(5691981815562240*x*RootOf(4822405632*_Z^4+78748917689772*_Z^2+32154
2101742580625)^5+83155176470593979136*RootOf(4822405632*_Z^4+7874891768...
```

3.85.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.78

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx = -\frac{\sqrt{341}(25\,x^4+30\,x^3+29\,x^2+12\,x+4)\sqrt{9297074375i\,\sqrt{31}-4009292404375}\log\left(\frac{\sqrt{341}\sqrt{2\,x^2-x+3}\sqrt{92970}}{}\right.}{}$$

input
```
integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/5075433088*(sqrt(341)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(929707
4375*I*sqrt(31) - 4009292404375)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(9
297074375*I*sqrt(31) - 4009292404375)*(123161*I*sqrt(31) + 809193) - 17578
45157500*sqrt(31)*(I*x - 6*I) + 33399057992500*x - 38672593465000)/x) - sq
rt(341)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(9297074375*I*sqrt(31) -
 4009292404375)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(9297074375*I*sqrt(
31) - 4009292404375)*(-123161*I*sqrt(31) - 809193) - 1757845157500*sqrt(31
)*(I*x - 6*I) + 33399057992500*x - 38672593465000)/x) - sqrt(341)*(25*x^4
+ 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(-9297074375*I*sqrt(31) - 4009292404375)
*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(123161*I*sqrt(31) - 809193)*sqrt(-929
7074375*I*sqrt(31) - 4009292404375) - 1757845157500*sqrt(31)*(-I*x + 6*I)
+ 33399057992500*x - 38672593465000)/x) + sqrt(341)*(25*x^4 + 30*x^3 + 29*
x^2 + 12*x + 4)*sqrt(-9297074375*I*sqrt(31) - 4009292404375)*log((sqrt(341
)*sqrt(2*x^2 - x + 3)*(-123161*I*sqrt(31) + 809193)*sqrt(-9297074375*I*sqr
t(31) - 4009292404375) - 1757845157500*sqrt(31)*(-I*x + 6*I) + 33399057992
500*x - 38672593465000)/x) - 2728*(431325*x^3 + 392765*x^2 + 341572*x + 59
044)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

3.85.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx = \int \frac{1}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)^3}\,dx$$

input
```
integrate(1/(5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```

3.85.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^3\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)
```

3.85.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)^3}\,dx = \int \frac{1}{\sqrt{2\,x^2-x+3}\,(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3),x)
```

output
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^3), x)
```

3.86 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}} \, dx$

3.86.1 Optimal result

Integrand size = 27, antiderivative size = 166

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}} \, dx = -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x^5\sqrt{3-x+2x^2} - \frac{310445587\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}}$$

output

```
-310445587/262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-14641/1472*(101+7
9*x)/(2*x^2-x+3)^(1/2)-31009685/65536*(2*x^2-x+3)^(1/2)-8992487/16384*x*(2
*x^2-x+3)^(1/2)-111315/2048*x^2*(2*x^2-x+3)^(1/2)+79425/512*x^3*(2*x^2-x+3
)^(1/2)+10075/96*x^4*(2*x^2-x+3)^(1/2)+625/24*x^5*(2*x^2-x+3)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}} \, dx = \frac{\frac{4(-10961697147-8859305979x-2534760678x^2-2613624504x^3+230669760x^4+1281670400x^5+831385600x^6+2}{\sqrt{3-x+2x^2}}}{18087936}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

output
```
((4*(-10961697147 - 8859305979*x - 2534760678*x^2 - 2613624504*x^3 + 23066
9760*x^4 + 1281670400*x^5 + 831385600*x^6 + 235520000*x^7))/Sqrt[3 - x + 2
*x^2] - 21420745503*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/180879
36
```

3.86.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(5x^2+3x+2\right)^4}{\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{23\left(40000x^6+116000x^5+148400x^4+49960x^3-84916x^2-57494x+122691\right)}{256\sqrt{2x^2-x+3}}dx-\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{128}\int \frac{40000x^6+116000x^5+148400x^4+49960x^3-84916x^2-57494x+122691}{\sqrt{2x^2-x+3}}dx-\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{128}\left(\frac{1}{12}\int \frac{4\left(403000x^5+295200x^4+149880x^3-254748x^2-172482x+368073\right)}{\sqrt{2x^2-x+3}}dx+\frac{10000}{3}\sqrt{2x^2-x+3}x^5\right)-\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{128}\left(\frac{1}{3}\int \frac{403000x^5+295200x^4+149880x^3-254748x^2-172482x+368073}{\sqrt{2x^2-x+3}}dx+\frac{10000}{3}\sqrt{2x^2-x+3}x^5\right)-\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 2192

$$\frac{1}{128}\left(\frac{1}{3}\left(\frac{1}{10}\int \frac{30(158850x^4-111240x^3-84916x^2-57494x+122691)}{\sqrt{2x^2-x+3}}dx+40300\sqrt{2x^2-x+3}x^4\right)+\frac{10000}{3}\sqrt{\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{128}\left(\frac{1}{3}\left(3\int \frac{158850x^4-111240x^3-84916x^2-57494x+122691}{\sqrt{2x^2-x+3}}dx+40300\sqrt{2x^2-x+3}x^4\right)+\frac{10000}{3}\sqrt{2x^2-\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 2192

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\int \frac{-333945x^3-2108978x^2-459952x+981528}{\sqrt{2x^2-x+3}}dx+\frac{79425}{4}\sqrt{2x^2-x+3}x^3\right)+40300\sqrt{2x^2-x-\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 2192

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{6}\int \frac{3(-8992487x^2-504028x+3926112)}{2\sqrt{2x^2-x+3}}dx-\frac{111315}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{79425}{4}\sqrt{2x^2-x+3\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\int \frac{-8992487x^2-504028x+3926112}{\sqrt{2x^2-x+3}}dx-\frac{111315}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{79425}{4}\sqrt{2x^2-x+3}x^3\right)\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 2192

3.86. $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx$

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{4}\int \frac{85363818-31009685x}{2\sqrt{2x^2-x+3}}dx-\frac{8992487}{4}x\sqrt{2x^2-x+3}\right)-\frac{111315}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{794}{4}\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\int \frac{85363818-31009685x}{\sqrt{2x^2-x+3}}dx-\frac{8992487}{4}x\sqrt{2x^2-x+3}\right)-\frac{111315}{2}x^2\sqrt{2x^2-x+3}\right)+\frac{794}{4}\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 1160

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{310445587}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{31009685}{2}\sqrt{2x^2-x+3}\right)-\frac{8992487}{4}x\sqrt{2x^2-x+3}\right)-\right.\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 1090

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{310445587\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{31009685}{2}\sqrt{2x^2-x+3}\right)-\frac{8992487}{4}x\sqrt{2x^2-x}\right.\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 222

$$\frac{1}{128}\left(\frac{1}{3}\left(3\left(\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(\frac{310445587\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{31009685}{2}\sqrt{2x^2-x+3}\right)-\frac{8992487}{4}x\sqrt{2x^2-x+3}\right)-\frac{11}{}\right.\right.\right.\right.$$
$$\frac{14641(79x+101)}{1472\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) + ((10000*x^5*Sqrt[3 - x
 + 2*x^2])/3 + (40300*x^4*Sqrt[3 - x + 2*x^2] + 3*((79425*x^3*Sqrt[3 - x +
2*x^2])/4 + ((-111315*x^2*Sqrt[3 - x + 2*x^2])/2 + ((-8992487*x*Sqrt[3 - x
 + 2*x^2])/4 + ((-31009685*Sqrt[3 - x + 2*x^2])/2 + (310445587*ArcSinh[(-1
 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/8)/4)/8))/3)/128
```

3.86. $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx$

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.86. $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx$

3.86.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$\frac{235520000x^7+831385600x^6+1281670400x^5+230669760x^4-2613624504x^3-2534760678x^2-8859305979x-10961697147}{4521984\sqrt{2x^2-x+3}}+\frac{310445}{}$
trager	$\frac{235520000x^7+831385600x^6+1281670400x^5+230669760x^4-2613624504x^3-2534760678x^2-8859305979x-10961697147}{4521984\sqrt{2x^2-x+3}}+\frac{310445}{}$
default	$\frac{-\frac{1234044515}{12058624}+\frac{1234044515x}{3014656}}{\sqrt{2x^2-x+3}}-\frac{1217267299}{524288\sqrt{2x^2-x+3}}+\frac{625x^7}{12\sqrt{2x^2-x+3}}+\frac{8825x^6}{48\sqrt{2x^2-x+3}}+\frac{217675x^5}{768\sqrt{2x^2-x+3}}+\frac{52235x^4}{1024\sqrt{2x^2-x+3}}-$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4521984*(235520000*x^7+831385600*x^6+1281670400*x^5+230669760*x^4-261362
4504*x^3-2534760678*x^2-8859305979*x-10961697147)/(2*x^2-x+3)^(1/2)+310445
587/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{21420745503\,\sqrt{2}(2\,x^2-x+3)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-\right.}{}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/36175872*(21420745503*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2
- x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(235520000*x^7 + 831385600*x^
6 + 1281670400*x^5 + 230669760*x^4 - 2613624504*x^3 - 2534760678*x^2 - 885
9305979*x - 10961697147)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

3.86.6 Sympy [F]

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(3/2), x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx &= \frac{625\,x^7}{12\,\sqrt{2\,x^2-x+3}} + \frac{8825\,x^6}{48\,\sqrt{2\,x^2-x+3}} \\ &+ \frac{217675\,x^5}{768\,\sqrt{2\,x^2-x+3}} + \frac{52235\,x^4}{1024\,\sqrt{2\,x^2-x+3}} - \frac{4734827\,x^3}{8192\,\sqrt{2\,x^2-x+3}} \\ &- \frac{18367831\,x^2}{32768\,\sqrt{2\,x^2-x+3}} + \frac{310445587}{262144}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) \\ &- \frac{2953101993\,x}{1507328\,\sqrt{2\,x^2-x+3}} - \frac{3653899049}{1507328\,\sqrt{2\,x^2-x+3}}\end{aligned}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
625/12*x^7/sqrt(2*x^2 - x + 3) + 8825/48*x^6/sqrt(2*x^2 - x + 3) + 217675/
768*x^5/sqrt(2*x^2 - x + 3) + 52235/1024*x^4/sqrt(2*x^2 - x + 3) - 4734827
/8192*x^3/sqrt(2*x^2 - x + 3) - 18367831/32768*x^2/sqrt(2*x^2 - x + 3) + 3
10445587/262144*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 2953101993/1507
328*x/sqrt(2*x^2 - x + 3) - 3653899049/1507328/sqrt(2*x^2 - x + 3)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = -\frac{310445587}{262144}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$+\frac{(46(4(40(20(16(100x+353)x+8707)x+31341)x-14204481)x-55103493)x-8859305979)x-109}{4521984\sqrt{2x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
-310445587/262144*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
 + 1) + 1/4521984*((46*(4*(40*(20*(16*(100*x + 353)*x + 8707)*x + 31341)*x
 - 14204481)*x - 55103493)*x - 8859305979)*x - 10961697147)/sqrt(2*x^2 - x
 + 3)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{3/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(3/2), x)
```

3.87 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx$

3.87.1 Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \frac{125}{16}x^3\sqrt{3-x+2x^2} + \frac{1168881\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

output
```
1168881/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1331/368*(17-45*x)/(2*
x^2-x+3)^(1/2)-181561/2048*(2*x^2-x+3)^(1/2)+15565/512*x*(2*x^2-x+3)^(1/2)
+1825/64*x^2*(2*x^2-x+3)^(1/2)+125/16*x^3*(2*x^2-x+3)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{\frac{4(-15423965+16138403x-5754186x^2+2624760x^3+2318400x^4+736000x^5)}{\sqrt{3-x+2x^2}} + 26884263\sqrt{2}\log\left(1-4\right.}{188416}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

output
```
((4*(-15423965 + 16138403*x - 5754186*x^2 + 2624760*x^3 + 2318400*x^4 + 73
6000*x^5))/Sqrt[3 - x + 2*x^2] + 26884263*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 -
 2*x + 4*x^2]])/188416
```

3.87.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int -\frac{23(-2000x^4-4600x^3-3860x^2+1658x+4795)}{64\sqrt{2x^2-x+3}}dx-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{32}\int \frac{-2000x^4-4600x^3-3860x^2+1658x+4795}{\sqrt{2x^2-x+3}}dx-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(250x^3\sqrt{2x^2-x+3}-\frac{1}{8}\int \frac{8(-5475x^3-1610x^2+1658x+4795)}{\sqrt{2x^2-x+3}}dx\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(250x^3\sqrt{2x^2-x+3}-\int \frac{-5475x^3-1610x^2+1658x+4795}{\sqrt{2x^2-x+3}}dx\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{32}\left(-\frac{1}{6}\int \frac{3(-15565x^2+28532x+19180)}{2\sqrt{2x^2-x+3}}dx+\frac{1825}{2}\sqrt{2x^2-x+3}x^2+250\sqrt{2x^2-x+3}x^3\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(-\frac{1}{4}\int \frac{-15565x^2+28532x+19180}{\sqrt{2x^2-x+3}}dx+\frac{1825}{2}\sqrt{2x^2-x+3}x^2+250\sqrt{2x^2-x+3}x^3\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 2192

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{15565}{4}x\sqrt{2x^2-x+3}-\frac{1}{4}\int \frac{181561x+246830}{2\sqrt{2x^2-x+3}}dx\right)+\frac{1825}{2}\sqrt{2x^2-x+3}x^2+250\sqrt{2x^2-x+3}x^3\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{15565}{4}x\sqrt{2x^2-x+3}-\frac{1}{8}\int \frac{181561x+246830}{\sqrt{2x^2-x+3}}dx\right)+\frac{1825}{2}\sqrt{2x^2-x+3}x^2+250\sqrt{2x^2-x+3}x^3\right)-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 1160

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{1168881}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{181561}{2}\sqrt{2x^2-x+3}\right)+\frac{15565}{4}\sqrt{2x^2-x+3}x\right)+\frac{1825}{2}\sqrt{2x^2-x\right.$$
$$\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 1090

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{1168881\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{181561}{2}\sqrt{2x^2-x+3}\right)+\frac{15565}{4}\sqrt{2x^2-x+3}x\right)+\frac{1825}{2}\sqrt{2\right.$$
$$\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

$\Big\downarrow$ 222

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{1168881\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{181561}{2}\sqrt{2x^2-x+3}\right)+\frac{15565}{4}\sqrt{2x^2-x+3}x\right)+\frac{1825}{2}\sqrt{2x^2-x+3\right.$$
$$\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

3.87. $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx$

output
```
(-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) + ((1825*x^2*Sqrt[3 - x + 2*
x^2])/2 + 250*x^3*Sqrt[3 - x + 2*x^2] + ((15565*x*Sqrt[3 - x + 2*x^2])/4 +
 ((-181561*Sqrt[3 - x + 2*x^2])/2 - (1168881*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(4*Sqrt[2]))/8)/4)/32
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.87.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965}{47104\sqrt{2x^2-x+3}}-\frac{1168881\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$
trager	$\frac{736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965}{47104\sqrt{2x^2-x+3}}-\frac{1168881\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+4\right.}{8192}$
default	$\frac{-\frac{5392543}{376832}+\frac{5392543x}{94208}}{\sqrt{2x^2-x+3}}-\frac{5130399}{16384\sqrt{2x^2-x+3}}+\frac{125x^5}{8\sqrt{2x^2-x+3}}+\frac{1575x^4}{32\sqrt{2x^2-x+3}}+\frac{14265x^3}{256\sqrt{2x^2-x+3}}-\frac{125091x^2}{1024\sqrt{2x^2-x+3}}+\frac{116}{4096\sqrt{}}$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/47104*(736000*x^5+2318400*x^4+2624760*x^3-5754186*x^2+16138403*x-1542396
5)/(2*x^2-x+3)^(1/2)-1168881/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{26884263\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)}{376}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/376832*(26884263*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x +
3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(736000*x^5 + 2318400*x^4 + 2624760
*x^3 - 5754186*x^2 + 16138403*x - 15423965)*sqrt(2*x^2 - x + 3))/(2*x^2 -
x + 3)
```

3.87.6 Sympy [F]

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{125\,x^5}{8\,\sqrt{2\,x^2-x+3}} + \frac{1575\,x^4}{32\,\sqrt{2\,x^2-x+3}} + \frac{14265\,x^3}{256\,\sqrt{2\,x^2-x+3}} - \frac{125091\,x^2}{1024\,\sqrt{2\,x^2-x+3}} - \frac{1168881}{8192}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{16138403\,x}{47104\,\sqrt{2\,x^2-x+3}} - \frac{15423965}{47104\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
125/8*x^5/sqrt(2*x^2 - x + 3) + 1575/32*x^4/sqrt(2*x^2 - x + 3) + 14265/25
6*x^3/sqrt(2*x^2 - x + 3) - 125091/1024*x^2/sqrt(2*x^2 - x + 3) - 1168881/
8192*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16138403/47104*x/sqrt(2*x^
2 - x + 3) - 15423965/47104/sqrt(2*x^2 - x + 3)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \frac{1168881}{8192}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{(46\,(20\,(40\,(20\,x+63)x+2853)x-125091)x+16138403)x-15423965}{47104\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
1168881/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
 + 1/47104*((46*(20*(40*(20*x + 63)*x + 2853)*x - 125091)*x + 16138403)*x
- 15423965)/sqrt(2*x^2 - x + 3)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^3}{(2\,x^2-x+3)^{3/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(3/2), x)
```

3.88 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx$

3.88.1 Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{415}{32}\sqrt{3-x+2x^2}+\frac{25}{8}x\sqrt{3-x+2x^2}-\frac{223\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

output
```
-223/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+121/92*(19-7*x)/(2*x^2-x+3
)^(1/2)+415/32*(2*x^2-x+3)^(1/2)+25/8*x*(2*x^2-x+3)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{47027-9421x+16790x^2+4600x^3}{736\sqrt{3-x+2x^2}}-\frac{223\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{64\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(47027 - 9421*x + 16790*x^2 + 4600*x^3)/(736*Sqrt[3 - x + 2*x^2]) - (223*L
og[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(64*Sqrt[2])
```

3.88.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{23(100x^2+170x+51)}{16\sqrt{2x^2-x+3}}dx+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{100x^2+170x+51}{\sqrt{2x^2-x+3}}dx+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{4}\int -\frac{2(48-415x)}{\sqrt{2x^2-x+3}}dx+25\sqrt{2x^2-x+3}x\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(25x\sqrt{2x^2-x+3}-\frac{1}{2}\int \frac{48-415x}{\sqrt{2x^2-x+3}}dx\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{223}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{415}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{223\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{415}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

$$\big\downarrow \text{222}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{223\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{415}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (25*x*Sqrt[3 - x + 2*x^2] + ((
415*Sqrt[3 - x + 2*x^2])/2 + (223*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]
))/2)/8
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.88.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result
risch	$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}+\frac{223\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{128}$
trager	$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}+\frac{223\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}-\operatorname{RootOf}\left(\_Z^2-2\right)\right)}{128}$
default	$-\frac{13713(-1+4x)}{5888\sqrt{2x^2-x+3}}+\frac{15761}{256\sqrt{2x^2-x+3}}+\frac{25x^3}{4\sqrt{2x^2-x+3}}+\frac{365x^2}{16\sqrt{2x^2-x+3}}-\frac{223x}{64\sqrt{2x^2-x+3}}+\frac{223\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{128}$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/736*(4600*x^3+16790*x^2-9421*x+47027)/(2*x^2-x+3)^(1/2)+223/128*2^(1/2)*
arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{5129\sqrt{2}(2\,x^2-x+3)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+8}{5888\,(2\,x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/5888*(5129*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4
*x - 1) - 32*x^2 + 16*x - 25) + 8*(4600*x^3 + 16790*x^2 - 9421*x + 47027)*
sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

3.88.6 Sympy [F]

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(3/2), x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{25\,x^3}{4\sqrt{2\,x^2-x+3}} + \frac{365\,x^2}{16\sqrt{2\,x^2-x+3}} + \frac{223}{128}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{9421\,x}{736\sqrt{2\,x^2-x+3}} + \frac{47027}{736\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
25/4*x^3/sqrt(2*x^2 - x + 3) + 365/16*x^2/sqrt(2*x^2 - x + 3) + 223/128*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 9421/736*x/sqrt(2*x^2 - x + 3) +
47027/736/sqrt(2*x^2 - x + 3)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = -\frac{223}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{(230\,(20\,x+73)x-9421)x+47027}{736\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
-223/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/736*((230*(20*x + 73)*x - 9421)*x + 47027)/sqrt(2*x^2 - x + 3)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^2}{(2\,x^2-x+3)^{3/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(3/2), x)
```

3.89 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx$

3.89.1 Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} - \frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

output
```
-5/4*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-11/23*(5+3*x)/(2*x^2-x+3)^(1/2
)
```

3.89.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}} - \frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{2\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*Log[1 - 4*x + 2*Sqrt[6 - 2*x
 + 4*x^2]])/(2*Sqrt[2])
```

3.89.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2191, 27, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{115}{4\sqrt{2x^2-x+3}}dx-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{2}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{5\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{2\sqrt{46}}-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{5\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) + (5*ArcSinh[(-1 + 4*x)/Sqrt[23]]
)/(2*Sqrt[2])
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.89.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{11(5+3x)}{23\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{4}$	35
trager	$-\frac{11(5+3x)}{23\sqrt{2x^2-x+3}}-\frac{5\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-4\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{2x^2-x+3}+\operatorname{RootOf}\left(\_Z^2-2\right)\right)}{4}$	60
default	$\frac{-\frac{49}{184}+\frac{49x}{46}}{\sqrt{2x^2-x+3}}-\frac{17}{8\sqrt{2x^2-x+3}}-\frac{5x}{2\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{4}$	64

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-11/23*(5+3*x)/(2*x^2-x+3)^(1/2)+5/4*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4)
)
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{115\sqrt{2}(2x^2-x+3)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)-88}{184\,(2x^2-x+3)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/184*(115*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) - 88*sqrt(2*x^2 - x + 3)*(3*x + 5))/(2*x^2 - x
 + 3)
```

3.89.6 Sympy [F]

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^2+3x+2}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{5}{4}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{33x}{23\sqrt{2x^2-x+3}} - \frac{55}{23\sqrt{2x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
5/4*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 33/23*x/sqrt(2*x^2 - x + 3)
 - 55/23/sqrt(2*x^2 - x + 3)
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = -\frac{5}{4}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)-\frac{11\,(3\,x+5)}{23\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
-5/4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/23
*(3*x + 5)/sqrt(2*x^2 - x + 3)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx = \frac{5\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,(2\,x-\frac{1}{2})}{2}\right)}{4} + \frac{3\,(2\,x-12)}{23\,\sqrt{2\,x^2-x+3}} - \frac{10\,\left(\frac{11\,x}{2}+\frac{3}{2}\right)}{23\,\sqrt{2\,x^2-x+3}} + \frac{16\,x-4}{23\,\sqrt{2\,x^2-x+3}}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

output
```
(5*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/4 + (3*(2
*x - 12))/(23*(2*x^2 - x + 3)^(1/2)) - (10*((11*x)/2 + 3/2))/(23*(2*x^2 -
x + 3)^(1/2)) + (16*x - 4)/(23*(2*x^2 - x + 3)^(1/2))
```

3.90 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx$

3.90.1 Optimal result

Integrand size = 27, antiderivative size = 176

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(61+4\sqrt{2}+\left(69+65\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \frac{1}{22}\sqrt{\frac{1}{682}\left(-247+500\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-247+500\sqrt{2}\right)}}\left(61-4\sqrt{2}+\left(69-65\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
1/253*(13-6*x)/(2*x^2-x+3)^(1/2)-1/15004*arctanh(1/31*(61+x*(69-65*2^(1/2)
)-4*2^(1/2))*341^(1/2)/(-247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-16845
4+341000*2^(1/2))^(1/2)+1/15004*arctan(1/31*(61+4*2^(1/2)+x*(69+65*2^(1/2)
))*341^(1/2)/(247+500*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(168454+341000*2^(
1/2))^(1/2)
```

3.90.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.13

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \frac{13-6x}{253\sqrt{3-x+2x^2}}$$
$$+\frac{1}{22}\text{RootSum}\Bigg[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3$$
$$-5\#1^4\&,\frac{23\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+16\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1-5\log\left(-\sqrt{\right.}\right.}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1
^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (23*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1] + 16*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 5*L
og[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9
*Sqrt[2]*#1^2 - 10*#1^3) & ]/22
```

3.90.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1305, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx$$
$$\Big\downarrow \text{1305}$$
$$\frac{13-6x}{253\sqrt{2x^2-x+3}}-\frac{\int -\frac{253(5x+8)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2783}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{22}\int \frac{5x+8}{\sqrt{2x^2-x+3}\,(5x^2+3x+2)}dx+\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$\downarrow$ 1368

$$\frac{1}{22}\left(\frac{\int -\frac{11\left(-\left(\left(13+5\sqrt{2}\right)x\right)-8\sqrt{2}+3\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(13-5\sqrt{2}\right)x\right)+8\sqrt{2}+3\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$\downarrow$ 27

$$\frac{1}{22}\left(\frac{\int \frac{-\left(\left(13-5\sqrt{2}\right)x\right)+8\sqrt{2}+3}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(13+5\sqrt{2}\right)x\right)-8\sqrt{2}+3}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$\downarrow$ 1362

$$\frac{1}{22}\left(\frac{\left(247-500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61\right)^2}{2x^2-x+3}-31\left(247-500\sqrt{2}\right)}d\frac{\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{\left(247+500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(69+65\sqrt{2}\right.\right.}{2x^2-}}}{}\right.$$

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$\downarrow$ 217

$$\frac{1}{22}\left(\frac{\left(247-500\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61\right)^2}{2x^2-x+3}-31\left(247-500\sqrt{2}\right)}d\frac{\left(69-65\sqrt{2}\right)x-4\sqrt{2}+61}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\arctan\right.$$

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

$\downarrow$ 219

$$\frac{1}{22}\left(\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(247-500\sqrt{2}\right)\operatorname{arctanh}\left(\right.}{\sqrt{68}}\right.$$

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}$$

3.90. $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx$

input
```
Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcT
an[(Sqrt[11/(31*(247 + 500*Sqrt[2]))]*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*
x))/Sqrt[3 - x + 2*x^2]] + ((247 - 500*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-247
 + 500*Sqrt[2]))]*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x))/Sqrt[3 - x + 2*x
^2]])/Sqrt[682*(-247 + 500*Sqrt[2])])/22
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.90.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.71 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.61

method	result
trager	$-\frac{-13+6x}{253\sqrt{2x^2-x+3}}-\frac{\mathrm{RootOf}\left(\_Z^2+150700176\,\mathrm{RootOf}\left(6103357128\_Z^4+6822387\_Z^2+15625\right)^2+168454\right)\ln\left(\frac{649244614491\,\mathrm{R}}{}\right.}{}$
risch	$-\frac{-13+6x}{253\sqrt{2x^2-x+3}}+\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(2197\sqrt{-775687+549362\sqrt{2}}\,\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\arctan\left(\right.\right.}{}$
default	$\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(2197\sqrt{-775687+549362\sqrt{2}}\,\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\,\arctan\left(\frac{\sqrt{-775687+549362\sqrt{2}}\,\sqrt{}}{}\right.\right.}{}$

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

3.90. $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx$

output
```
-1/253*(-13+6*x)/(2*x^2-x+3)^(1/2)-1/15004*RootOf(_Z^2+150700176*RootOf(61
03357128*_Z^4+6822387*_Z^2+15625)^2+168454)*ln((649244614491*RootOf(_Z^2+1
50700176*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^2+168454)*RootOf(61033
57128*_Z^4+6822387*_Z^2+15625)^4*x-2033209431*RootOf(6103357128*_Z^4+68223
87*_Z^2+15625)^2*RootOf(_Z^2+150700176*RootOf(6103357128*_Z^4+6822387*_Z^2
+15625)^2+168454)*x+1608394722165*RootOf(6103357128*_Z^4+6822387*_Z^2+1562
5)^2*(2*x^2-x+3)^(1/2)+3040381575*RootOf(6103357128*_Z^4+6822387*_Z^2+1562
5)^2*RootOf(_Z^2+150700176*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^2+16
8454)+1509120*RootOf(_Z^2+150700176*RootOf(6103357128*_Z^4+6822387*_Z^2+15
625)^2+168454)*x+1021170535*(2*x^2-x+3)^(1/2)-5845875*RootOf(_Z^2+15070017
6*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^2+168454))/(110484*x*RootOf(6
103357128*_Z^4+6822387*_Z^2+15625)^2+151*x+119))-9/11*RootOf(6103357128*_Z
^4+6822387*_Z^2+15625)*ln((373964897946816*x*RootOf(6103357128*_Z^4+682238
7*_Z^2+15625)^5+2007171224784*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^3
*x-1751259787200*RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^3-75467201040*
RootOf(6103357128*_Z^4+6822387*_Z^2+15625)^2*(2*x^2-x+3)^(1/2)+2645619075*
RootOf(6103357128*_Z^4+6822387*_Z^2+15625)*x-5324797800*RootOf(6103357128*
_Z^4+6822387*_Z^2+15625)-36443750*(2*x^2-x+3)^(1/2))/(220968*x*RootOf(6103
357128*_Z^4+6822387*_Z^2+15625)^2-55*x-238))
```

3.90.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.92

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx =$$

$$-\frac{23\sqrt{341}(2x^2-x+3)\sqrt{119i\sqrt{31}-247}\log\left(\frac{\sqrt{341}\sqrt{2x^2-x+3}\sqrt{119i\sqrt{31}-247}\left(61i\sqrt{31}+93\right)-7750\sqrt{31}(ix-6i)+147250x-}{x}\right.}{}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
-1/690184*(23*sqrt(341)*(2*x^2 - x + 3)*sqrt(119*I*sqrt(31) - 247)*log((sq
rt(341)*sqrt(2*x^2 - x + 3)*sqrt(119*I*sqrt(31) - 247)*(61*I*sqrt(31) + 93
) - 7750*sqrt(31)*(I*x - 6*I) + 147250*x - 170500)/x) - 23*sqrt(341)*(2*x^
2 - x + 3)*sqrt(119*I*sqrt(31) - 247)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*s
qrt(119*I*sqrt(31) - 247)*(-61*I*sqrt(31) - 93) - 7750*sqrt(31)*(I*x - 6*I
) + 147250*x - 170500)/x) - 23*sqrt(341)*(2*x^2 - x + 3)*sqrt(-119*I*sqrt(
31) - 247)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(61*I*sqrt(31) - 93)*sqrt(-1
19*I*sqrt(31) - 247) - 7750*sqrt(31)*(-I*x + 6*I) + 147250*x - 170500)/x)
+ 23*sqrt(341)*(2*x^2 - x + 3)*sqrt(-119*I*sqrt(31) - 247)*log((sqrt(341)*
sqrt(2*x^2 - x + 3)*(-61*I*sqrt(31) + 93)*sqrt(-119*I*sqrt(31) - 247) - 77
50*sqrt(31)*(-I*x + 6*I) + 147250*x - 170500)/x) + 2728*sqrt(2*x^2 - x + 3
)*(6*x - 13))/(2*x^2 - x + 3)
```

3.90.6 Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^2+3x+2)}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)), x)
```

3.90.7 Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(5x^2+3x+2)(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(3/2)), x)
```

3.90.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(2\,x^2-x+3)^{3/2}\,(5\,x^2+3\,x+2)}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)),x)
```

output
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)), x)
```

3.91 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx$

3.91.1 Optimal result

Integrand size = 27, antiderivative size = 211

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx = -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}}+\frac{4+65x}{682\sqrt{3-x+2x^2}\,(2+3x+5x^2)} + \frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(129694447+103775000\sqrt{2}\right)}}\left(12611+16454\sqrt{2}+\left(45519+29065\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{30008} - \frac{\sqrt{\frac{1}{682}\left(-129694447+103775000\sqrt{2}\right)}\text{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-129694447+103775000\sqrt{2}\right)}}\left(12611-16454\sqrt{2}+\left(45519-29065\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{30008}$$

output
```
1/345092*(-6315+2306*x)/(2*x^2-x+3)^(1/2)+1/682*(4+65*x)/(5*x^2+3*x+2)/(2*
x^2-x+3)^(1/2)-1/20465456*arctanh(1/31*(12611+x*(45519-29065*2^(1/2))-1645
4*2^(1/2))*341^(1/2)/(-129694447+103775000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2
))*(-88451612854+70774550000*2^(1/2))^(1/2)+1/20465456*arctan(1/31*(12611+
16454*2^(1/2)+x*(45519+29065*2^(1/2)))*341^(1/2)/(129694447+103775000*2^(1
/2))^(1/2)/(2*x^2-x+3)^(1/2))*(88451612854+70774550000*2^(1/2))^(1/2)
```

3.91.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.96

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx = \frac{\sqrt{3-x+2x^2}(-10606+18557x-24657x^2+11530x^3)}{345092(6+7x+16x^2+x^3+10x^4)} - \frac{1}{484}\text{RootSum}\Big[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3 -5\#1^4\&, \frac{225\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+8\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)\#1-15\log\left(-\right.}{-13\sqrt{2}+17\#1+9\sqrt{2}\#1^2-10\#1^3} + \frac{\text{RootSum}\Big[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{8623\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+9624\log\left(-\sqrt{2}x+\sqrt{3-\right.}}{-13\sqrt{2}+17\#1+}}{30008\sqrt{2}}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(Sqrt[3 - x + 2*x^2]*(-10606 + 18557*x - 24657*x^2 + 11530*x^3))/(345092*(
6 + 7*x + 16*x^2 + x^3 + 10*x^4)) - RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2
+ 6*Sqrt[2]*#1^3 - 5*#1^4 & , (225*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2]
- #1] + 8*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 15*Log
[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*S
qrt[2]*#1^2 - 10*#1^3) & ]/484 + RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6
*Sqrt[2]*#1^3 - 5*#1^4 & , (8623*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2
*x^2] - #1] + 9624*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 1565*
Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] +
17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/(30008*Sqrt[2])
```

3.91.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1305, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}dx$$
$$\Big\downarrow \text{1305}$$
$$\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}-\frac{\int -\frac{11(520x^2-303x+324)}{2(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx}{7502}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{520x^2-303x+324}{(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$
$$\Big\downarrow \text{2135}$$
$$\frac{\frac{\int \frac{253(2158-2495x)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2783}-\frac{6315-2306x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\frac{1}{22}\int \frac{2158-2495x}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx-\frac{6315-2306x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$
$$\Big\downarrow \text{1368}$$
$$\frac{\frac{1}{22}\left(\frac{\int -\frac{11\left(\left(337+2495\sqrt{2}\right)x-2158\sqrt{2}+4653\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(337-2495\sqrt{2}\right)x+2158\sqrt{2}+4653\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)-\frac{6315-2306x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\frac{1}{22}\left(\frac{\int \frac{\left(337-2495\sqrt{2}\right)x+2158\sqrt{2}+4653}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(337+2495\sqrt{2}\right)x-2158\sqrt{2}+4653}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)-\frac{6315-2306x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{65x+4}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}$$
$$\Big\downarrow \text{1362}$$

3.91. $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}dx$

$$\frac{\frac{1}{22}\left(\frac{\left(129694447-103775000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611\right)^2}{2x^2-x+3}-31\left(129694447-103775000\sqrt{2}\right)}d\frac{\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611}{\sqrt{2x^2-x+3}}}{\sqrt{2}} -\right.}{}$$

$$\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}$$

$\big\downarrow$ 217

$$\frac{\frac{1}{22}\left(\frac{\left(129694447-103775000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611\right)^2}{2x^2-x+3}-31\left(129694447-103775000\sqrt{2}\right)}d\frac{\left(45519-29065\sqrt{2}\right)x-16454\sqrt{2}+12611}{\sqrt{2x^2-x+3}}}{\sqrt{2}} +\right.}{}$$

$$\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{22}\left(\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(129694447+103775000\sqrt{2}\right)}}\left(\left(45519+29065\sqrt{2}\right)x+16454\sqrt{2}+12611\right)}{\sqrt{2x^2-x+3}}\right)+\frac{(12}{}\right.}{1364}$$

$$\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(4 + 65*x)/(682*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (-1/253*(6315 - 2
306*x)/Sqrt[3 - x + 2*x^2] + (Sqrt[(129694447 + 103775000*Sqrt[2])/682]*Ar
cTan[(Sqrt[11/(31*(129694447 + 103775000*Sqrt[2]))]*(12611 + 16454*Sqrt[2]
 + (45519 + 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] + ((129694447 - 103775
000*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-129694447 + 103775000*Sqrt[2]))]*(1261
1 - 16454*Sqrt[2] + (45519 - 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt
[682*(-129694447 + 103775000*Sqrt[2])])/22)/1364
```

3.91. $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx$

3.91.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

3.91. $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx$

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.91. $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}\,dx$

3.91.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.17 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.33

method	result
trager	Expression too large to display
risch	$\frac{11530x^3-24657x^2+18557x-10606}{345092(5x^2+3x+2)\sqrt{2x^2-x+3}} + \frac{\sqrt{\frac{8(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(1173047\sqrt{-775687+549362\sqrt{2}}\,\sqrt{2}\,\sqrt{-886\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/345092*(11530*x^3-24657*x^2+18557*x-10606)/(10*x^4+x^3+16*x^2+7*x+6)*(2*
x^2-x+3)^(1/2)-9/7502*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541
015625)*ln((-1593373465716747264*x*RootOf(24413428512*_Z^4+3582290320587*_
Z^2+168269541015625)^5-356051747055336070464*RootOf(24413428512*_Z^4+35822
90320587*_Z^2+168269541015625)^3*x+3247545983585953896000*RootOf(244134285
12*_Z^4+3582290320587*_Z^2+168269541015625)^2*(2*x^2-x+3)^(1/2)-1933844983
82205292800*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^3-
8600066701843343049675*RootOf(24413428512*_Z^4+3582290320587*_Z^2+16826954
1015625)*x+244659713848830018593750*(2*x^2-x+3)^(1/2)-37885378432347118747
800*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541015625))/(883872*x
*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^2+83351945*x+
24672962))-1/20465456*RootOf(_Z^2+602800704*RootOf(24413428512*_Z^4+358229
0320587*_Z^2+168269541015625)^2+88451612854)*ln(-(691568344495116*x*RootOf
(24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^4*RootOf(_Z^2+602800
704*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^2+88451612
854)+48417413991953391*RootOf(24413428512*_Z^4+3582290320587*_Z^2+16826954
1015625)^2*RootOf(_Z^2+602800704*RootOf(24413428512*_Z^4+3582290320587*_Z^
2+168269541015625)^2+88451612854)*x+34606661887587821204250*RootOf(2441342
8512*_Z^4+3582290320587*_Z^2+168269541015625)^2*(2*x^2-x+3)^(1/2)-83934244
089498825*RootOf(24413428512*_Z^4+3582290320587*_Z^2+168269541015625)^2...
```

3.91.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.84

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx = \frac{23\sqrt{341}(10\,x^4+x^3+16\,x^2+7\,x+6)\sqrt{12336481i\sqrt{31}-129694}}{}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/941410976*(23*sqrt(341)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(12336481*
I*sqrt(31) - 129694447)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(12336481*I
*sqrt(31) - 129694447)*(12611*I*sqrt(31) - 144243) - 1608512500*sqrt(31)*(
-I*x + 6*I) + 30561737500*x - 35387275000)/x) - 23*sqrt(341)*(10*x^4 + x^3
 + 16*x^2 + 7*x + 6)*sqrt(12336481*I*sqrt(31) - 129694447)*log((sqrt(341)*
sqrt(2*x^2 - x + 3)*sqrt(12336481*I*sqrt(31) - 129694447)*(-12611*I*sqrt(3
1) + 144243) - 1608512500*sqrt(31)*(-I*x + 6*I) + 30561737500*x - 35387275
000)/x) - 23*sqrt(341)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(-12336481*I*
sqrt(31) - 129694447)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(12611*I*sqrt(31)
 + 144243)*sqrt(-12336481*I*sqrt(31) - 129694447) - 1608512500*sqrt(31)*(I
*x - 6*I) + 30561737500*x - 35387275000)/x) + 23*sqrt(341)*(10*x^4 + x^3 +
 16*x^2 + 7*x + 6)*sqrt(-12336481*I*sqrt(31) - 129694447)*log((sqrt(341)*s
qrt(2*x^2 - x + 3)*(-12611*I*sqrt(31) - 144243)*sqrt(-12336481*I*sqrt(31)
- 129694447) - 1608512500*sqrt(31)*(I*x - 6*I) + 30561737500*x - 353872750
00)/x) + 2728*(11530*x^3 - 24657*x^2 + 18557*x - 10606)*sqrt(2*x^2 - x + 3
))/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)
```

3.91.6 Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{3}{2}}\,(5x^2+3x+2)^2}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2), x)
```

3.91.7 Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^2(2\,x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(3/2)), x)
```

3.91.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^2}\,dx = \int \frac{1}{(2\,x^2-x+3)^{3/2}\,(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2),x)
```

output
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^2), x)
```

3.92 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx$

3.92.1 Optimal result

Integrand size = 27, antiderivative size = 246

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\,(2+3x+5x^2)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2}\,(2+3x+5x^2)}$$

$$+ \frac{3\sqrt{\frac{1}{682}\left(13874275807943+9819738650000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13874275807943+9819738650000\sqrt{2}\right)}}\left(5538393+4123702\sqrt{2}+(13\right.}{\sqrt{3-x+2x^2}}\right.}{81861824}$$

$$- \frac{3\sqrt{\frac{1}{682}\left(-13874275807943+9819738650000\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-13874275807943+9819738650000\sqrt{2}\right)}}\left(5538393-4123702\sqrt{2}\right.}{\sqrt{3-x+2x^2}}\right.}{81861824}$$

output
```
1/941410976*(-4353943+6508666*x)/(2*x^2-x+3)^(1/2)+1/1364*(4+65*x)/(5*x^2+
3*x+2)^2/(2*x^2-x+3)^(1/2)+5/1860496*(7318+17315*x)/(5*x^2+3*x+2)/(2*x^2-x
+3)^(1/2)-3/55829763968*arctanh(1/31*(5538393+x*(13785797-9662095*2^(1/2))
-4123702*2^(1/2))*341^(1/2)/(-13874275807943+9819738650000*2^(1/2))^(1/2)/
(2*x^2-x+3)^(1/2))*(-9462256101017126+6697061759300000*2^(1/2))^(1/2)+3/55
829763968*arctan(1/31*(5538393+4123702*2^(1/2)+x*(13785797+9662095*2^(1/2)
))*341^(1/2)/(13874275807943+9819738650000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2
))*(9462256101017126+6697061759300000*2^(1/2))^(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.06 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.47

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \frac{\frac{4(22374044+161806828x+175833195x^2+277167774x^3+86411405x^4+162716650x^5)}{\sqrt{3-x+2x^2}(2+3x+5x^2)^2} - 1}{}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
((4*(22374044 + 161806828*x + 175833195*x^2 + 277167774*x^3 + 86411405*x^4
 + 162716650*x^5))/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) - 176824*Sqrt
[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-
491*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 208*Log[-(Sqrt[
2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 5*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3
 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3)
 & ] + 124*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3
- 5*#1^4 & , (7194481*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]
 - 3798456*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 575915*Sqrt[2
]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1
+ 9*Sqrt[2]*#1^2 - 10*#1^3) & ] - 7*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 +
17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (143178771*Sqrt[2]*Log[-(Sqrt[2]*x)
+ Sqrt[3 - x + 2*x^2] - #1] - 105962920*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*
x^2] - #1]*#1 + 6180225*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #
1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/3765643904
```

3.92.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1305, 27, 2135, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{3/2}(5x^2+3x+2)^3}\,dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}-\frac{\int -\frac{11\left(1040x^2-687x+1042\right)}{2\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}dx}{15004}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{1040x^2-687x+1042}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}dx}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 2135

$$\frac{\frac{\int \frac{11\left(692600x^2-28425x+483914\right)}{2\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{7502}+\frac{5(17315x+7318)}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{692600x^2-28425x+483914}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{1364}+\frac{5(17315x+7318)}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 2135

$$\frac{\frac{\frac{\int \frac{759(847654-395185x)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2783}-\frac{4353943-6508666x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{5(17315x+7318)}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3}{22}\int \frac{847654-395185x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx-\frac{4353943-6508666x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{5(17315x+7318)}{682\sqrt{2x^2-x+3}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 1368

$$\frac{\frac{\frac{3}{22}\left(\frac{\int -\frac{11\left(-\left(\left(452469-395185\sqrt{2}\right)x\right)-847654\sqrt{2}+1242839\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(-\left(\left(452469+395185\sqrt{2}\right)x\right)+847654\sqrt{2}+1242839\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)-\frac{4353943-6508666x}{253\sqrt{2x^2-x+3}}}{1364}+682\surd}{2728}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\Big\downarrow$ 27

3.92. $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}dx$

$$\frac{\frac{\frac{3}{22}\left(\frac{\int \frac{-\left(\left(452469+395185\sqrt{2}\right)x\right)+847654\sqrt{2}+1242839}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{-\left(\left(452469-395185\sqrt{2}\right)x\right)-847654\sqrt{2}+1242839}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)-\frac{4353943-6508666x}{253\sqrt{2x^2-x+3}}}{1364}+\frac{5(17315x+7}{682\sqrt{2x^2-x+3}(5}}{2728}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 1362

$$\frac{\frac{3}{22}\left(\frac{\left(13874275807943-9819738650000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13785797-9662095\sqrt{2}\right)x-4123702\sqrt{2}+5538393\right)^2}{2x^2-x+3}-31\left(13874275807943-9819738650000\sqrt{2}\right)}d\left(13785797-9662\right.}{\sqrt{2}}\right.}{}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 217

$$\frac{\frac{3}{22}\left(\frac{\left(13874275807943-9819738650000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(13785797-9662095\sqrt{2}\right)x-4123702\sqrt{2}+5538393\right)^2}{2x^2-x+3}-31\left(13874275807943-9819738650000\sqrt{2}\right)}d\left(13785797-9662\right.}{\sqrt{2}}\right.}{}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 219

$$\frac{\frac{3}{22}\left(\sqrt{\frac{1}{682}\left(13874275807943+9819738650000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(13874275807943+9819738650000\sqrt{2}\right)}}\left(\left(13785797+9662095\sqrt{2}\right)x+4123702\sqrt{2}+5538393\right)}{\sqrt{2x^2-x+3}}\right)\right.}{}$$

$$\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}$$

input
```
Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

3.92. $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx$

output
```
(4 + 65*x)/(1364*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + ((5*(7318 + 17
315*x))/(682*Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (-1/253*(4353943 - 6
508666*x)/Sqrt[3 - x + 2*x^2] + (3*(Sqrt[(13874275807943 + 9819738650000*S
qrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2]))
]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 - x
 + 2*x^2]] + ((13874275807943 - 9819738650000*Sqrt[2])*ArcTanh[(Sqrt[11/(3
1*(-13874275807943 + 9819738650000*Sqrt[2]))]*(5538393 - 4123702*Sqrt[2] +
 (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-1387427
5807943 + 9819738650000*Sqrt[2])]))/22)/1364)/2728
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.92. $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx$

3.92.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.56 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.00

method	result
trager	Expression too large to display
risch	$\frac{162716650x^5+86411405x^4+277167774x^3+175833195x^2+161806828x+22374044}{941410976(5x^2+3x+2)^2\sqrt{2x^2-x+3}}+\frac{3\sqrt{\frac{8(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}\cdots}}{\cdots}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/941410976*(162716650*x^5+86411405*x^4+277167774*x^3+175833195*x^2+161806
828*x+22374044)/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2)+27/10232728*RootOf(97653
714048*_Z^4+383221372091193603*_Z^2+376669012321499306640625)*ln((22056286
34920723955712*x*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+376669012
321499306640625)^5+106257539646851603556713080 32*RootOf(97653714048*_Z^4+3
83221372091193603*_Z^2+376669012321499306640625)^3*x+5837873149767416564 02
406400*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+3766690123214993066
40625)^3+536130689864192083507898620 8000*RootOf(97653714048*_Z^4+383221372
091193603*_Z^2+376669012321499306640625)^2*(2*x^2-x+3)^(1/2)+1213869077110
296083725473727981 1025*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+376
669012321499306640625)*x+172528717649345479225805050579554 00*RootOf(9765371
4048*_Z^4+383221372091193603*_Z^2+376669012321499306640625)+10542686598921
071198677006035680781250*(2*x^2-x+3)^(1/2))/(3535488*x*RootOf(97653714048*
_Z^4+383221372091193603*_Z^2+376669012321499306640625)^2+7098559162705*x+2
15228344978))-3/55829763968*RootOf(_Z^2+2411202816*RootOf(97653714048*_Z^4
+383221372091193603*_Z^2+376669012321499306640625)^2+9462256101017126)*ln(
-(239326023754418832*RootOf(_Z^2+2411202816*RootOf(97653714048*_Z^4+383221
372091193603*_Z^2+376669012321499306640625)^2+9462256101017126)*RootOf(976
53714048*_Z^4+383221372091193603*_Z^2+376669012321499306640625)^4*x+725400
753296830666567677*RootOf(97653714048*_Z^4+383221372091193603*_Z^2+3766...
```

3.92.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.86

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^3}\,dx = \frac{23\,\sqrt{341}(50\,x^6+35\,x^5+103\,x^4+85\,x^3+83\,x^2+32\,x+12)\sqrt{968}}{}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/2568169142528*(23*sqrt(341)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2
 + 32*x + 12)*sqrt(968527552401*I*sqrt(31) - 124868482271487)*log((sqrt(34
1)*sqrt(2*x^2 - x + 3)*sqrt(968527552401*I*sqrt(31) - 124868482271487)*(55
38393*I*sqrt(31) - 38528009) - 456617847225000*sqrt(31)*(-I*x + 6*I) + 867
5739097275000*x - 10045592638950000)/x) - 23*sqrt(341)*(50*x^6 + 35*x^5 +
103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*sqrt(968527552401*I*sqrt(31) - 1248
68482271487)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(968527552401*I*sqrt(3
1) - 124868482271487)*(-5538393*I*sqrt(31) + 38528009) - 456617847225000*s
qrt(31)*(-I*x + 6*I) + 8675739097275000*x - 10045592638950000)/x) - 23*sqr
t(341)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*sqrt(-968
527552401*I*sqrt(31) - 124868482271487)*log((sqrt(341)*sqrt(2*x^2 - x + 3)
*(5538393*I*sqrt(31) + 38528009)*sqrt(-968527552401*I*sqrt(31) - 124868482
271487) - 456617847225000*sqrt(31)*(I*x - 6*I) + 8675739097275000*x - 1004
5592638950000)/x) + 23*sqrt(341)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*
x^2 + 32*x + 12)*sqrt(-968527552401*I*sqrt(31) - 124868482271487)*log((sqr
t(341)*sqrt(2*x^2 - x + 3)*(-5538393*I*sqrt(31) - 38528009)*sqrt(-96852755
2401*I*sqrt(31) - 124868482271487) - 456617847225000*sqrt(31)*(I*x - 6*I)
+ 8675739097275000*x - 10045592638950000)/x) + 2728*(162716650*x^5 + 86411
405*x^4 + 277167774*x^3 + 175833195*x^2 + 161806828*x + 22374044)*sqrt(2*x
^2 - x + 3))/(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

3.92.6 Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}\,(2+3x+5x^2)^3}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{3}{2}}\,(5x^2+3x+2)^3}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

3.92.7 Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^3(2\,x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(3/2)), x)
```

3.92.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(2\,x^2-x+3)^{3/2}\,(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3),x)
```

output
```
int(1/((2*x^2 - x + 3)^(3/2)*(3*x + 5*x^2 + 2)^3), x)
```

3.93 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx$

3.93.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = -\frac{14641(101+79x)}{4416\,(3-x+2x^2)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{526075x\sqrt{3-x+2x^2}}{3072} + \frac{38375}{384}x^2\sqrt{3-x+2x^2} + \frac{625}{32}x^3\sqrt{3-x+2x^2} + \frac{16955197\,\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}$$

output
```
-14641/4416*(101+79*x)/(2*x^2-x+3)^(3/2)+16955197/16384*arcsinh(1/23*(1-4*
x)*23^(1/2))*2^(1/2)+1331/101568*(7409+116368*x)/(2*x^2-x+3)^(1/2)-1308645
/4096*(2*x^2-x+3)^(1/2)+526075/3072*x*(2*x^2-x+3)^(1/2)+38375/384*x^2*(2*x
^2-x+3)^(1/2)+625/32*x^3*(2*x^2-x+3)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{-18974698519 + 49883864262x - 36481630395x^2 + 39848900984x^3 - 507678126}{6500352\,(3-x+2x^2)^{3/2}} + \frac{16955197\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8192\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 507678
1260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 -
x + 2*x^2)^(3/2)) + (16955197*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(819
2*Sqrt[2])
```

3.93.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {2191, 27, 2191, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int \frac{2760000x^6+8004000x^5+10239600x^4+3447240x^3-5859204x^2-3967086x+3839123}{256(2x^2-x+3)^{3/2}}dx-\frac{14641(79x+101)}{4416(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2760000x^6+8004000x^5+10239600x^4+3447240x^3-5859204x^2-3967086x+3839123}{(2x^2-x+3)^{3/2}}dx}{8832}-\frac{14641(79x+101)}{4416(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{2}{23}\int -\frac{1587(-10000x^4-34000x^3-39100x^2+18960x+89359)}{\sqrt{2x^2-x+3}}dx+\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}}{8832}-\frac{14641(79x+101)}{4416(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\int \frac{-10000x^4-34000x^3-39100x^2+18960x+89359}{\sqrt{2x^2-x+3}}dx}{8832}-\frac{14641(79x+101)}{4416(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{8}\int \frac{8(-38375x^3-27850x^2+18960x+89359)}{\sqrt{2x^2-x+3}}dx-1250x^3\sqrt{2x^2-x+3}\right)}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\int \frac{-38375x^3-27850x^2+18960x+89359}{\sqrt{2x^2-x+3}}dx-1250x^3\sqrt{2x^2-x+3}\right)}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{2192}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{6}\int \frac{-526075x^2+688020x+1072308}{2\sqrt{2x^2-x+3}}dx-\frac{38375}{6}\sqrt{2x^2-x+3}x^2-1250\sqrt{2x^2-x+3}x^3\right)}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\int \frac{-526075x^2+688020x+1072308}{\sqrt{2x^2-x+3}}dx-\frac{38375}{6}\sqrt{2x^2-x+3}x^2-1250\sqrt{2x^2-x+3}x^3\right)}{8832}-\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{2192}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{1}{4}\int \frac{3(1308645x+3911638)}{2\sqrt{2x^2-x+3}}dx-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)-\frac{38375}{6}\sqrt{2x^2-x+3}x^2-1250\sqrt{2}\right.}{8832}$$

$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{3}{8}\int \frac{1308645x+3911638}{\sqrt{2x^2-x+3}}dx-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)-\frac{38375}{6}\sqrt{2x^2-x+3}x^2-1250\sqrt{2x^2}\right.}{8832}$$

$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1160}$$

3.93. $\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^{5/2}}\,dx$

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{16955197}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1308645}{2}\sqrt{2x^2-x+3}\right)-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)-\frac{38375}{6}\sqrt{}\right.}{8832}$$

$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 1090

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{16955197\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{1308645}{2}\sqrt{2x^2-x+3}\right)-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)\right.}{8832}$$

$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 222

$$\frac{\frac{2662(116368x+7409)}{23\sqrt{2x^2-x+3}}-138\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{16955197\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{1308645}{2}\sqrt{2x^2-x+3}\right)-\frac{526075}{4}x\sqrt{2x^2-x+3}\right)-\frac{38375}{6}\right.}{8832}$$

$$\frac{14641(79x+101)}{4416\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + ((2662*(7409 + 116368
*x))/(23*Sqrt[3 - x + 2*x^2]) - 138*((-38375*x^2*Sqrt[3 - x + 2*x^2])/6 -
1250*x^3*Sqrt[3 - x + 2*x^2] + ((-526075*x*Sqrt[3 - x + 2*x^2])/4 + (3*((1
308645*Sqrt[3 - x + 2*x^2])/2 + (16955197*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4
*Sqrt[2])))/8)/12))/8832
```

3.93.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.93. $\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^{5/2}}\,dx$

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.93.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$\frac{507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+49883864262x-18974698519}{6500352(2x^2-x+3)^{\frac{3}{2}}} -$
trager	$\frac{507840000x^7+2090608000x^6+3504730800x^5-5076781260x^4+39848900984x^3-36481630395x^2+49883864262x-18974698519}{6500352(2x^2-x+3)^{\frac{3}{2}}} +$
default	$\frac{16955197x}{8192\sqrt{2x^2-x+3}} + \frac{-\frac{992926033}{13000704}+\frac{992926033x}{3250176}}{\sqrt{2x^2-x+3}} - \frac{2149616639}{524288(2x^2-x+3)^{\frac{3}{2}}} + \frac{16955197}{32768\sqrt{2x^2-x+3}} + \frac{625x^7}{8(2x^2-x+3)^{\frac{3}{2}}} - \frac{67488035x}{16384(2x^2-x+}$

input
```
int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6500352*(507840000*x^7+2090608000*x^6+3504730800*x^5-5076781260*x^4+3984
8900984*x^3-36481630395*x^2+49883864262*x-18974698519)/(2*x^2-x+3)^(3/2)-1
6955197/16384*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{26907897639\,\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-\right.}{}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
1/52002816*(26907897639*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(4*s
qrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(507840000*
x^7 + 2090608000*x^6 + 3504730800*x^5 - 5076781260*x^4 + 39848900984*x^3 -
 36481630395*x^2 + 49883864262*x - 18974698519)*sqrt(2*x^2 - x + 3))/(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.93.6 Sympy [F]

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5x^2+3x+2)^4}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(5/2), x)
```

3.93.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(118) = 236.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.72

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{625\,x^7}{8\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{30875\,x^6}{96\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{138025\,x^5}{256\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{799745\,x^4}{1024\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{16955197}{13000704}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right) - \frac{16955197}{16384}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{1203818987}{6500352}\,\sqrt{2\,x^2-x+3} + \frac{3536205583\,x}{3250176\,\sqrt{2\,x^2-x+3}} - \frac{2638851\,x^2}{512\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{257773037}{1083392\,\sqrt{2\,x^2-x+3}} + \frac{29484067\,x}{23552\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{374445479}{70656\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
625/8*x^7/(2*x^2 - x + 3)^(3/2) + 30875/96*x^6/(2*x^2 - x + 3)^(3/2) + 138
025/256*x^5/(2*x^2 - x + 3)^(3/2) - 799745/1024*x^4/(2*x^2 - x + 3)^(3/2)
- 16955197/13000704*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3
)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x
^2 - x + 3)^(3/2)) - 16955197/16384*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1
)) + 1203818987/6500352*sqrt(2*x^2 - x + 3) + 3536205583/3250176*x/sqrt(2*
x^2 - x + 3) - 2638851/512*x^2/(2*x^2 - x + 3)^(3/2) + 257773037/1083392/s
qrt(2*x^2 - x + 3) + 29484067/23552*x/(2*x^2 - x + 3)^(3/2) - 374445479/70
656/(2*x^2 - x + 3)^(3/2)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{16955197}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$
$$+\frac{((4\,(2645\,(20\,(40\,(60\,x+247)x+16563)x-479847)x+9962225246)x-36481630395)x+49883864262)}{6500352\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
16955197/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) +
1) + 1/6500352*(((4*(2645*(20*(40*(60*x + 247)*x + 16563)*x - 479847)*x +
9962225246)*x - 36481630395)*x + 49883864262)*x - 18974698519)/(2*x^2 - x
+ 3)^(3/2)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^4}{(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^4/(2*x^2 - x + 3)^(5/2), x)
```

3.94 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx$

3.94.1 Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = -\frac{1331(17-45x)}{1104\,(3-x+2x^2)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}} + \frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3-x+2x^2}-\frac{7495\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

output
```
-1331/1104*(17-45*x)/(2*x^2-x+3)^(3/2)-7495/256*arcsinh(1/23*(1-4*x)*23^(1
/2))*2^(1/2)+121/8464*(10679-6744*x)/(2*x^2-x+3)^(1/2)+3175/64*(2*x^2-x+3)
^(1/2)+125/16*x*(2*x^2-x+3)^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \frac{89784565-62463282x+101546529x^2-29423976x^3+16980900x^4+3174000x^5}{101568\,(3-x+2x^2)^{3/2}} - \frac{7495\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{128\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(89784565 - 62463282*x + 101546529*x^2 - 29423976*x^3 + 16980900*x^4 + 317
4000*x^5)/(101568*(3 - x + 2*x^2)^(3/2)) - (7495*Log[1 - 4*x + 2*Sqrt[6 -
2*x + 4*x^2]])/(128*Sqrt[2])
```

3.94.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2191, 27, 2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int -\frac{3(-46000x^4-105800x^3-88780x^2+38134x+30425)}{64\,(2x^2-x+3)^{3/2}}dx-\frac{1331(17-45x)}{1104\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{736}\int \frac{-46000x^4-105800x^3-88780x^2+38134x+30425}{(2x^2-x+3)^{3/2}}dx-\frac{1331(17-45x)}{1104\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{736}\left(\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}-\frac{2}{23}\int -\frac{2645(100x^2+280x+183)}{\sqrt{2x^2-x+3}}dx\right)-\frac{1331(17-45x)}{1104\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{736}\left(230\int \frac{100x^2+280x+183}{\sqrt{2x^2-x+3}}dx+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{736}\left(230\left(\frac{1}{4}\int \frac{2(635x+216)}{\sqrt{2x^2-x+3}}dx+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{736}\left(230\left(\frac{1}{2}\int \frac{635x+216}{\sqrt{2x^2-x+3}}dx+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{736}\left(230\left(\frac{1}{2}\left(\frac{1499}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{635}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{736}\left(230\left(\frac{1}{2}\left(\frac{1499\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{635}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x}{23\sqrt{2x^2-x+3}}\right.$$
$$\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{736}\left(230\left(\frac{1}{2}\left(\frac{1499\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{635}{2}\sqrt{2x^2-x+3}\right)+25\sqrt{2x^2-x+3}x\right)+\frac{242(10679-6744x)}{23\sqrt{2x^2-x+3}}\right)-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + ((242*(10679 - 6744*x))
/(23*Sqrt[3 - x + 2*x^2]) + 230*(25*x*Sqrt[3 - x + 2*x^2] + ((635*Sqrt[3 -
 x + 2*x^2])/2 + (1499*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/2))/736
```

3.94. $\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^{5/2}}\,dx$

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.94. $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx$

3.94.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result
risch	$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568(2x^2-x+3)^{\frac{3}{2}}}+\frac{7495\sqrt{2}\,\text{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}$
trager	$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568(2x^2-x+3)^{\frac{3}{2}}}+\frac{7495\,\text{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\text{RootOf}\left(\_Z^2-2\right)x\right.}{256}$
default	$-\frac{14081711(-1+4x)}{565248(2x^2-x+3)^{\frac{3}{2}}}-\frac{3391139(-1+4x)}{203136\sqrt{2x^2-x+3}}+\frac{20961031}{24576(2x^2-x+3)^{\frac{3}{2}}}+\frac{125x^5}{4(2x^2-x+3)^{\frac{3}{2}}}+\frac{2675x^4}{16(2x^2-x+3)^{\frac{3}{2}}}-\frac{7495x^3}{192(2x^2-x+3)^{\frac{3}{2}}}$

input
```
int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/101568*(3174000*x^5+16980900*x^4-29423976*x^3+101546529*x^2-62463282*x+8
9784565)/(2*x^2-x+3)^(3/2)+7495/256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \frac{11894565\,\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-3\right.}{}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
1/812544*(11894565*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(
2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(3174000*x^5 +
16980900*x^4 - 29423976*x^3 + 101546529*x^2 - 62463282*x + 89784565)*sqrt(
2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.94.6 Sympy [F]

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5x^2+3x+2)^3}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(5/2), x)
```

3.94.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(84) = 168$.

Time = 0.27 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.09

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \frac{125\,x^5}{4\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{2675\,x^4}{16\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{7495}{203136}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right) + \frac{7495}{256}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{532145}{101568}\,\sqrt{2\,x^2-x+3} - \frac{4515389\,x}{50784\,\sqrt{2\,x^2-x+3}} + \frac{7197\,x^2}{8\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{396211}{50784\,\sqrt{2\,x^2-x+3}} - \frac{269783\,x}{1104\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{1002137}{1104\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
125/4*x^5/(2*x^2 - x + 3)^(3/2) + 2675/16*x^4/(2*x^2 - x + 3)^(3/2) + 7495
/203136*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71
/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^
(3/2)) + 7495/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 532145/101568
*sqrt(2*x^2 - x + 3) - 4515389/50784*x/sqrt(2*x^2 - x + 3) + 7197/8*x^2/(2
*x^2 - x + 3)^(3/2) + 396211/50784/sqrt(2*x^2 - x + 3) - 269783/1104*x/(2*
x^2 - x + 3)^(3/2) + 1002137/1104/(2*x^2 - x + 3)^(3/2)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = -\frac{7495}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{3\left(\left(4\left(13225\left(20\,x+107\right)x-2451998\right)x+33848843\right)x-20821094\right)x+89784565}{101568\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
-7495/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
1/101568*(3*((4*(13225*(20*x + 107)*x - 2451998)*x + 33848843)*x - 2082109
4)*x + 89784565)/(2*x^2 - x + 3)^(3/2)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^3}{(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^3/(2*x^2 - x + 3)^(5/2), x)
```

3.95 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx$

3.95.1 Optimal result

Integrand size = 27, antiderivative size = 68

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{121(19-7x)}{276\,(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} - \frac{25\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

output
```
121/276*(19-7*x)/(2*x^2-x+3)^(3/2)-25/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(
1/2)-11/6348*(7351+2336*x)/(2*x^2-x+3)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = -\frac{11(8623+714x+6183x^2+2336x^3)}{3174\,(3-x+2x^2)^{3/2}} - \frac{25\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{4\sqrt{2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-11*(8623 + 714*x + 6183*x^2 + 2336*x^3))/(3174*(3 - x + 2*x^2)^(3/2)) -
(25*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(4*Sqrt[2])
```

3.95.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2191, 27, 2191, 27, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int \frac{6900x^2+11730x+131}{16(2x^2-x+3)^{3/2}}dx+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{552}\int \frac{6900x^2+11730x+131}{(2x^2-x+3)^{3/2}}dx+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{552}\left(\frac{2}{23}\int \frac{39675}{\sqrt{2x^2-x+3}}dx-\frac{22(2336x+7351)}{23\sqrt{2x^2-x+3}}\right)+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{552}\left(3450\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{22(2336x+7351)}{23\sqrt{2x^2-x+3}}\right)+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{552}\left(75\sqrt{46}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{22(2336x+7351)}{23\sqrt{2x^2-x+3}}\right)+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{552}\left(1725\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{22(2336x+7351)}{23\sqrt{2x^2-x+3}}\right)+\frac{121(19-7x)}{276(2x^2-x+3)^{3/2}}$$

input
```
Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) + ((-22*(7351 + 2336*x))/(23*
Sqrt[3 - x + 2*x^2]) + 1725*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/552
```

3.95.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.95.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{11\left(2336 x^{3}+6183 x^{2}+714 x+8623\right)}{3174\left(2 x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{25 \sqrt{2}\, \operatorname{arcsinh}\left(\frac{4 \sqrt{23}\, \left(x-\frac{1}{4}\right)}{23}\right)}{8}$
trager	$-\frac{11\left(2336 x^{3}+6183 x^{2}+714 x+8623\right)}{3174\left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{25 \operatorname{RootOf}\left(\_Z^{2}-2\right) \ln\left(-4 \operatorname{RootOf}\left(\_Z^{2}-2\right) x+4 \sqrt{2 x^{2}-x+3}+\operatorname{RootOf}\left(\_Z^{2}-2\right)\right)}{8}$
default	$\frac{-\frac{8493}{5888}+\frac{8493 x}{1472}}{\left(2 x^{2}-x+3\right)^{\frac{3}{2}}}+\frac{-\frac{2267}{2116}+\frac{2267 x}{529}}{\sqrt{2 x^{2}-x+3}}-\frac{15775}{768\left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{25 x^{3}}{6\left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{145 x^{2}}{8\left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{319 x}{64\left(2 x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{25 x}{4\sqrt{2 x^{2}-x+3}}$

input
```
int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-11/3174*(2336*x^3+6183*x^2+714*x+8623)/(2*x^2-x+3)^(3/2)+25/8*2^(1/2)*arc
sinh(4/23*23^(1/2)*(x-1/4))
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(55) = 110$.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{39675\,\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x\right.}{25392\,(4\,x^4-4\,x^3+13\,x}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
1/25392*(39675*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 88*(2336*x^3 + 6183*x
^2 + 714*x + 8623)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.95.6 Sympy [F]

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5x^2+3x+2)^2}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(5/2), x)
```

3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(55) = 110$.

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.72

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{25}{6348}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \right.$$
$$+ \frac{25}{8}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{1775}{3174}\sqrt{2\,x^2-x+3} + \frac{1017\,x}{529\sqrt{2\,x^2-x+3}}$$
$$- \frac{15\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{6413}{3174\sqrt{2\,x^2-x+3}} - \frac{x}{138\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{2593}{138\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
25/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71
/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^
(3/2)) + 25/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 1775/3174*sqrt(2*
x^2 - x + 3) + 1017/529*x/sqrt(2*x^2 - x + 3) - 15*x^2/(2*x^2 - x + 3)^(3/
2) - 6413/3174/sqrt(2*x^2 - x + 3) - 1/138*x/(2*x^2 - x + 3)^(3/2) - 2593/
138/(2*x^2 - x + 3)^(3/2)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = -\frac{25}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) - \frac{11\,(((2336\,x+6183)x+714)x+8623)}{3174\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
-25/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/3
174*(((2336*x + 6183)*x + 714)*x + 8623)/(2*x^2 - x + 3)^(3/2)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(5\,x^2+3\,x+2)^2}{(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int((3*x + 5*x^2 + 2)^2/(2*x^2 - x + 3)^(5/2), x)
```

3.96 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx$

3.96.1 Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = -\frac{11(5+3x)}{69\,(3-x+2x^2)^{3/2}} - \frac{71(1-4x)}{529\sqrt{3-x+2x^2}}$$

output
```
-11/69*(5+3*x)/(2*x^2-x+3)^(3/2)-71/529*(1-4*x)/(2*x^2-x+3)^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{2(-952+1005x-639x^2+852x^3)}{1587\,(3-x+2x^2)^{3/2}}$$

input
```
Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(2*(-952 + 1005*x - 639*x^2 + 852*x^3))/(1587*(3 - x + 2*x^2)^(3/2))
```

3.96.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2191, 27, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+3x+2}{(2x^2-x+3)^{5/2}}\,dx$$

$$\Big\downarrow \; 2191$$

$$\frac{2}{69}\int \frac{213}{4\,(2x^2-x+3)^{3/2}}dx-\frac{11(3x+5)}{69\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{71}{46}\int \frac{1}{(2x^2-x+3)^{3/2}}dx-\frac{11(3x+5)}{69\,(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \; 1088$$

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69\,(2x^2-x+3)^{3/2}}$$

input
```
Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 -
x + 2*x^2])
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.96.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
trager	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
risch	$\frac{\frac{568}{529}x^3-\frac{426}{529}x^2+\frac{670}{529}x-\frac{1904}{1587}}{(2x^2-x+3)^{\frac{3}{2}}}$	30
default	$\frac{-\frac{71}{368}+\frac{71x}{92}}{(2x^2-x+3)^{\frac{3}{2}}}+\frac{-\frac{71}{529}+\frac{284x}{529}}{\sqrt{2x^2-x+3}}-\frac{29}{48(2x^2-x+3)^{\frac{3}{2}}}-\frac{5x}{4(2x^2-x+3)^{\frac{3}{2}}}$	69

input
```
int((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/1587/(2*x^2-x+3)^(3/2)*(852*x^3-639*x^2+1005*x-952)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{2\,(852\,x^3-639\,x^2+1005\,x-952)\sqrt{2\,x^2-x+3}}{1587\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
2/1587*(852*x^3 - 639*x^2 + 1005*x - 952)*sqrt(2*x^2 - x + 3)/(4*x^4 - 4*x
^3 + 13*x^2 - 6*x + 9)
```

3.96.6 Sympy [F]

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^2+3x+2}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{284\,x}{529\,\sqrt{2\,x^2-x+3}} - \frac{71}{529\,\sqrt{2\,x^2-x+3}} - \frac{11\,x}{23\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{55}{69\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
284/529*x/sqrt(2*x^2 - x + 3) - 71/529/sqrt(2*x^2 - x + 3) - 11/23*x/(2*x^
2 - x + 3)^(3/2) - 55/69/(2*x^2 - x + 3)^(3/2)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{2\,(3\,(71\,(4\,x-3)x+335)x-952)}{1587\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
2/1587*(3*(71*(4*x - 3)*x + 335)*x - 952)/(2*x^2 - x + 3)^(3/2)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{2+3x+5x^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{2\,(852\,x^3-639\,x^2+1005\,x-952)}{1587\,(2\,x^2-x+3)^{3/2}}$$

input
```
int((3*x + 5*x^2 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

output
```
(2*(1005*x - 639*x^2 + 852*x^3 - 952))/(1587*(2*x^2 - x + 3)^(3/2))
```

3.97 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx$

3.97.1 Optimal result

Integrand size = 27, antiderivative size = 199

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \frac{13-6x}{759(3-x+2x^2)^{3/2}}+\frac{3603-658x}{128018\sqrt{3-x+2x^2}}$$
$$+\frac{1}{484}\sqrt{\frac{1}{682}\left(-15457+25000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(-15457+25000\sqrt{2}\right)}}\left(443-98\sqrt{2}+\left(247+345\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$
$$-\frac{1}{484}\sqrt{\frac{1}{682}\left(15457+25000\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(15457+25000\sqrt{2}\right)}}\left(443+98\sqrt{2}+\left(247-345\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)$$

output
```
1/759*(13-6*x)/(2*x^2-x+3)^(3/2)+1/128018*(3603-658*x)/(2*x^2-x+3)^(1/2)+1
/330088*arctan(1/31*(443-98*2^(1/2)+x*(247+345*2^(1/2)))*341^(1/2)/(-15457
+25000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-10541674+17050000*2^(1/2))^(1/2
)-1/330088*arctanh(1/31*(443+x*(247-345*2^(1/2))+98*2^(1/2))*341^(1/2)/(15
457+25000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(10541674+17050000*2^(1/2))^(1
/2)
```

3.97.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.64 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.05

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \frac{39005-19767x+23592x^2-3948x^3}{384054(3-x+2x^2)^{3/2}} + \frac{1}{484}\text{RootSum}\Big[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&, \frac{249\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+108\sqrt{}}{}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(39005 - 19767*x + 23592*x^2 - 3948*x^3)/(384054*(3 - x + 2*x^2)^(3/2)) +
RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (249*L
og[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 108*Sqrt[2]*Log[-(Sqrt[2]*x)
 + Sqrt[3 - x + 2*x^2] - #1]*#1 - 65*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2
] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/484
```

3.97.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1305, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{5/2}(5x^2+3x+2)}\,dx$$

$$\Big\downarrow \; 1305$$

$$\frac{13-6x}{759(2x^2-x+3)^{3/2}} - \frac{\int -\frac{33(-40x^2+91x+168)}{2(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx}{8349}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{506}\int \frac{-40x^2+91x+168}{(2x^2-x+3)^{3/2}(5x^2+3x+2)}dx + \frac{13-6x}{759(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \; 2135$$

$$\frac{1}{506}\left(\frac{\int \frac{5819(65x+54)}{2\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2783}+\frac{3603-658x}{253\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{506}\left(\frac{23}{22}\int \frac{65x+54}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx+\frac{3603-658x}{253\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 1368$$

$$\frac{1}{506}\left(\frac{23}{22}\left(\frac{\int \frac{11\left(\left(119+65\sqrt{2}\right)x+54\sqrt{2}+11\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}-\frac{\int \frac{11\left(\left(119-65\sqrt{2}\right)x-54\sqrt{2}+11\right)}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{22\sqrt{2}}\right)+\frac{3603-658x}{253\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{506}\left(\frac{23}{22}\left(\frac{\int \frac{\left(119+65\sqrt{2}\right)x+54\sqrt{2}+11}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(119-65\sqrt{2}\right)x-54\sqrt{2}+11}{\sqrt{2x^2-x+3}(5x^2+3x+2)}dx}{2\sqrt{2}}\right)+\frac{3603-658x}{253\sqrt{2x^2-x+3}}\right)+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 1362$$

$$\frac{1}{506}\left(\frac{23}{22}\left(\frac{\left(15457+25000\sqrt{2}\right)\int \frac{1}{31\left(15457+25000\sqrt{2}\right)-\frac{11\left(\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443\right)^2}{2x^2-x+3}}d\left(-\frac{\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443}{\sqrt{2x^2-x+3}}\right)}{\sqrt{2}}-\frac{(154\ldots}{\ldots}\right.\right.$$

$$\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{506}\left(\frac{23}{22}\left(\frac{\left(15457+25000\sqrt{2}\right)\int \frac{1}{31\left(15457+25000\sqrt{2}\right)-\frac{11\left(\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443\right)^2}{2x^2-x+3}}d\left(-\frac{\left(247-345\sqrt{2}\right)x+98\sqrt{2}+443}{\sqrt{2x^2-x+3}}\right)}{\sqrt{2}}-\frac{(154\ldots}{\ldots}\right.\right.$$

$$\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

3.97. $\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)}\,dx$

$$\Big\downarrow 219$$

$$\frac{1}{506}\left(\frac{23}{22}\left(-\frac{\left(15457-25000\sqrt{2}\right)\arctan\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left(\left(247+345\sqrt{2}\right)x-98\sqrt{2}+443\right)}{\sqrt{2x^2-x+3}}\right)}{\sqrt{682\left(25000\sqrt{2}-15457\right)}}-\sqrt{\frac{1}{682}\left(15457+25000\sqrt{\right.}}\right.\right.$$

$$\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

output
```
(13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + ((3603 - 658*x)/(253*Sqrt[3 - x +
 2*x^2]) + (23*(-(((15457 - 25000*Sqrt[2])*ArcTan[(Sqrt[11/(31*(-15457 + 2
5000*Sqrt[2]))]*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/Sqrt[682*(-15457 + 25000*Sqrt[2])]) - Sqrt[(15457 + 25000*Sqrt[2])
/682]*ArcTanh[(Sqrt[11/(31*(15457 + 25000*Sqrt[2]))]*(443 + 98*Sqrt[2] + (
247 - 345*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]))/22)/506
```

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.97. $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.97.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.64 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.37

method	result
trager	$-\frac{3948x^3-23592x^2+19767x-39005}{384054(2x^2-x+3)^{\frac{3}{2}}}+\frac{\operatorname{RootOf}\left(\_Z^2+1356301584\operatorname{RootOf}\left(494371927368\_Z^4-3842440173\_Z^2+39062500\right)^2 \cdots\right.}{\cdots}$
default	$\frac{\sqrt{\frac{8\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+\frac{3\sqrt{2}\left(\sqrt{2}-1+x\right)^2}{\left(\sqrt{2}+1-x\right)^2}+8-3\sqrt{2}}\,\sqrt{2}\left(10111\sqrt{-775687+549362\sqrt{2}}\,\sqrt{2}\,\sqrt{-8866+6820\sqrt{2}}\arctan\left(\frac{\sqrt{-775687+549362\sqrt{2}}\cdots}{\sqrt{\cdots}}\right.\right.}{\cdots}$

input
```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/384054*(3948*x^3-23592*x^2+19767*x-39005)/(2*x^2-x+3)^(3/2)+1/330088*Ro
otOf(_Z^2+1356301584*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^2-
10541674)*ln(-(58416228676213*x*RootOf(494371927368*_Z^4-3842440173*_Z^2+
39062500)^4*RootOf(_Z^2+1356301584*RootOf(494371927368*_Z^4-3842440173*_Z^
2+39062500)^2-10541674)-13400087377743*RootOf(494371927368*_Z^4-3842440173
*_Z^2+39062500)^2*RootOf(_Z^2+1356301584*RootOf(494371927368*_Z^4-38424401
73*_Z^2+39062500)^2-10541674)*x-34735243833848250*RootOf(494371927368*_Z^4
-3842440173*_Z^2+39062500)^2*(2*x^2-x+3)^(1/2)+14587333029225*RootOf(49437
1927368*_Z^4-3842440173*_Z^2+39062500)^2*RootOf(_Z^2+1356301584*RootOf(494
371927368*_Z^4-3842440173*_Z^2+39062500)^2-10541674)+74861069500*RootOf(_Z
^2+1356301584*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^2-1054167
4)*x+120911390237000*(2*x^2-x+3)^(1/2)-140419212500*RootOf(_Z^2+1356301584
*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^2-10541674))/(994356*x
*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^2+419*x+5711))+27/242*
RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)*ln(-(100943233115249606
4*x*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^5+7463993424020496*
RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^3*x-25206911474500800*R
ootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)^3+1629806162292000*RootO
f(494371927368*_Z^4-3842440173*_Z^2+39062500)^2*(2*x^2-x+3)^(1/2)+10367533
373175*RootOf(494371927368*_Z^4-3842440173*_Z^2+39062500)*x-46727026216...
```

3.97.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.00

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx =$$

$$-\frac{1587\sqrt{341}(4x^4-4x^3+13x^2-6x+9)\sqrt{5711i\sqrt{31}+15457}\log\left(\frac{\sqrt{341}\sqrt{2x^2-x+3}\sqrt{5711i\sqrt{31}+15457}\left(443i\sqrt{31}-\right.}{x}\right.}{}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
-1/1047699312*(1587*sqrt(341)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(5711
*I*sqrt(31) + 15457)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(5711*I*sqrt(3
1) + 15457)*(443*I*sqrt(31) - 341) - 387500*sqrt(31)*(I*x - 6*I) + 7362500
*x - 8525000)/x) - 1587*sqrt(341)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(
5711*I*sqrt(31) + 15457)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(5711*I*sq
rt(31) + 15457)*(-443*I*sqrt(31) + 341) - 387500*sqrt(31)*(I*x - 6*I) + 73
62500*x - 8525000)/x) - 1587*sqrt(341)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*
sqrt(-5711*I*sqrt(31) + 15457)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(443*I*s
qrt(31) + 341)*sqrt(-5711*I*sqrt(31) + 15457) - 387500*sqrt(31)*(-I*x + 6*
I) + 7362500*x - 8525000)/x) + 1587*sqrt(341)*(4*x^4 - 4*x^3 + 13*x^2 - 6*
x + 9)*sqrt(-5711*I*sqrt(31) + 15457)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(
-443*I*sqrt(31) - 341)*sqrt(-5711*I*sqrt(31) + 15457) - 387500*sqrt(31)*(-
I*x + 6*I) + 7362500*x - 8525000)/x) + 2728*(3948*x^3 - 23592*x^2 + 19767*
x - 39005)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

3.97.6 Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{5}{2}}\cdot(5x^2+3x+2)}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)), x)
```

3.97.7 Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)(2\,x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(5/2)), x)
```

3.97.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{5/2}\,(2+3x+5x^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{5/2}\,(2+3x+5x^2)}\,dx = \int \frac{1}{(2\,x^2-x+3)^{5/2}\,(5\,x^2+3\,x+2)}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)),x)
```

output
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)), x)
```

3.98 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx$

3.98.1 Optimal result

Integrand size = 27, antiderivative size = 234

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = -\frac{15101-8654x}{1035276(3-x+2x^2)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+65x}{682(3-x+2x^2)^{3/2}(2+3x+5x^2)} + \frac{625\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(30463+23600\sqrt{2}\right)}}\left(203+242\sqrt{2}+\left(687+445\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{660176} - \frac{625\sqrt{\frac{1}{682}\left(-30463+23600\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-30463+23600\sqrt{2}\right)}}\left(203-242\sqrt{2}+\left(687-445\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)}{660176}$$

output
```
1/1035276*(-15101+8654*x)/(2*x^2-x+3)^(3/2)+1/682*(4+65*x)/(2*x^2-x+3)^(3/
2)/(5*x^2+3*x+2)+1/523849656*(-3133427-1352542*x)/(2*x^2-x+3)^(1/2)-625/45
0240032*arctanh(1/31*(203+x*(687-445*2^(1/2))-242*2^(1/2))*341^(1/2)/(-304
63+23600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(-20775766+16095200*2^(1/2))^(1
/2)+625/450240032*arctan(1/31*(203+242*2^(1/2)+x*(687+445*2^(1/2)))*341^(1
/2)/(30463+23600*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(20775766+16095200*2^(1
/2))^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.07 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.78

$$\int \frac{1}{(3-x+2x^2)^{5/2}\,(2+3x+5x^2)^2}\,dx = \frac{-31010342+5712309x-84671384x^2-2879479x^3-32686812x}{523849656\,(3-x+2x^2)^{3/2}\,(2+3x+5x^2)}$$
$$+\frac{\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{-1376\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+106\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-}\right.}{-13\sqrt{2}+17\#1+9\sqrt{}}\right.}{5324}$$
$$+\frac{\text{RootSum}\left[-56-26\sqrt{2}\#1+17\#1^2+6\sqrt{2}\#1^3-5\#1^4\&,\frac{126249\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{3-x+2x^2}-\#1\right)+58712\log\left(-\sqrt{2}x+\sqrt{}\right.}{-13\sqrt{2}+17\#1}\right.}{660176\sqrt{2}}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

output
```
(-31010342 + 5712309*x - 84671384*x^2 - 2879479*x^3 - 32686812*x^4 - 13525
420*x^5)/(523849656*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + RootSum[-56
 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-1376*Log[-(Sqrt
[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 106*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3
 - x + 2*x^2] - #1]*#1 + 95*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#
1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/5324 + RootSum[-
56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (126249*Sqrt[2]
*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 58712*Log[-(Sqrt[2]*x) + S
qrt[3 - x + 2*x^2] - #1]*#1 + 10095*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x
+ 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ]/
(660176*Sqrt[2])
```

3.98.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {1305, 27, 2135, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{5/2}\,(5x^2+3x+2)^2}\,dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}-\frac{\int -\frac{11\left(1040x^2-401x+316\right)}{2(2x^2-x+3)^{5/2}(5x^2+3x+2)}dx}{7502}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1040x^2-401x+316}{(2x^2-x+3)^{5/2}(5x^2+3x+2)}dx}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\int \frac{11\left(173080x^2-284277x+155482\right)}{2\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{8349}-\frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{173080x^2-284277x+155482}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{1518}-\frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\frac{\int \frac{10910625(34-35x)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2783}-\frac{1352542x+3133427}{253\sqrt{2x^2-x+3}}}{1518}-\frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{43125}{22}\int \frac{34-35x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx-\frac{1352542x+3133427}{253\sqrt{2x^2-x+3}}}{1518}-\frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{1368}$$

$$\frac{\frac{\frac{43125}{22}\left(\frac{\int -\frac{11\left(\left(1+35\sqrt{2}\right)x-34\sqrt{2}+69\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(1-35\sqrt{2}\right)x+34\sqrt{2}+69\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)-\frac{1352542x+3133427}{253\sqrt{2x^2-x+3}}}{1518}-\frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$$\Big\downarrow \text{27}$$

3.98. $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx$

$$\frac{\frac{\frac{43125}{22}\left(\frac{\int \frac{\left(1-35\sqrt{2}\right)x+34\sqrt{2}+69}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(1+35\sqrt{2}\right)x-34\sqrt{2}+69}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)-\frac{1352542x+3133427}{253\sqrt{2x^2-x+3}}}{1518}-\frac{15101-8654x}{759(2x^2-x+3)^{3/2}}}{1364}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$\downarrow$ 1362

$$\frac{\frac{\frac{43125}{22}\left(\frac{\left(30463-23600\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203\right)^2}{2x^2-x+3}-31\left(30463-23600\sqrt{2}\right)}d\frac{\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203}{\sqrt{2x^2-x+3}}}{\sqrt{2}}-\frac{\left(30463+23600\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(687+445\sqrt{\cdots}\right.\right.}{2x\cdots}}\cdots}{\cdots}\right.}{1518}\cdots}{1364}\cdots+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$\downarrow$ 217

$$\frac{\frac{\frac{43125}{22}\left(\frac{\left(30463-23600\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203\right)^2}{2x^2-x+3}-31\left(30463-23600\sqrt{2}\right)}d\frac{\left(687-445\sqrt{2}\right)x-242\sqrt{2}+203}{\sqrt{2x^2-x+3}}}{\sqrt{2}}+\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\arctan\left(\cdots\right.\right.}{1518}\cdots}{1364}\cdots+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

$\downarrow$ 219

$$\frac{\frac{\frac{43125}{22}\left(\sqrt{\frac{1}{682}\left(30463+23600\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(30463+23600\sqrt{2}\right)}}\left(\left(687+445\sqrt{2}\right)x+242\sqrt{2}+203\right)}{\sqrt{2x^2-x+3}}\right)+\frac{\left(30463-23600\sqrt{2}\right)\text{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(23600\sqrt{2}-30463\right)\cdots}}\cdots}{\cdots}\right.}{\sqrt{682\left(23600\sqrt{2}-30463\cdots\right.}}\right.}{1518}\cdots}{1364}\cdots+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}$$

input
```
Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

3.98. $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx$

output
```
(4 + 65*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (-1/759*(15101
- 8654*x)/(3 - x + 2*x^2)^(3/2) + (-1/253*(3133427 + 1352542*x)/Sqrt[3 - x
 + 2*x^2] + (43125*(Sqrt[(30463 + 23600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*
(30463 + 23600*Sqrt[2]))]*(203 + 242*Sqrt[2] + (687 + 445*Sqrt[2])*x))/Sqr
t[3 - x + 2*x^2]] + ((30463 - 23600*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-30463
+ 23600*Sqrt[2]))]*(203 - 242*Sqrt[2] + (687 - 445*Sqrt[2])*x))/Sqrt[3 - x
 + 2*x^2]])/Sqrt[682*(-30463 + 23600*Sqrt[2])]))/22)/1518)/1364
```

3.98.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.98. $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx$

3.98.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.96 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.11

method	result
trager	$-\frac{13525420x^5+32686812x^4+2879479x^3+84671384x^2-5712309x+31010342}{523849656(2x^2-x+3)^{\frac{3}{2}}(5x^2+3x+2)} - \frac{16875\,\mathrm{RootOf}\left(1977487709472\_Z^4+7572766707\right.}{}$
risch	$-\frac{13525420x^5+32686812x^4+2879479x^3+84671384x^2-5712309x+31010342}{523849656(2x^2-x+3)^{\frac{3}{2}}(5x^2+3x+2)} + \frac{625\sqrt{\frac{8(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+\frac{3\sqrt{2}\,(\sqrt{2}-1+x)^2}{(\sqrt{2}+1-x)^2}+8-3\sqrt{2}\,\sqrt{2}}}{}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/523849656*(13525420*x^5+32686812*x^4+2879479*x^3+84671384*x^2-5712309*x
+31010342)/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)-16875/165044*RootOf(19774877094
72*_Z^4+7572766707*_Z^2+8702500)*ln((-522645255642946 8672*x*RootOf(1977487
709472*_Z^4+7572766707*_Z^2+8702500)^5-30671894532413376*RootOf(1977487709
472*_Z^4+7572766707*_Z^2+8702500)^3*x+1319515725838464*(2*x^2-x+3)^(1/2)*R
ootOf(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2-1399155782048 6400*Root
Of(1977487709472*_Z^4+7572766707*_Z^2+8702500)^3-23459476566825*RootOf(197
7487709472*_Z^4+7572766707*_Z^2+8702500)*x+2583860307500*(2*x^2-x+3)^(1/2)
-69460188136200*RootOf(1977487709472*_Z^4+7572766707*_Z^2+8702500))/(79548
48*x*RootOf(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2+18905*x+4898))+6
25/450240032*RootOf(_Z^2+5425206336*RootOf(1977487709472*_Z^4+7572766707*_
Z^2+8702500)^2+20775766)*ln(-(-75614186290 9356*RootOf(_Z^2+5425206336*Root
Of(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2+20775766)*RootOf(19774877
09472*_Z^4+7572766707*_Z^2+8702500)^4*x-1353788597799*RootOf(1977487709472
*_Z^4+7572766707*_Z^2+8702500)^2*RootOf(_Z^2+5425206336*RootOf(19774877094
72*_Z^4+7572766707*_Z^2+8702500)^2+20775766)*x+14061089453466132*(2*x^2-x+
3)^(1/2)*RootOf(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2+202424158282
5*RootOf(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2*RootOf(_Z^2+5425206
336*RootOf(1977487709472*_Z^4+7572766707*_Z^2+8702500)^2+20775766)+2510468
172*RootOf(_Z^2+5425206336*RootOf(1977487709472*_Z^4+7572766707*_Z^2+87...
```

3.98.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.91

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx = \frac{1587\,\sqrt{341}\left(20\,x^6-8\,x^5+61\,x^4+x^3+53\,x^2+15\,x+18\right)\sqrt{95664}}{}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/1429061861568*(1587*sqrt(341)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 +
15*x + 18)*sqrt(956640625*I*sqrt(31) - 11899609375)*log((sqrt(341)*sqrt(2*
x^2 - x + 3)*sqrt(956640625*I*sqrt(31) - 11899609375)*(203*I*sqrt(31) - 21
39) - 228625000*sqrt(31)*(-I*x + 6*I) + 4343875000*x - 5029750000)/x) - 15
87*sqrt(341)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(956
640625*I*sqrt(31) - 11899609375)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(9
56640625*I*sqrt(31) - 11899609375)*(-203*I*sqrt(31) + 2139) - 228625000*sq
rt(31)*(-I*x + 6*I) + 4343875000*x - 5029750000)/x) - 1587*sqrt(341)*(20*x
^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(-956640625*I*sqrt(31)
 - 11899609375)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*(203*I*sqrt(31) + 2139)
*sqrt(-956640625*I*sqrt(31) - 11899609375) - 228625000*sqrt(31)*(I*x - 6*I
) + 4343875000*x - 5029750000)/x) + 1587*sqrt(341)*(20*x^6 - 8*x^5 + 61*x^
4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(-956640625*I*sqrt(31) - 11899609375)*lo
g((sqrt(341)*sqrt(2*x^2 - x + 3)*(-203*I*sqrt(31) - 2139)*sqrt(-956640625*
I*sqrt(31) - 11899609375) - 228625000*sqrt(31)*(I*x - 6*I) + 4343875000*x
- 5029750000)/x) - 2728*(13525420*x^5 + 32686812*x^4 + 2879479*x^3 + 84671
384*x^2 - 5712309*x + 31010342)*sqrt(2*x^2 - x + 3))/(20*x^6 - 8*x^5 + 61*
x^4 + x^3 + 53*x^2 + 15*x + 18)
```

3.98.6 Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{5}{2}}(5x^2+3x+2)^2}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)
```

3.98.7 Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^2(2\,x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)
```

3.98.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^2}\,dx = \int \frac{1}{(2\,x^2-x+3)^{5/2}\,(5\,x^2+3\,x+2)^2}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2),x)
```

output
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^2), x)
```

3.99 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx$

3.99.1 Optimal result

Integrand size = 27, antiderivative size = 269

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx =$$

$$-\frac{12280939-19536786x}{2824232928(3-x+2x^2)^{3/2}}-\frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}}$$

$$+\frac{4+65x}{1364(3-x+2x^2)^{3/2}(2+3x+5x^2)^2}+\frac{46386+86885x}{1860496(3-x+2x^2)^{3/2}(2+3x+5x^2)}$$

$$+\frac{35\sqrt{\frac{1}{682}\left(2243059557247+2011748500000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(2243059557247+2011748500000\sqrt{2}\right)}}\left(1432939+2428746\sqrt{2}+\left(629\right.\right.}{\sqrt{3-x+2x^2}}\right.}{1800960128}$$

$$-\frac{35\sqrt{\frac{1}{682}\left(-2243059557247+2011748500000\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\sqrt{\frac{11}{31\left(-2243059557247+2011748500000\sqrt{2}\right)}}\left(1432939-2428746\sqrt{2}-\right.}{\sqrt{3-x+2x^2}}\right.}{1800960128}$$

output
```
1/2824232928*(-12280939+19536786*x)/(2*x^2-x+3)^(3/2)+1/1364*(4+65*x)/(2*x
^2-x+3)^(3/2)/(5*x^2+3*x+2)^2+1/1860496*(46386+86885*x)/(2*x^2-x+3)^(3/2)/
(5*x^2+3*x+2)+1/476353953856*(-1134826571+1504660754*x)/(2*x^2-x+3)^(1/2)-
35/1228254807296*arctanh(1/31*(1432939+x*(6290431-3861685*2^(1/2))-2428746
*2^(1/2))*341^(1/2)/(-2243059557247+2011748500000*2^(1/2))^(1/2)/(2*x^2-x+
3)^(1/2))*(-1529766618042454+1372012477000000*2^(1/2))^(1/2)+35/1228254807
296*arctan(1/31*(1432939+2428746*2^(1/2)+x*(6290431+3861685*2^(1/2)))*341^
(1/2)/(2243059557247+2011748500000*2^(1/2))^(1/2)/(2*x^2-x+3)^(1/2))*(1529
766618042454+1372012477000000*2^(1/2))^(1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.17 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.25

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \frac{\frac{4\sqrt{3-x+2x^2}(9739335532+218659985088x+178650961091x^2+519223213785x^3+1742416}{(6+7x+16x^2+x^3+10x^4)^2}}{}$$

input
```
Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

output
```
((4*Sqrt[3 - x + 2*x^2]*(9739335532 + 218659985088*x + 178650961091*x^2 +
519223213785*x^3 + 174241614961*x^4 + 592923725931*x^5 - 12234606480*x^6 +
 225699113100*x^7))/(6 + 7*x + 16*x^2 + x^3 + 10*x^4)^2 - 2976*RootSum[-56
 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 & , (-26154346*Log[-(
Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 37230166*Sqrt[2]*Log[-(Sqrt[2]*x)
 + Sqrt[3 - x + 2*x^2] - #1]*#1 - 1193705*Log[-(Sqrt[2]*x) + Sqrt[3 - x +
2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ] -
24401712*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3 - 5*#1^4 &
 , (-3647*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1] + 3172*Sqrt[2]*Log[
-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 - 485*Log[-(Sqrt[2]*x) + Sqrt[
3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3
) & ] + 15*Sqrt[2]*RootSum[-56 - 26*Sqrt[2]*#1 + 17*#1^2 + 6*Sqrt[2]*#1^3
- 5*#1^4 & , (-9138129081*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] -
 #1] + 16445754136*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1 + 10044
12885*Sqrt[2]*Log[-(Sqrt[2]*x) + Sqrt[3 - x + 2*x^2] - #1]*#1^2)/(-13*Sqrt
[2] + 17*#1 + 9*Sqrt[2]*#1^2 - 10*#1^3) & ])/5716247446272
```

3.99.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {1305, 27, 2135, 27, 2135, 27, 2135, 27, 1368, 27, 1362, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x^2-x+3)^{5/2}(5x^2+3x+2)^3}dx$$

$$\Big\downarrow \ 1305$$

$$\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}-\frac{\int -\frac{11(1560x^2-785x+1034)}{2(2x^2-x+3)^{5/2}(5x^2+3x+2)^2}dx}{15004}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1560x^2-785x+1034}{(2x^2-x+3)^{5/2}(5x^2+3x+2)^2}dx}{2728}+\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}$$

$$\Big\downarrow \ 2135$$

$$\frac{\frac{\int \frac{11\left(1390160x^2+284771x+462194\right)}{2\left(2x^2-x+3\right)^{5/2}\left(5x^2+3x+2\right)}dx}{7502}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{1390160x^2+284771x+462194}{\left(2x^2-x+3\right)^{5/2}\left(5x^2+3x+2\right)}dx}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}$$

$$\Big\downarrow \ 2135$$

$$\frac{\frac{\frac{\int \frac{33\left(130245240x^2+4179719x+60094966\right)}{2\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx}{8349}-\frac{12280939-19536786x}{759\left(2x^2-x+3\right)^{3/2}}}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\,(2x^2-x+3)^{3/2}(5x^2+3x+2)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{1}{506}\int \frac{130245240x^2+4179719x+60094966}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}dx-\frac{12280939-19536786x}{759\left(2x^2-x+3\right)^{3/2}}}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 2135$$

$$\frac{\frac{\frac{1}{506}\left(\frac{\int \frac{203665(263242-409755x)}{2\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2783}-\frac{1134826571-1504660754x}{253\sqrt{2x^2-x+3}}\right)-\frac{12280939-19536786x}{759\left(2x^2-x+3\right)^{3/2}}}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{506}\left(\frac{805}{22}\int \frac{263242-409755x}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx-\frac{1134826571-1504660754x}{253\sqrt{2x^2-x+3}}\right)-\frac{12280939-19536786x}{759\left(2x^2-x+3\right)^{3/2}}}{1364}+\frac{86885x+46386}{682(2x^2-x+3)^{3/2}(5x^2+3x+2)}}{2728}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 1368$$

$$\frac{\frac{\frac{1}{506}\left(\frac{805}{22}\left(\frac{\int -\frac{11\left(\left(146513+409755\sqrt{2}\right)x-263242\sqrt{2}+672997\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}-\frac{\int -\frac{11\left(\left(146513-409755\sqrt{2}\right)x+263242\sqrt{2}+672997\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{22\sqrt{2}}\right)-\frac{1134826571-1504660754x}{253\sqrt{2x^2-x+3}}\right)-\frac{122\cdots}{75\cdots}}{1364}\cdots}{2728}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{506}\left(\frac{805}{22}\left(\frac{\int \frac{\left(146513-409755\sqrt{2}\right)x+263242\sqrt{2}+672997}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}-\frac{\int \frac{\left(146513+409755\sqrt{2}\right)x-263242\sqrt{2}+672997}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}dx}{2\sqrt{2}}\right)-\frac{1134826571-1504660754x}{253\sqrt{2x^2-x+3}}\right)-\frac{12280939-19536\cdots}{759\left(2x^2-x+3\right)\cdots}}{1364}\cdots}{2728}+\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$$\Big\downarrow \; 1362$$

3.99. $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx$

$$\frac{1}{506}\left(\frac{805}{22}\left(\frac{\left(2243059557247-2011748500000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(6290431-3861685\sqrt{2}\right)x-2428746\sqrt{2}+1432939\right)^2}{2x^2-x+3}-31\left(2243059557247-2011748500000\sqrt{2}\right)}d\frac{\left(6290431-3\right.}{}}{\sqrt{2}}\right.\right.$$

$$\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 217

$$\frac{1}{506}\left(\frac{805}{22}\left(\frac{\left(2243059557247-2011748500000\sqrt{2}\right)\int \frac{1}{-\frac{11\left(\left(6290431-3861685\sqrt{2}\right)x-2428746\sqrt{2}+1432939\right)^2}{2x^2-x+3}-31\left(2243059557247-2011748500000\sqrt{2}\right)}d\frac{\left(6290431-3\right.}{}}{\sqrt{2}}\right.\right.$$

$$\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

$\downarrow$ 219

$$\frac{1}{506}\left(\frac{805}{22}\left(\sqrt{\frac{1}{682}\left(2243059557247+2011748500000\sqrt{2}\right)}\arctan\left(\frac{\sqrt{\frac{11}{31\left(2243059557247+2011748500000\sqrt{2}\right)}}\left(\left(6290431+3861685\sqrt{2}\right)x+2428746\sqrt{2}+1432939\right.}{\sqrt{2x^2-x+3}}\right.\right.\right.$$

$$\frac{65x+4}{1364\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}$$

input

```
Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

3.99. $\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx$

output
```
(4 + 65*x)/(1364*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + ((46386 + 86
885*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (-1/759*(12280939 -
 19536786*x)/(3 - x + 2*x^2)^(3/2) + (-1/253*(1134826571 - 1504660754*x)/S
qrt[3 - x + 2*x^2] + (805*(Sqrt[(2243059557247 + 2011748500000*Sqrt[2])/68
2]*ArcTan[(Sqrt[11/(31*(2243059557247 + 2011748500000*Sqrt[2]))]*(1432939
+ 2428746*Sqrt[2] + (6290431 + 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] +
 ((2243059557247 - 2011748500000*Sqrt[2])*ArcTanh[(Sqrt[11/(31*(-224305955
7247 + 2011748500000*Sqrt[2]))]*(1432939 - 2428746*Sqrt[2] + (6290431 - 38
61685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/Sqrt[682*(-2243059557247 + 201174
8500000*Sqrt[2])]))/22)/506)/1364)/2728
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

rule 1368
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*d -
a*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqr
t[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[Simp[h*(b*d - a*e) - g*(c*
d - a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2
- 4*a*c]
```

3.99. $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.99.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.63 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.90

method	result
trager	Expression too large to display
risch	$\frac{225699113100x^7-12234606480x^6+592923725931x^5+174241614961x^4+519223213785x^3+178650961091x^2+218659985088x+97393}{1429061861568(5x^2+3x+2)^2(2x^2-x+3)^{\frac{3}{2}}}$
default	Expression too large to display

input
```
int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1429061861568*(225699113100*x^7-12234606480*x^6+592923725931*x^5+1742416
14961*x^4+519223213785*x^3+178650961091*x^2+218659985088*x+9739335532)/(10
*x^4+x^3+16*x^2+7*x+6)^2*(2*x^2-x+3)^(1/2)+35/1228254807296*RootOf(_Z^2+21
700825344*RootOf(7909950837888*_Z^4+5575999322764744483*_Z^2+15809109481454
101562500)^2+1529766618042454)*ln((3609214479402775056*RootOf(_Z^2+2170082
5344*RootOf(7909950837888*_Z^4+5575999322764744483*_Z^2+1580910948145410156
2500)^2+1529766618042454)*RootOf(7909950837888*_Z^4+5575999322764744483*_Z^
2+15809109481454101562500)^4*x+13326758501298020522 1621*RootOf(79099508378
88*_Z^4+5575999322764744483*_Z^2+15809109481454101562500)^2*RootOf(_Z^2+217
00825344*RootOf(7909950837888*_Z^4+5575999322764744483*_Z^2+158091094814541
01562500)^2+1529766618042454)*x-3100358606898847120 26075*RootOf(7909950837
888*_Z^4+5575999322764744483*_Z^2+15809109481454101562500)^2*RootOf(_Z^2+21
700825344*RootOf(7909950837888*_Z^4+5575999322764744483*_Z^2+15809109481454
101562500)^2+1529766618042454)-139858266288457675541555869050 00*RootOf(790
9950837888*_Z^4+5575999322764744483*_Z^2+15809109481454101562500)^2*(2*x^2-
x+3)^(1/2)-751077958109653657562967250 0*RootOf(_Z^2+21700825344*RootOf(790
9950837888*_Z^4+5575999322764744483*_Z^2+15809109481454101562500)^2+1529766
618042454)*x+953365811550878078022668750 0*RootOf(_Z^2+21700825344*RootOf(7
909950837888*_Z^4+5575999322764744483*_Z^2+15809109481454101562500)^2+15297
66618042454)-4734581933484903840340584668369 80000*(2*x^2-x+3)^(1/2))/(1...
```

3.99.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.93

$$\int \frac{1}{(3-x+2x^2)^{5/2}\,(2+3x+5x^2)^3}\,dx = \frac{1587\sqrt{341}(100\,x^8+20\,x^7+321\,x^6+172\,x^5+390\,x^4+236\,x^3+ \cdots}{\cdots}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/3898480758357504*(1587*sqrt(341)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 +
 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(385057019579225*I*sqrt(31)
- 2747747957627575)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(38505701957922
5*I*sqrt(31) - 2747747957627575)*(1432939*I*sqrt(31) - 20862907) - 1091373
561250000*sqrt(31)*(-I*x + 6*I) + 20736097663750000*x - 24010218347500000)
/x) - 1587*sqrt(341)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236
*x^3 + 241*x^2 + 84*x + 36)*sqrt(385057019579225*I*sqrt(31) - 274774795762
7575)*log((sqrt(341)*sqrt(2*x^2 - x + 3)*sqrt(385057019579225*I*sqrt(31) -
 2747747957627575)*(-1432939*I*sqrt(31) + 20862907) - 1091373561250000*sqr
t(31)*(-I*x + 6*I) + 20736097663750000*x - 24010218347500000)/x) - 1587*sq
rt(341)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^
2 + 84*x + 36)*sqrt(-385057019579225*I*sqrt(31) - 2747747957627575)*log((s
qrt(341)*sqrt(2*x^2 - x + 3)*(1432939*I*sqrt(31) + 20862907)*sqrt(-3850570
19579225*I*sqrt(31) - 2747747957627575) - 1091373561250000*sqrt(31)*(I*x -
 6*I) + 20736097663750000*x - 24010218347500000)/x) + 1587*sqrt(341)*(100*
x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)*sqrt(-385057019579225*I*sqrt(31) - 2747747957627575)*log((sqrt(341)*sqrt
(2*x^2 - x + 3)*(-1432939*I*sqrt(31) - 20862907)*sqrt(-385057019579225*I*s
qrt(31) - 2747747957627575) - 1091373561250000*sqrt(31)*(I*x - 6*I) + 2073
6097663750000*x - 24010218347500000)/x) + 2728*(225699113100*x^7 - 1223...
```

3.99.6 Sympy [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(2x^2-x+3)^{\frac{5}{2}}(5x^2+3x+2)^3}\,dx$$

input
```
integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

output
```
Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)
```

3.99.7 Maxima [F]

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(5\,x^2+3\,x+2)^3(2\,x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(5/2)), x)
```

3.99.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Francis algorithm failure for[-1.0,
infinity,infinity,infinity,infinity]proot error [1.0,infinity,infinity,inf
inity,inf
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)^3}\,dx = \int \frac{1}{(2\,x^2-x+3)^{5/2}\,(5\,x^2+3\,x+2)^3}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3),x)
```

output
```
int(1/((2*x^2 - x + 3)^(5/2)*(3*x + 5*x^2 + 2)^3), x)
```

3.100 $\int \sqrt{a+bx+cx^2}(d+ex+fx^2)^2\,dx$

3.100.1 Optimal result

Integrand size = 27, antiderivative size = 436

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$$
$$= \frac{(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+8c^2(12abef+2a^2f^2+5b^2(e^2+2df))}{512c^5}$$
$$+\frac{(640c^3de-105b^3f^2+28bcf(10be+7af)-8c^2(32aef+25b(e^2+2df)))\left(a+bx+cx^2\right)^{3/2}}{960c^4}$$
$$+\frac{(21b^2f^2-4cf(14be+5af)+40c^2(e^2+2df))\,x\left(a+bx+cx^2\right)^{3/2}}{160c^3}$$
$$+\frac{f(8ce-3bf)x^2\left(a+bx+cx^2\right)^{3/2}}{20c^2}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$
$$-\frac{(b^2-4ac)\,(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3(4bde+a(e^2+2df))+8c^2(12abef+2a^2f^2+5}{1024c^{11/2}}$$

output
```
1/960*(640*c^3*d*e-105*b^3*f^2+28*b*c*f*(7*a*f+10*b*e)-8*c^2*(32*a*e*f+25*
b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(3/2)/c^4+1/160*(21*b^2*f^2-4*c*f*(5*a*f+14*
b*e)+40*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(3/2)/c^3+1/20*f*(-3*b*f+8*c*e)*x
^2*(c*x^2+b*x+a)^(3/2)/c^2+1/6*f^2*x^3*(c*x^2+b*x+a)^(3/2)/c-1/1024*(-4*a*
c+b^2)*(128*c^4*d^2+21*b^4*f^2-56*b^2*c*f*(a*f+b*e)-32*c^3*(4*b*d*e+a*(2*d
*f+e^2))+8*c^2*(12*a*b*e*f+2*a^2*f^2+5*b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/512*(128*c^4*d^2+21*b^4*f^2-5
6*b^2*c*f*(a*f+b*e)-32*c^3*(4*b*d*e+a*(2*d*f+e^2))+8*c^2*(12*a*b*e*f+2*a^2
*f^2+5*b^2*(2*d*f+e^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

3.100.2 Mathematica [A] (verified)

Time = 5.40 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.05

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$$
$$=\frac{\sqrt{c}\sqrt{a+x(b+cx)}(315b^5f^2-210b^4cf(4e+fx)-16b^2c^2(-2af(115e+28fx)+c(120de+25e^2x+50dj}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^5*f^2 - 210*b^4*c*f*(4*e + f*x) - 16
*b^2*c^2*(-2*a*f*(115*e + 28*f*x) + c*(120*d*e + 25*e^2*x + 50*d*f*x + 28*
e*f*x^2 + 9*f^2*x^3)) + 8*b^3*c*(-210*a*f^2 + c*(75*e^2 + 70*e*f*x + 3*f*(
50*d + 7*f*x^2))) + 16*b*c^2*(113*a^2*f^2 - 2*a*c*(65*e^2 + 58*e*f*x + f*(
130*d + 17*f*x^2)) + 4*c^2*(30*d^2 + 10*d*x*(2*e + f*x) + x^2*(5*e^2 + 6*e
*f*x + 2*f^2*x^2))) - 32*c^3*(a^2*f*(64*e + 15*f*x) - 2*a*c*(80*d*e + 15*e
^2*x + 30*d*f*x + 16*e*f*x^2 + 5*f^2*x^3) - 4*c^2*x*(30*d^2 + 10*d*x*(4*e
+ 3*f*x) + x^2*(15*e^2 + 24*e*f*x + 10*f^2*x^2)))) - 15*(b^2 - 4*a*c)*(128
*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2
+ 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*ArcTanh[
(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(7680*c^(11/2))
```

3.100.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int \frac{3}{2}\sqrt{cx^2+bx+a}\left(f(8ce-3bf)x^3-2\left(af^2-2c\left(e^2+2df\right)\right)x^2+8cdex+4cd^2\right)dx}{6c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\int \sqrt{cx^2+bx+a}\left(f(8ce-3bf)x^3-2\left(af^2-2c\left(e^2+2df\right)\right)x^2+8cdex+4cd^2\right)dx}{4c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \textcolor{blue}{2192}$$

$$\frac{\frac{\int \frac{1}{2}\sqrt{cx^2+bx+a}\left(40c^2d^2+\left(40\left(e^2+2df\right)c^2-4f(14be+5af)c+21b^2f^2\right)x^2+4\left(20dec^2-8aefc+3abf^2\right)x\right)dx}{5c}+\frac{fx^2\left(a+bx+cx^2\right)^{3/2}(8ce-3bf)}{5c}}{4c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{\int \sqrt{cx^2+bx+a}\left(40c^2d^2+\left(40\left(e^2+2df\right)c^2-4f(14be+5af)c+21b^2f^2\right)x^2+4\left(20dec^2-8aefc+3abf^2\right)x\right)dx}{10c}+\frac{fx^2\left(a+bx+cx^2\right)^{3/2}(8ce-3bf)}{5c}}{4c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \textcolor{blue}{2192}$$

$$\frac{\frac{\frac{\int \frac{1}{2}\left(320d^2c^3-80a\left(e^2+2df\right)c^2+8af(14be+5af)c-42ab^2f^2+\left(-105f^2b^3+28cf(10be+7af)b+640c^3de-8c^2\left(32aef+25b\left(e^2+2df\right)\right)\right)x\right)\sqrt{cx^2+bx+a}dx}{4c}+\frac{x\left(a+bx+c\cdots\right.}{\cdots}}{10c}}{4c}$$

$$+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{\frac{\frac{\int \left(2\left(160d^2c^3-40a\left(e^2+2df\right)c^2+4af(14be+5af)c-21ab^2f^2\right)+\left(-105f^2b^3+28cf(10be+7af)b+640c^3de-8c^2\left(32aef+25b\left(e^2+2df\right)\right)\right)x\right)\sqrt{cx^2+bx+a}dx}{8c}+\frac{x\left(a+bx+\cdots\right.}{\cdots}}{10c}}{4c}$$

$$+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$$\Big\downarrow \textcolor{blue}{1160}$$

$$\frac{\frac{\frac{\frac{5\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+128c^4d^2\right)\int \sqrt{cx^2+bx+a}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}\left(-8c^2\left(32aef+25b\left(2df\cdots\right.\right.\right.}{\cdots}}{8c}}{10c}}{4c}$$

$$+\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

3.100. $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$

$\downarrow$ 1087

$$\frac{\frac{\frac{5\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+128c^4d^2\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}+\ldots}{8c}}{10c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$\downarrow$ 1092

$$\frac{\frac{5\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+128c^4d^2\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b\ldots}{\sqrt{cx\ldots}}}{4c}\ldots\right.}{2c}}{8c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

$\downarrow$ 219

$$\frac{\frac{5\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b^4f^2+\ldots\right.}{2c}}{8c}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c}$$

input
```
Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

output
```
(f^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) + ((f*(8*c*e - 3*b*f)*x^2*(a + b*x
 + c*x^2)^(3/2))/(5*c) + (((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(
e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (((640*c^3*d*e - 105*b^3*
f^2 + 28*b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(
a + b*x + c*x^2)^(3/2))/(3*c) + (5*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*
(b*e + a*f) - 32*c^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a
^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c)
- ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(3/2))))/(2*c))/(8*c))/(10*c))/(4*c)
```

3.100. $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.100.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.48

method	result
risch	$\frac{(1280f^2c^5x^5+128b\,c^4f^2x^4+3072c^5ef\,x^4+320a\,c^4f^2x^3-144b^2c^3f^2x^3+384b\,c^4ef\,x^3+3840c^5df\,x^3+1920c^5e^2x^3-544ab\,c^3f^2x^2+102}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7680*(1280*c^5*f^2*x^5+128*b*c^4*f^2*x^4+3072*c^5*e*f*x^4+320*a*c^4*f^2*
x^3-144*b^2*c^3*f^2*x^3+384*b*c^4*e*f*x^3+3840*c^5*d*f*x^3+1920*c^5*e^2*x^
3-544*a*b*c^3*f^2*x^2+1024*a*c^4*e*f*x^2+168*b^3*c^2*f^2*x^2-448*b^2*c^3*e
*f*x^2+640*b*c^4*d*f*x^2+320*b*c^4*e^2*x^2+5120*c^5*d*e*x^2-480*a^2*c^3*f^
2*x+896*a*b^2*c^2*f^2*x-1856*a*b*c^3*e*f*x+1920*a*c^4*d*f*x+960*a*c^4*e^2*
x-210*b^4*c*f^2*x+560*b^3*c^2*e*f*x-800*b^2*c^3*d*f*x-400*b^2*c^3*e^2*x+12
80*b*c^4*d*e*x+3840*c^5*d^2*x+1808*a^2*b*c^2*f^2-2048*a^2*c^3*e*f-1680*a*b
^3*c*f^2+3680*a*b^2*c^2*e*f-4160*a*b*c^3*d*f-2080*a*b*c^3*e^2+5120*a*c^4*d
*e+315*b^5*f^2-840*b^4*c*e*f+1200*b^3*c^2*d*f+600*b^3*c^2*e^2-1920*b^2*c^3
*d*e+1920*b*c^4*d^2)*(c*x^2+b*x+a)^(1/2)/c^5+1/1024*(64*a^3*c^3*f^2-240*a^
2*b^2*c^2*f^2+384*a^2*b*c^3*e*f-256*a^2*c^4*d*f-128*a^2*c^4*e^2+140*a*b^4*
c*f^2-320*a*b^3*c^2*e*f+384*a*b^2*c^3*d*f+192*a*b^2*c^3*e^2-512*a*b*c^4*d*
e+512*a*c^5*d^2-21*b^6*f^2+56*b^5*c*e*f-80*b^4*c^2*d*f-40*b^4*c^2*e^2+128*
b^3*c^3*d*e-128*b^2*c^4*d^2)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1269, normalized size of antiderivative = 2.91

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/30720*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a*b*c^4)*d*e
 + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140*a*b^4*c +
 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a
^2*c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*sqrt(c)*log(-8*c
^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a
*c) - 4*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*
x^4 + 16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*
e)*f)*x^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e
^2 + (315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e +
40*b*c^5*e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4
- 16*a*c^5)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 -
 460*a*b^2*c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*
(5*b^2*c^4 - 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f
^2 - 8*(10*(5*b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*
sqrt(c*x^2 + b*x + a))/c^6, 1/15360*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128
*(b^3*c^3 - 4*a*b*c^4)*d*e + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2
 + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4
*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c
^3)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(2...
```

3.100.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. $2(444) = 888$.

Time = 0.74 (sec) , antiderivative size = 1544, normalized size of antiderivative = 3.54

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)**2,x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f**2*x**5/6 + x**4*(b*f**2/12 + 2*c*e*f
)/(5*c) + x**3*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*
c*d*f + c*e**2)/(4*c) + x**2*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c) +
2*b*d*f + b*e**2 - 7*b*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10
*c) + 2*c*d*f + c*e**2)/(8*c) + 2*c*d*e)/(3*c) + x*(2*a*d*f + a*e**2 - 3*a
*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f + c*e**2
)/(4*c) + 2*b*d*e - 5*b*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c) + 2*b*d
*f + b*e**2 - 7*b*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) +
 2*c*d*f + c*e**2)/(8*c) + 2*c*d*e)/(6*c) + c*d**2)/(2*c) + (2*a*d*e - 2*a
*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c) + 2*b*d*f + b*e**2 - 7*b*(a*f*
*2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f + c*e**2)/(8*c
) + 2*c*d*e)/(3*c) + b*d**2 - 3*b*(2*a*d*f + a*e**2 - 3*a*(a*f**2/6 + 2*b*
e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f + c*e**2)/(4*c) + 2*b*d*e
 - 5*b*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c) + 2*b*d*f + b*e**2 - 7*b
*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f + c*e**2
)/(8*c) + 2*c*d*e)/(6*c) + c*d**2)/(4*c))/c) + (a*d**2 - a*(2*a*d*f + a*e*
*2 - 3*a*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/(10*c) + 2*c*d*f
+ c*e**2)/(4*c) + 2*b*d*e - 5*b*(2*a*e*f - 4*a*(b*f**2/12 + 2*c*e*f)/(5*c)
 + 2*b*d*f + b*e**2 - 7*b*(a*f**2/6 + 2*b*e*f - 9*b*(b*f**2/12 + 2*c*e*f)/
(10*c) + 2*c*d*f + c*e**2)/(8*c) + 2*c*d*e)/(6*c) + c*d**2)/(2*c) - b*(...
```

3.100.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.44

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$$
$$=\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,f^2x+\frac{24\,c^5ef+bc^4f^2}{c^5}\right)x+\frac{120\,c^5e^2+240\,c^5df+24\,bc^4ef-9\,b^2c^3}{c^5}\right.\right.\right.\right.$$
$$+\frac{(128\,b^2c^4d^2-512\,ac^5d^2-128\,b^3c^3de+512\,abc^4de+40\,b^4c^2e^2-192\,ab^2c^3e^2+128\,a^2c^4e^2+80\,b^4c^2df-}{}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^2*x + (24*c^5*e*f + b*c^4*f
^2)/c^5)*x + (120*c^5*e^2 + 240*c^5*d*f + 24*b*c^4*e*f - 9*b^2*c^3*f^2 + 2
0*a*c^4*f^2)/c^5)*x + (640*c^5*d*e + 40*b*c^4*e^2 + 80*b*c^4*d*f - 56*b^2*
c^3*e*f + 128*a*c^4*e*f + 21*b^3*c^2*f^2 - 68*a*b*c^3*f^2)/c^5)*x + (1920*
c^5*d^2 + 640*b*c^4*d*e - 200*b^2*c^3*e^2 + 480*a*c^4*e^2 - 400*b^2*c^3*d*
f + 960*a*c^4*d*f + 280*b^3*c^2*e*f - 928*a*b*c^3*e*f - 105*b^4*c*f^2 + 44
8*a*b^2*c^2*f^2 - 240*a^2*c^3*f^2)/c^5)*x + (1920*b*c^4*d^2 - 1920*b^2*c^3
*d*e + 5120*a*c^4*d*e + 600*b^3*c^2*e^2 - 2080*a*b*c^3*e^2 + 1200*b^3*c^2*
d*f - 4160*a*b*c^3*d*f - 840*b^4*c*e*f + 3680*a*b^2*c^2*e*f - 2048*a^2*c^3
*e*f + 315*b^5*f^2 - 1680*a*b^3*c*f^2 + 1808*a^2*b*c^2*f^2)/c^5) + 1/1024*
(128*b^2*c^4*d^2 - 512*a*c^5*d^2 - 128*b^3*c^3*d*e + 512*a*b*c^4*d*e + 40*
b^4*c^2*e^2 - 192*a*b^2*c^3*e^2 + 128*a^2*c^4*e^2 + 80*b^4*c^2*d*f - 384*a
*b^2*c^3*d*f + 256*a^2*c^4*d*f - 56*b^5*c*e*f + 320*a*b^3*c^2*e*f - 384*a^
2*b*c^3*e*f + 21*b^6*f^2 - 140*a*b^4*c*f^2 + 240*a^2*b^2*c^2*f^2 - 64*a^3*
c^3*f^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11
/2)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 15.15 (sec) , antiderivative size = 1299, normalized size of antiderivative = 2.98

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx=\text{Too large to display}$$

input
```
int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2,x)
```

output
```
d^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e^2*x*(a + b*x + c*x^2)^(3/
2))/(4*c) + (a*f^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)
*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(
4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1
/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c)
- (3*b*f^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/
2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(
a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*
c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2
) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) -
 (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^
(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (f
^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (a*e^2*((x/2 + b/(4*c))*(a + b*x +
 c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (d^2*log((b/2 + c*x)/c^(1/2) + (a + b*x + c
*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) - (5*b*e^2*((log((b + 2*c*x)/c^(1/
2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a +
 c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - ...
```

3.100. $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$

3.101 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

3.101.1 Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx &= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} \\ &\quad+\frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c} \\ &\quad-\frac{(b^2-4ac)\left(16c^2d+5b^2f-4c(2be+af)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}\end{aligned}$$

output

```
1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-
1/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c
^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
```

3.101.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{c}\sqrt{a+x(b+cx)}(15b^3f-2b^2c(12e+5fx)+4bc(-13af+2c(6d+2ex+fx^2))+8c^2(a(8e+3fx)+2}{192c^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c*
(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e
+ a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(192*c^(7
/2))
```

3.101.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2192, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \text{ 2192}$$
$$\frac{\int \frac{1}{2}(8cd-2af+(8ce-5bf)x)\sqrt{cx^2+bx+a}dx}{4c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int (2(4cd-af)+(8ce-5bf)x)\sqrt{cx^2+bx+a}dx}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$
$$\Big\downarrow \text{ 1160}$$
$$\frac{\frac{\left(-4acf+5b^2f-8bce+16c^2d\right)\int \sqrt{cx^2+bx+a}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{\left(-4acf+5b^2f-8bce+16c^2d\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{\left(-4acf+5b^2f-8bce+16c^2d\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)\left(-4acf+5b^2f-8bce+16c^2d\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

input
```
Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (((8*c*e - 5*b*f)*(a + b*x + c*x^2)^
(3/2))/(3*c) + ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(((b + 2*c*x)*Sqr
t[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(2*c))/(8*c)
```

3.101.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.101. $\int \sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.101.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{(-48 f c^3 x^3 - 8 b c^2 f x^2 - 64 c^3 e x^2 - 24 a c^2 f x + 10 b^2 c f x - 16 b c^2 e x - 96 c^3 d x + 52 a b c f - 64 a c^2 e - 15 b^3 f + 24 b^2 c e - 48 b d c^2)\sqrt{c x^2 + b x + c}}{192 c^3}$
default	$d\left(\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c} + \frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right) + f\left(\frac{x(c x^2+bx+a)^{\frac{3}{2}}}{4c} - \frac{5b\left(\frac{(c x^2+bx+a)^{\frac{3}{2}}}{3c} - \frac{b\left(\frac{(2cx+b)}{}\right.}{}\right.}{}\right.$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*c^3*f*x^3-8*b*c^2*f*x^2-64*c^3*e*x^2-24*a*c^2*f*x+10*b^2*c*f*x
-16*b*c^2*e*x-96*c^3*d*x+52*a*b*c*f-64*a*c^2*e-15*b^3*f+24*b^2*c*e-48*b*c^
2*d)*(c*x^2+b*x+a)^(1/2)/c^3-1/128*(16*a^2*c^2*f-24*a*b^2*c*f+32*a*b*c^2*e
-64*a*c^3*d+5*b^4*f-8*b^3*c*e+16*b^2*c^2*d)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.66

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{\ldots}\right.}{}\right.$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 2
4*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^
3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c -
52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqr
t(c*x^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4
*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c
^4*f*x^3 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3
)*e + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 1
2*a*c^3)*f)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

3.101.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(168) = 336$.

Time = 0.54 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.19

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fx^3}{4} + \frac{x^2\left(\frac{bf}{8}+ce\right)}{3c} + \frac{x\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{2c} + \frac{ae-\frac{2a\left(\frac{bf}{8}+ce\right)}{3c}+bd-\frac{3b\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{4c}}{c}\right) + \Bigg(\\ \frac{2\left(\frac{f(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{(a+bx)^{\frac{5}{2}}(-2af+be)}{5b^2}+\frac{(a+bx)^{\frac{3}{2}}\left(a^2f-abe+b^2d\right)}{3b^2}\right)}{b} \\ \sqrt{a}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*x**3/4 + x**2*(b*f/8 + c*e)/(3*c) + x
*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) + (a*e - 2*a*(b*f/8 +
 c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(4*c
))/c) + (a*d - a*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) - b*(
a*e - 2*a*(b*f/8 + c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)
/(6*c) + c*d)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*
x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c)
 + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*(a + b*x)**(7/2)/
(7*b**2) + (a + b*x)**(5/2)*(-2*a*f + b*e)/(5*b**2) + (a + b*x)**(3/2)*(a*
*2*f - a*b*e + b**2*d)/(3*b**2))/b, Ne(b, 0)), (sqrt(a)*(d*x + e*x**2/2 +
f*x**3/3), True))
```

3.101.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{8\,c^3e+bc^2f}{c^3}\right)x+\frac{48\,c^3d+8\,bc^2e-5\,b^2cf+12\,ac^2f}{c^3}\right)x+\frac{48\,bc^2d-2\cdots}{\cdots}\right.$$
$$+\frac{\left(16\,b^2c^2d-64\,ac^3d-8\,b^3ce+32\,abc^2e+5\,b^4f-24\,ab^2cf+16\,a^2c^2f\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{\cdots}\right.\right.}{128\,c^{\frac{7}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (8*c^3*e + b*c^2*f)/c^3)*x + (4
8*c^3*d + 8*b*c^2*e - 5*b^2*c*f + 12*a*c^2*f)/c^3)*x + (48*b*c^2*d - 24*b^
2*c*e + 64*a*c^2*e + 15*b^3*f - 52*a*b*c*f)/c^3) + 1/128*(16*b^2*c^2*d - 6
4*a*c^3*d - 8*b^3*c*e + 32*a*b*c^2*e + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2
*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.83

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= d\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a} - \frac{a\,f\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} + \frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}} + \frac{e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}} - \frac{5\,b\,f\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c} + \frac{e\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\sqrt{c\,x^2+b\,x+a}}{24\,c^2} + \frac{f\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}$$

input
```
int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
 + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c
*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(
a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b +
2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
+ ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(8*c) + (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(
24*c^2) + (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

3.102 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

3.102.1 Optimal result

Integrand size = 27, antiderivative size = 431

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$
$$-\frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\,\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{\ldots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-\ldots}}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$
$$+\frac{\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}\,\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{\ldots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-\ldots}}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

output
```
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1
/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(
1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e
^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d
*f+e^2)^(1/2)))^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/4*(4*a*f-
b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^
2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))
^(1/2))*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/
2))))^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)
```

3.102.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)+\operatorname{RootSum}\left[c^2d-bce+b^2f+2\sqrt{a}ce\#1-4\sqrt{a}bf\#1-2cd\#1^2+be\#1^2\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

output
```
(2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])] + RootS
um[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^
2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*d*Log[x])
 + b*c*e*Log[x] - b^2*f*Log[x] + a*c*f*Log[x] + c^2*d*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1] - b*c*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*
#1] + b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - a*c*f*Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c*e*Log[x]*#1 + 2*Sqrt[a]*
b*f*Log[x]*#1 + 2*Sqrt[a]*c*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]
*#1 - 2*Sqrt[a]*b*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c*d*
Log[x]*#1^2 - a*f*Log[x]*#1^2 - c*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1]*#1^2 + a*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sq
rt[a]*c*e) + 2*Sqrt[a]*b*f + 2*c*d*#1 - b*e*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1
^2 - 2*d*#1^3) & ])/f
```

3.102.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1320, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$\Big\downarrow \; 1320$$

$$\frac{c\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \textbf{1092}$$

$$\frac{2c\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \textbf{1365}$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\frac{\left(2f(cd-af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2\cdots}}}{\sqrt{e^2-4df}}}{f}$$

$$\Big\downarrow \textbf{1154}$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\frac{2\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-\cdots}{\cdots}}{\sqrt{e^2-4df}}\cdots}{\cdots}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\frac{\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2f(cd-af)-\left(e-\sqrt{e^2-4df}\right)(ce-\cdots\right.}{\sqrt{2}\sqrt{e^2\cdots}}}{f}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

3.102. $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

output
```
(Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (-(((
2*f*(c*d - a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*
(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*
Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + ((2*f*(c*d - a*f) -
(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*
d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
 (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/f
```

3.102.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1320
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f
  Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(376) = 752$.

Time = 0.88 (sec) , antiderivative size = 1547, normalized size of antiderivative = 3.59

method	result	size
default	Expression too large to display	1547

input
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c
*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*
d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1
/2)+1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4*d*f+
e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4
/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*
f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/(-4*d*f+e^2)^(1/2)*(1/2*(4
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+e^2)^(1...
```

3.102.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.102.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.102.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.102.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

3.103 $\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2} dx$

3.103.1 Optimal result

Integrand size = 27, antiderivative size = 488

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2} dx = -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)(d+ex+fx^2)} - \frac{(f(be-4af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right))\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\,(e^2-4df)^{3/2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{(f(be-4af)-(ce-bf)\left(e+\sqrt{e^2-4df}\right))\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\,(e^2-4df)^{3/2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
-(2*f*x+e)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)-1/2*arctanh(1/4*
(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2
)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)
^(1/2))^(1/2))*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+
e^2)^(3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1
/2))^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(
-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*
f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(-4*a*f+b*e)-(-b*f+c*e)*(e+(-
4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2
+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.07 (sec) , antiderivative size = 1691, normalized size of antiderivative = 3.47

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

output
```
-1/2*(2*d^3*e*Sqrt[a + x*(b + c*x)] + 4*d^3*f*x*Sqrt[a + x*(b + c*x)] - 2*
(e^2 - 4*d*f)*(d + x*(e + f*x))*RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*
c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]
*e*#1^3 + d*#1^4 & , (-(b^2*d^2*Log[x]) - 3*a*c*d^2*Log[x] + 5*a*b*d*e*Log
[x] - 4*a^2*e^2*Log[x] + 4*a^2*d*f*Log[x] + b^2*d^2*Log[-Sqrt[a] + Sqrt[a
+ b*x + c*x^2] - x*#1] + 3*a*c*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
x*#1] - 5*a*b*d*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 4*a^2*e^2
*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a^2*d*f*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1] + 2*Sqrt[a]*b*d^2*Log[x]*#1 - 2*a^(3/2)*d*e*L
og[x]*#1 - 2*Sqrt[a]*b*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1
 + 2*a^(3/2)*d*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - a*d^2*L
og[x]*#1^2 + a*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(S
qrt[a]*c*e) + 2*Sqrt[a]*b*f + 2*c*d*#1 - b*e*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#
1^2 - 2*d*#1^3) & ] + (d + x*(e + f*x))*RootSum[c^2*d - b*c*e + b^2*f + 2*
Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2
*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(b*c*d^3*e*Log[x]) + 2*b^2*d^2*e^2*Log[x] +
 6*a*c*d^2*e^2*Log[x] - 10*a*b*d*e^3*Log[x] + 8*a^2*e^4*Log[x] - 6*b^2*d^3
*f*Log[x] - 28*a*c*d^3*f*Log[x] + 40*a*b*d^2*e*f*Log[x] - 40*a^2*d*e^2*f*L
og[x] + 32*a^2*d^2*f^2*Log[x] + b*c*d^3*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*
x^2] - x*#1] - 2*b^2*d^2*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#...
```

3.103.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1302, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx$$

$$\Big\downarrow\ 1302$$

$$\frac{\int \frac{be-4af+2(ce-bf)x}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{e^2-4df}-\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)}$$

$$\Big\downarrow\ 27$$

$$\frac{\int \frac{be-4af+2(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{2\,(e^2-4df)}-\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)}$$

$$\Big\downarrow\ 1365$$

$$\frac{\frac{2\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{2\left(f(be-4af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{c}}}{\sqrt{e^2-4df}}}{2\,(e^2-4df)}$$
$$\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)}$$

$$\Big\downarrow\ 1154$$

$$\frac{4\left(f(be-4af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-}{}}{\sqrt{e^2-4df}}$$

$$\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)}$$

$$\Big\downarrow\ 219$$

$$\frac{\frac{\sqrt{2}\left(f(be-4af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}\left(f(be-4af)-\left(e-\sqrt{e^2-4d}\right.\right.}{\sqrt{}}}{2\,(e^2-4df)}$$

$$\frac{(e+2fx)\sqrt{a+bx+cx^2}}{(e^2-4df)\,(d+ex+fx^2)}$$

input

```
Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

3.103. $\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx$

output
```
-(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) +
 (-((Sqrt[2]*(f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTa
nh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f])
)*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + (Sqrt[2]*(f*(b
*e - 4*a*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sq
rt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a
*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/(2*(e^2 - 4*d*f))
```

3.103.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4690 vs. $2(439) = 878$.

Time = 0.98 (sec) , antiderivative size = 4691, normalized size of antiderivative = 9.61

method	result	size
default	Expression too large to display	4691

input
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(4*d*f-e^2)*(-2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)*f^2/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*((x-1/2/f*(-e+(
-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(3/2)+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*f/(b*f*(-
4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(1/2*
(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/
f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+e^2)^(1/
2)+b*f-c*e)/f*ln((1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f+c*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2))))/c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*
f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)
)/c^(1/2)-1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+c*e^2)/f^2*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2
)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/...
```

3.103.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

output
```
Timed out
```

3.103.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(fx^2+ex+d)^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d)^2, x)
```

3.103.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{(d+ex+fx^2)^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{(f\,x^2+e\,x+d)^2}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2,x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2)^2, x)
```

3.104 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx$

3.104.1 Optimal result

Integrand size = 27, antiderivative size = 564

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx =$$

$$-\frac{(b^2-4ac)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6bde+a(e^2+2df))+16c^2(24abef+3a^2f^2+\right.}{16384c^6}$$

$$+\frac{\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6bde+a(e^2+2df))+16c^2(24abef+3a^2f^2+14b^2(e^2+\right.}{6144c^5}$$

$$+\frac{(5376c^3de-693b^3f^2+36bcf(56be+31af)-32c^2(48aef+49b(e^2+2df)))\left(a+bx+cx^2\right)^{5/2}}{13440c^4}$$

$$+\frac{(99b^2f^2-12cf(24be+7af)+224c^2(e^2+2df))\,x(a+bx+cx^2)^{5/2}}{1344c^3}$$

$$+\frac{f(32ce-11bf)x^2(a+bx+cx^2)^{5/2}}{112c^2}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$$+\frac{(b^2-4ac)^2\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3(6bde+a(e^2+2df))+16c^2(24abef+3a^2f^2\right.}{32768c^{13/2}}$$

output
```
1/6144*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(3*a*f+4*b*e)-128*c^3*(6*b*d*e+a
*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+14*b^2*(2*d*f+e^2)))*(2*c*x+b)*
(c*x^2+b*x+a)^(3/2)/c^5+1/13440*(5376*c^3*d*e-693*b^3*f^2+36*b*c*f*(31*a*f
+56*b*e)-32*c^2*(48*a*e*f+49*b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(5/2)/c^4+1/134
4*(99*b^2*f^2-12*c*f*(7*a*f+24*b*e)+224*c^2*(2*d*f+e^2))*x*(c*x^2+b*x+a)^(
5/2)/c^3+1/112*f*(-11*b*f+32*c*e)*x^2*(c*x^2+b*x+a)^(5/2)/c^2+1/8*f^2*x^3*
(c*x^2+b*x+a)^(5/2)/c+1/32768*(-4*a*c+b^2)^2*(768*c^4*d^2+99*b^4*f^2-72*b^
2*c*f*(3*a*f+4*b*e)-128*c^3*(6*b*d*e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a
^2*f^2+14*b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(13/2)-1/16384*(-4*a*c+b^2)*(768*c^4*d^2+99*b^4*f^2-72*b^2*c*f*(3*a
*f+4*b*e)-128*c^3*(6*b*d*e+a*(2*d*f+e^2))+16*c^2*(24*a*b*e*f+3*a^2*f^2+14*
b^2*(2*d*f+e^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6
```

3.104.2 Mathematica [A] (verified)

Time = 11.12 (sec) , antiderivative size = 829, normalized size of antiderivative = 1.47

$$\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)^2 \, dx = \frac{430080d^2(b+2cx)(a+x(b+cx))^{3/2}+1376256de(a+x(b+cx))^{5/2}+573440(e^2+2df)\,x(a+\cdots}{\cdots}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

output
```
(430080*d^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 1376256*d*e*(a + x*(b +
c*x))^(5/2) + 573440*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(5/2) + 983040*e*f*
x^2*(a + x*(b + c*x))^(5/2) + 430080*f^2*x^3*(a + x*(b + c*x))^(5/2) + (80
640*(b^2 - 4*a*c)*d^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2
 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2)
 - (26880*b*d*e*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 -
 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (96*e*f*(-2
56*c^(5/2)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3
*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4
*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(9/2) - (224*(e^2 +
2*d*f)*(1792*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*c^(
3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b +
 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))))/c^(7/2) - (3*f^2*(112640*b*c^(9/2)*x^2*(a +
 x*(b + c*x))^(5/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280
*a*c^2*x)*(a + x*(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*
(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[
c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x...
```

3.104.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int \frac{1}{2}\left(cx^2+bx+a\right)^{3/2} \left(f(32ce-11bf)x^3-2\left(3af^2-8c\left(e^2+2df\right)\right)x^2+32cdex+16cd^2\right)dx}{8c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \left(cx^2+bx+a\right)^{3/2}\left(f(32ce-11bf)x^3-2\left(3af^2-8c\left(e^2+2df\right)\right)x^2+32cdex+16cd^2\right)dx}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\downarrow$ 2192

$$\frac{\frac{\int \frac{1}{2}\left(cx^2+bx+a\right)^{3/2}\left(224c^2d^2+\left(224\left(e^2+2df\right)c^2-12f(24be+7af)c+99b^2f^2\right)x^2+4\left(112dec^2-32aefc+11abf^2\right)x\right)dx}{7c}+\frac{fx^2\left(a+bx+cx^2\right)^{5/2}(32ce-}{7c}}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\downarrow$ 27

$$\frac{\frac{\int \left(cx^2+bx+a\right)^{3/2}\left(224c^2d^2+\left(224\left(e^2+2df\right)c^2-12f(24be+7af)c+99b^2f^2\right)x^2+4\left(112dec^2-32aefc+11abf^2\right)x\right)dx}{14c}+\frac{fx^2\left(a+bx+cx^2\right)^{5/2}(32ce-11}{7c}}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{1}{2}\left(2688d^2c^3-448a\left(e^2+2df\right)c^2+24af(24be+7af)c-198ab^2f^2+\left(-693f^2b^3+36cf(56be+31af)b+5376c^3de-32c^2\left(48aef+49b\left(e^2+2df\right)\right)\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{6c}+}{14c}}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \left(2\left(1344d^2c^3-224a\left(e^2+2df\right)c^2+12af(24be+7af)c-99ab^2f^2\right)+\left(-693f^2b^3+36cf(56be+31af)b+5376c^3de-32c^2\left(48aef+49b\left(e^2+2df\right)\right)\right)x\right)\left(cx^2+bx+a\right)^{3/2}dx}{12c}-}{14c}}{16c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\downarrow$ 1160

$$\frac{\frac{\frac{7\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{2c}+\left(a+bx+cx^2\right)^{5/2}\left(-32c^2\left(48ae\right.\right.}{12c}}{14c}+\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\downarrow$ 1087

3.104. $\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx$

$$\frac{\frac{\frac{7\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int\sqrt{cx^2+bx+\cdots}}{16c}\cdots\right.}{2c}\cdots}{12c}\cdots}{\cdots}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 1087

$$\frac{\frac{\frac{7\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a\cdots}}{4c}\cdots\right.}{\cdots}\right.}{2c}\cdots}{12c}\cdots}{\cdots}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\frac{7\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)+99b^4f^2+768c^4d^2\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a\cdots}}{4c}\cdots\right.}{\cdots}\right.}{2c}\cdots}{12c}\cdots}{\cdots}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{7\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf\cdots\right.}{2c}\cdots}{12c}\cdots}{\cdots}$$

$$\frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c}$$

3.104. $\int\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx$

input
```
Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

output
```
(f^2*x^3*(a + b*x + c*x^2)^(5/2))/(8*c) + ((f*(32*c*e - 11*b*f)*x^2*(a + b
*x + c*x^2)^(5/2))/(7*c) + (((99*b^2*f^2 - 12*c*f*(24*b*e + 7*a*f) + 224*c
^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(5/2))/(6*c) + (((5376*c^3*d*e - 693
*b^3*f^2 + 36*b*c*f*(56*b*e + 31*a*f) - 32*c^2*(48*a*e*f + 49*b*(e^2 + 2*d
*f)))*(a + b*x + c*x^2)^(5/2))/(5*c) + (7*(768*c^4*d^2 + 99*b^4*f^2 - 72*b
^2*c*f*(4*b*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*
a*b*e*f + 3*a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*(((b + 2*c*x)*(a + b*x + c*x^
2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4
*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(8*c^(3/2))))/(16*c)))/(2*c))/(12*c))/(14*c))/(16*c)
```

3.104.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(530) = 1060$.

Time = 0.99 (sec) , antiderivative size = 1200, normalized size of antiderivative = 2.13

method	result	size
risch	Expression too large to display	1200
default	Expression too large to display	1654

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1720320/c^6*(215040*c^7*f^2*x^7+261120*b*c^6*f^2*x^6+491520*c^7*e*f*x^6+
322560*a*c^6*f^2*x^5+3840*b^2*c^5*f^2*x^5+614400*b*c^6*e*f*x^5+573440*c^7*
d*f*x^5+286720*c^7*e^2*x^5+19968*a*b*c^5*f^2*x^4+786432*a*c^6*e*f*x^4-4224
*b^3*c^4*f^2*x^4+12288*b^2*c^5*e*f*x^4+745472*b*c^6*d*f*x^4+372736*b*c^6*e
^2*x^4+688128*c^7*d*e*x^4+26880*a^2*c^5*f^2*x^3-27264*a*b^2*c^4*f^2*x^3+67
584*a*b*c^5*e*f*x^3+1003520*a*c^6*d*f*x^3+501760*a*c^6*e^2*x^3+4752*b^4*c^
3*f^2*x^3-13824*b^3*c^4*e*f*x^3+21504*b^2*c^5*d*f*x^3+10752*b^2*c^5*e^2*x^
3+946176*b*c^6*d*e*x^3+430080*c^7*d^2*x^3-57984*a^2*b*c^4*f^2*x^2+98304*a^
2*c^5*e*f*x^2+37440*a*b^3*c^3*f^2*x^2-95232*a*b^2*c^4*e*f*x^2+129024*a*b*c
^5*d*f*x^2+64512*a*b*c^5*e^2*x^2+1376256*a*c^6*d*e*x^2-5544*b^5*c^2*f^2*x^
2+16128*b^4*c^3*e*f*x^2-25088*b^3*c^4*d*f*x^2-12544*b^3*c^4*e^2*x^2+43008*
b^2*c^5*d*e*x^2+645120*b*c^6*d^2*x^2-40320*a^3*c^4*f^2*x+113376*a^2*b^2*c^
3*f^2*x-224256*a^2*b*c^4*e*f*x+215040*a^2*c^5*d*f*x+107520*a^2*c^5*e^2*x-5
3928*a*b^4*c^2*f^2*x+139776*a*b^3*c^3*e*f*x-193536*a*b^2*c^4*d*f*x-96768*a
*b^2*c^4*e^2*x+301056*a*b*c^5*d*e*x+1075200*a*c^6*d^2*x+6930*b^6*c*f^2*x-2
0160*b^5*c^2*e*f*x+31360*b^4*c^3*d*f*x+15680*b^4*c^3*e^2*x-53760*b^3*c^4*d
*e*x+53760*b^2*c^5*d^2*x+176448*a^3*b*c^3*f^2-196608*a^3*c^4*e*f-244944*a^
2*b^3*c^2*f^2+526848*a^2*b^2*c^3*e*f-580608*a^2*b*c^4*d*f-290304*a^2*b*c^4
*e^2+688128*a^2*c^5*d*e+91980*a*b^5*c*f^2-241920*a*b^4*c^2*e*f+340480*a*b^
3*c^3*d*f+170240*a*b^3*c^3*e^2-537600*a*b^2*c^4*d*e+537600*a*b*c^5*d^2-...
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(530) = 1060$.

Time = 0.84 (sec) , antiderivative size = 2179, normalized size of antiderivative = 3.86

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
[1/6881280*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 - 768*(b^5*c
^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*
a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2
 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 - 60*a*b^4*c^3 +
 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*
c^3 - 64*a^3*b*c^4)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(215040*c^8*f^2*x^7 + 15
360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^6 + 84*
a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + 2912*
b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2*c^6
 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*b^2*
c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 +
32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 -
26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 12
8*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 -
3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2
 + 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 3
6*a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 3
2*(14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2
*c^6)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)...
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5942 vs. $2(580) = 1160$.

Time = 0.86 (sec) , antiderivative size = 5942, normalized size of antiderivative = 10.54

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)**2,x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f**2*x**7/8 + x**6*(17*b*c*f**2/16 +
2*c**2*e*f)/(7*c) + x**5*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f - 13*b*(17*
b*c*f**2/16 + 2*c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(6*c) + x**4*(2
*a*b*f**2 + 4*a*c*e*f - 6*a*(17*b*c*f**2/16 + 2*c**2*e*f)/(7*c) + 2*b**2*e
*f + 4*b*c*d*f + 2*b*c*e**2 - 11*b*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f -
 13*b*(17*b*c*f**2/16 + 2*c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(12*c
) + 2*c**2*d*e)/(5*c) + x**3*(a**2*f**2 + 4*a*b*e*f + 4*a*c*d*f + 2*a*c*e*
*2 - 5*a*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f - 13*b*(17*b*c*f**2/16 + 2*
c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(6*c) + 2*b**2*d*f + b**2*e**2
+ 4*b*c*d*e - 9*b*(2*a*b*f**2 + 4*a*c*e*f - 6*a*(17*b*c*f**2/16 + 2*c**2*e
*f)/(7*c) + 2*b**2*e*f + 4*b*c*d*f + 2*b*c*e**2 - 11*b*(9*a*c*f**2/8 + b**
2*f**2 + 4*b*c*e*f - 13*b*(17*b*c*f**2/16 + 2*c**2*e*f)/(14*c) + 2*c**2*d*
f + c**2*e**2)/(12*c) + 2*c**2*d*e)/(10*c) + c**2*d**2)/(4*c) + x**2*(2*a*
*2*e*f + 4*a*b*d*f + 2*a*b*e**2 + 4*a*c*d*e - 4*a*(2*a*b*f**2 + 4*a*c*e*f
- 6*a*(17*b*c*f**2/16 + 2*c**2*e*f)/(7*c) + 2*b**2*e*f + 4*b*c*d*f + 2*b*c
*e**2 - 11*b*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f - 13*b*(17*b*c*f**2/16
+ 2*c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(12*c) + 2*c**2*d*e)/(5*c)
+ 2*b**2*d*e + 2*b*c*d**2 - 7*b*(a**2*f**2 + 4*a*b*e*f + 4*a*c*d*f + 2*a*c
*e**2 - 5*a*(9*a*c*f**2/8 + b**2*f**2 + 4*b*c*e*f - 13*b*(17*b*c*f**2/16 +
 2*c**2*e*f)/(14*c) + 2*c**2*d*f + c**2*e**2)/(6*c) + 2*b**2*d*f + b**2...
```

3.104.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(530) = 1060$.

Time = 0.30 (sec) , antiderivative size = 1132, normalized size of antiderivative = 2.01

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f^2*x + (32*c^8*
e*f + 17*b*c^7*f^2)/c^7)*x + (224*c^8*e^2 + 448*c^8*d*f + 480*b*c^7*e*f +
3*b^2*c^6*f^2 + 252*a*c^7*f^2)/c^7)*x + (5376*c^8*d*e + 2912*b*c^7*e^2 + 5
824*b*c^7*d*f + 96*b^2*c^6*e*f + 6144*a*c^7*e*f - 33*b^3*c^5*f^2 + 156*a*b
*c^6*f^2)/c^7)*x + (26880*c^8*d^2 + 59136*b*c^7*d*e + 672*b^2*c^6*e^2 + 31
360*a*c^7*e^2 + 1344*b^2*c^6*d*f + 62720*a*c^7*d*f - 864*b^3*c^5*e*f + 422
4*a*b*c^6*e*f + 297*b^4*c^4*f^2 - 1704*a*b^2*c^5*f^2 + 1680*a^2*c^6*f^2)/c
^7)*x + (80640*b*c^7*d^2 + 5376*b^2*c^6*d*e + 172032*a*c^7*d*e - 1568*b^3*
c^5*e^2 + 8064*a*b*c^6*e^2 - 3136*b^3*c^5*d*f + 16128*a*b*c^6*d*f + 2016*b
^4*c^4*e*f - 11904*a*b^2*c^5*e*f + 12288*a^2*c^6*e*f - 693*b^5*c^3*f^2 + 4
680*a*b^3*c^4*f^2 - 7248*a^2*b*c^5*f^2)/c^7)*x + (26880*b^2*c^6*d^2 + 5376
00*a*c^7*d^2 - 26880*b^3*c^5*d*e + 150528*a*b*c^6*d*e + 7840*b^4*c^4*e^2 -
 48384*a*b^2*c^5*e^2 + 53760*a^2*c^6*e^2 + 15680*b^4*c^4*d*f - 96768*a*b^2
*c^5*d*f + 107520*a^2*c^6*d*f - 10080*b^5*c^3*e*f + 69888*a*b^3*c^4*e*f -
112128*a^2*b*c^5*e*f + 3465*b^6*c^2*f^2 - 26964*a*b^4*c^3*f^2 + 56688*a^2*
b^2*c^4*f^2 - 20160*a^3*c^5*f^2)/c^7)*x - (80640*b^3*c^5*d^2 - 537600*a*b*
c^6*d^2 - 80640*b^4*c^4*d*e + 537600*a*b^2*c^5*d*e - 688128*a^2*c^6*d*e +
23520*b^5*c^3*e^2 - 170240*a*b^3*c^4*e^2 + 290304*a^2*b*c^5*e^2 + 47040*b^
5*c^3*d*f - 340480*a*b^3*c^4*d*f + 580608*a^2*b*c^5*d*f - 30240*b^6*c^2*e*
f + 241920*a*b^4*c^3*e*f - 526848*a^2*b^2*c^4*e*f + 196608*a^3*c^5*e*f ...
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx = \int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)^2 \,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x)
```

output
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2, x)
```

3.105 $\int (a+bx+cx^2)^{3/2} (d+ex+fx^2) \, dx$

3.105.1 Optimal result

Integrand size = 25, antiderivative size = 236

$$\begin{aligned}\int (a+bx+cx^2)^{3/2} (d+ex+fx^2) \, dx = & -\frac{(b^2-4ac)(24c^2d+7b^2f-4c(3be+af))(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} \\ & + \frac{(24c^2d-12bce+7b^2f-4acf)(b+2cx)(a+bx+cx^2)^{3/2}}{192c^3} \\ & + \frac{(12ce-7bf)(a+bx+cx^2)^{5/2}}{60c^2} + \frac{fx(a+bx+cx^2)^{5/2}}{6c} \\ & + \frac{(b^2-4ac)^2(24c^2d+7b^2f-4c(3be+af))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}\end{aligned}$$

output
```
1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c
^3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*x^2+b*x+a)^(5/2
)/c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*arctanh(1/2*(
2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d
+7*b^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

3.105.2 Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.24

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5 f+10b^4c(18e+7fx)-8b^3c(45cd-95af+cx(15e+7fx))+48b^2}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*f + 10*b^4*c*(18*e + 7*f*x) - 8*b
^3*c*(45*c*d - 95*a*f + c*x*(15*e + 7*f*x)) + 48*b^2*c^2*(-(a*(25*e + 9*f*
x)) + c*x*(5*d + x*(2*e + f*x))) + 16*b*c^2*(-81*a^2*f + 6*a*c*(25*d + x*(
7*e + 3*f*x)) + 4*c^2*x^2*(45*d + x*(33*e + 26*f*x))) + 32*c^3*(3*a^2*(16*
e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*
e + 35*f*x)))) + 15*(b^2 - 4*a*c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f
))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(7680*c^(9/2))
```

3.105.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2192, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int \frac{1}{2}(12cd-2af+(12ce-7bf)x)\left(cx^2+bx+a\right)^{3/2}dx}{6c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (2(6cd-af)+(12ce-7bf)x)\left(cx^2+bx+a\right)^{3/2}dx}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \; 1087$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+$$
$$\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \; 1087$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce}{5c}}{12c}$$
$$\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{2c}+(a+bx+}{12c}$$
$$\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(-4acf+7b^2f-12bce+24c^2d\right)}{2c}+(a+bx+c}{12c}$$
$$\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

3.105. $\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\, dx$

input
```
Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + (((12*c*e - 7*b*f)*(a + b*x + c*x^2)
^(5/2))/(5*c) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(((b + 2*c*x)*(
a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*
x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(2*c))/(12*c)
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.105.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{(-1280c^5 f x^5-1664b c^4 f x^4-1536c^5 e x^4-2240a c^4 f x^3-48b^2c^3 f x^3-2112b c^4 e x^3-1920c^5 d x^3-288ab c^3 f x^2-3072a c^4 e x^2+56b$
default	$d\left(\frac{(2cx+b)(c x^2+bx+a)^{\frac{3}{2}}}{8c}+\frac{3(4ac-b^2)\left(\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+f\left(\frac{x(c x^2+bx+a)^{\frac{5}{2}}}{6c}\right.$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/7680/c^4*(-1280*c^5*f*x^5-1664*b*c^4*f*x^4-1536*c^5*e*x^4-2240*a*c^4*f*
x^3-48*b^2*c^3*f*x^3-2112*b*c^4*e*x^3-1920*c^5*d*x^3-288*a*b*c^3*f*x^2-307
2*a*c^4*e*x^2+56*b^3*c^2*f*x^2-96*b^2*c^3*e*x^2-2880*b*c^4*d*x^2-480*a^2*c
^3*f*x+432*a*b^2*c^2*f*x-672*a*b*c^3*e*x-4800*a*c^4*d*x-70*b^4*c*f*x+120*b
^3*c^2*e*x-240*b^2*c^3*d*x+1296*a^2*b*c^2*f-1536*a^2*c^3*e-760*a*b^3*c*f+1
200*a*b^2*c^2*e-2400*a*b*c^3*d+105*b^5*f-180*b^4*c*e+360*b^3*c^2*d)*(c*x^2
+b*x+a)^(1/2)-1/1024*(64*a^3*c^3*f-144*a^2*b^2*c^2*f+192*a^2*b*c^3*e-384*a
^2*c^4*d+60*a*b^4*c*f-96*a*b^3*c^2*e+192*a*b^2*c^3*d-7*b^6*f+12*b^5*c*e-24
*b^4*c^2*d)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.56

$$\int (a+bx+cx^2)^{3/2}(d+ex+fx^2)\,dx = \left[-\frac{15\,(24\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)d-12\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)e+(7\,b^6-60\,ab^4c+144\,a^2}{}\right.$$

$$-\frac{15\,(24\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)d-12\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)e+(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3}{}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^
3*c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c
^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 +
 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*
x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 12
8*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^
2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*
b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
a*c)) - 2*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*
d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^
2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 2
0*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*
c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12
*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

3.105.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(230) = 460$.

Time = 0.63 (sec) , antiderivative size = 1360, normalized size of antiderivative = 5.76

$$\int \left(a + bx + cx^2\right)^{3/2} \left(d + ex + fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*x**5/6 + x**4*(13*b*c*f/12 + c**2*e
)/(5*c) + x**3*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/
(10*c) + c**2*d)/(4*c) + x**2*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2
*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13
*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(3*c) + x*(a**2*f + 2*a*b*e +
2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(
10*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12
 + c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e -
9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(2*c) + (a**2*e
+ 2*a*b*d - 2*a*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b*
*2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c*
*2*e)/(10*c) + c**2*d)/(8*c))/(3*c) - 3*b*(a**2*f + 2*a*b*e + 2*a*c*d - 3*
a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2
*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(
5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*
f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(4*c))/c) + (a**2*d - a*(a**
2*f + 2*a*b*e + 2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*
f/12 + c**2*e)/(10*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e -
4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**
2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c...
```

3.105.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.71

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{12\,c^6e+13\,bc^5f}{c^5}\right)x+\frac{120\,c^6d+132\,bc^5e+3\,b^2c^4f}{c^5}\right.\right.\right.\right.$$
$$-\frac{\left(24\,b^4c^2d-192\,ab^2c^3d+384\,a^2c^4d-12\,b^5ce+96\,ab^3c^2e-192\,a^2bc^3e+7\,b^6f-60\,ab^4cf+144\,a^2b^2c^2f-\right.}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (12*c^6*e + 13*b*c^5*
f)/c^5)*x + (120*c^6*d + 132*b*c^5*e + 3*b^2*c^4*f + 140*a*c^5*f)/c^5)*x +
 (360*b*c^5*d + 12*b^2*c^4*e + 384*a*c^5*e - 7*b^3*c^3*f + 36*a*b*c^4*f)/c
^5)*x + (120*b^2*c^4*d + 2400*a*c^5*d - 60*b^3*c^3*e + 336*a*b*c^4*e + 35*
b^4*c^2*f - 216*a*b^2*c^3*f + 240*a^2*c^4*f)/c^5)*x - (360*b^3*c^3*d - 240
0*a*b*c^4*d - 180*b^4*c^2*e + 1200*a*b^2*c^3*e - 1536*a^2*c^4*e + 105*b^5*
c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f)/c^5) - 1/1024*(24*b^4*c^2*d - 19
2*a*b^2*c^3*d + 384*a^2*c^4*d - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^
3*e + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^3*c^3*f)*log(abs(2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) dx$$

input
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

3.106 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

3.106.1 Optimal result

Integrand size = 27, antiderivative size = 679

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2}$$
$$+\frac{(3b^2f^2-12cf(be-af)+8c^2(e^2-df))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^3}$$
$$+\frac{\left((ce-bf)\left(e-\sqrt{e^2-4df}\right)(f(be-2af)-c(e^2-2df))-2f(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)}}$$
$$-\frac{\left((ce-bf)\left(e+\sqrt{e^2-4df}\right)(f(be-2af)-c(e^2-2df))-2f(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf}}$$

output
```
1/8*(3*b^2*f^2-12*c*f*(-a*f+b*e)+8*c^2*(-d*f+e^2))*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/f^3/c^(1/2)-1/4*(-2*c*f*x-5*b*f+4*c*e)*(c*x^2+
b*x+a)^(1/2)/f^2+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))
-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*
f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c*d*f*(-a*f+b*e)-
f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2))+(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+
e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*
f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*
f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*
x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2
))^(1/2))*(-2*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2))+(
-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^3*2^(1
/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2
)^(1/2))^(1/2)
```

3.106.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.92 (sec) , antiderivative size = 1472, normalized size of antiderivative = 2.17

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

output
```
(f*(-4*c*e + 5*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + ((3*b^2*f^2 + 12*c*f
*(-(b*e) + a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[
a + x*(b + c*x)])])/Sqrt[c] - 4*RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*
c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]
*e*#1^3 + d*#1^4 & , (c^3*d*e^2*Log[x] - b*c^2*e^3*Log[x] - c^3*d^2*f*Log[
x] + 2*b^2*c*e^2*f*Log[x] - b^2*c*d*f^2*Log[x] + 2*a*c^2*d*f^2*Log[x] - b^
3*e*f^2*Log[x] - 2*a*b*c*e*f^2*Log[x] + 2*a*b^2*f^3*Log[x] - a^2*c*f^3*Log
[x] - c^3*d*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b*c^2*e^3*L
og[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + c^3*d^2*f*Log[-Sqrt[a] + Sqr
t[a + b*x + c*x^2] - x*#1] - 2*b^2*c*e^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c
*x^2] - x*#1] + b^2*c*d*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] -
 2*a*c^2*d*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b^3*e*f^2*Lo
g[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2*a*b*c*e*f^2*Log[-Sqrt[a] +
Sqrt[a + b*x + c*x^2] - x*#1] - 2*a*b^2*f^3*Log[-Sqrt[a] + Sqrt[a + b*x +
c*x^2] - x*#1] + a^2*c*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] +
2*Sqrt[a]*c^2*e^3*Log[x]*#1 - 4*Sqrt[a]*c^2*d*e*f*Log[x]*#1 - 4*Sqrt[a]*b*
c*e^2*f*Log[x]*#1 + 4*Sqrt[a]*b*c*d*f^2*Log[x]*#1 + 2*Sqrt[a]*b^2*e*f^2*Lo
g[x]*#1 + 4*a^(3/2)*c*e*f^2*Log[x]*#1 - 4*a^(3/2)*b*f^3*Log[x]*#1 - 2*Sqrt
[a]*c^2*e^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 4*Sqrt[a]*c^
2*d*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 4*Sqrt[a]*b*c...
```

3.106.3 Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1308, 27, 2143, 27, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}dx$$

$$\Big\downarrow \text{1308}$$

$$-\frac{\int -\frac{-5dfb^2+4cdeb+\left(8(e^2-df)c^2-12f(be-af)c+3b^2f^2\right)x^2-4af(cd-2af)+\left(8dec^2+4(be^2-afe-4bdf)c-bf(5be-16af)\right)x}{4\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{2f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-5dfb^2+4cdeb+\left(8\left(e^2-df\right)c^2-12f(be-af)c+3b^2f^2\right)x^2-4af(cd-2af)+\left(8dec^2-4aefc+4b\left(e^2-4df\right)c-bf(5be-16af)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{8f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 2143$$

$$\frac{\frac{\int \frac{8\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)+(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)x\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{\left(-12cf(be-af)+3b^2f^2+8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{8\int \frac{-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)-(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{\left(-12cf(be-af)+3b^2f^2+8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8f^2}-\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{8\int \frac{-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)-(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{2\left(-12cf(be-af)+3b^2f^2+8c^2\left(e^2-df\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+}{\sqrt{cx^2}}}{f}}{8f^2}$$

$$\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{8\int \frac{-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)-(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12cf(be-af)+3b^2f^2+8c^2\left(e^2-\right.\right.}{\sqrt{c}f}}{8f^2}$$

$$\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$$\Big\downarrow \; 1365$$

3.106. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$8\left(\frac{\left(-2f^3\left(b^2d-a^2f\right)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+4cdf^2(be-af)-2c^2df\left(e^2-df\right)\right)\int\frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(-2f^3\left(b^2d-\right.\right.}{}\right.$$

$$f$$

$$\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$\downarrow$ 1154

$$8\left(\frac{2\left(-2f^3\left(b^2d-a^2f\right)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+4cdf^2(be-af)-2c^2df\left(e^2-df\right)\right)\int\frac{}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-}}{\sqrt{e^2-4df}}\right.$$

$$\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

$\downarrow$ 219

$$8\left(\frac{\left(-2f^3\left(b^2d-a^2f\right)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+4cdf^2(be-af)-2c^2df\left(e^2-df\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\cdot\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-b}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right.$$

$$\frac{\sqrt{a+bx+cx^2}(-5bf+4ce-2cfx)}{4f^2}$$

input

```
Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

3.106. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

output
```
-1/4*((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/f^2 + (((3*b^2*f^2
- 12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*f) + (8*(-(((4*c*d*f^2*(b*e - a*f) - 2*f^
3*(b^2*d - a^2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*
d*f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x
 + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a
*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + ((4*c*d*f^2*(b*e - a*f) - 2*f^3*
(b^2*d - a^2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*
f])*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2
- 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 -
 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])))/f)/(8*f^2)
```

3.106.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1308
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p +
 q)*x)*(a + b*x + c*x^2)^(p - 1)*((d + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*
(2*p + 2*q + 1))), x] - Simp[1/(2*f^2*(p + q)*(2*p + 2*q + 1))   Int[(a + b
*x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p
)*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
 + c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p +
q) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*
(2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(
p + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
 - 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*
a*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2
*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1327 vs. $2(618) = 1236$.

Time = 1.03 (sec) , antiderivative size = 1328, normalized size of antiderivative = 1.96

method	result	size
risch	Expression too large to display	1328
default	Expression too large to display	2860

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*f*x+5*b*f-4*c*e)*(c*x^2+b*x+a)^(1/2)/f^2+1/8/f^2*(1/f*(12*a*c*f^2
+3*b^2*f^2-12*b*c*e*f-8*c^2*d*f+8*c^2*e^2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))/c^(1/2)-1/2*(16*a*b*f^3*(-4*d*f+e^2)^(1/2)-16*a*c*e*f^2*(-4*d
*f+e^2)^(1/2)-8*b^2*e*f^2*(-4*d*f+e^2)^(1/2)-16*b*c*d*f^2*(-4*d*f+e^2)^(1/
2)+16*b*c*e^2*f*(-4*d*f+e^2)^(1/2)+16*c^2*d*e*f*(-4*d*f+e^2)^(1/2)-8*c^2*e
^3*(-4*d*f+e^2)^(1/2)-16*a^2*f^4+16*a*b*f^3*e+32*a*c*d*f^3-16*a*c*e^2*f^2+
16*b^2*d*f^3-8*b^2*e^2*f^2-48*b*c*d*e*f^2+16*b*c*e^3*f-16*c^2*d^2*f^2+32*c
^2*d*e^2*f-8*c^2*e^4)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(
1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-
b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)
/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c
*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*
d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1
/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(16*a*b*f^3*(-4*d*f+e^2)^(1/2)-
16*a*c*e*f^2*(-4*d*f+e^2)^(1/2)-8*b^2*e*f^2*(-4*d*f+e^2)^(1/2)-16*b*c*d*f^
2*(-4*d*f+e^2)^(1/2)+16*b*c*e^2*f*(-4*d*f+e^2)^(1/2)+16*c^2*d*e*f*(-4*d*f+
e^2)^(1/2)-8*c^2*e^3*(-4*d*f+e^2)^(1/2)+16*a^2*f^4-16*a*b*f^3*e-32*a*c*d*f
^3+16*a*c*e^2*f^2-16*b^2*d*f^3+8*b^2*e^2*f^2+48*b*c*d*e*f^2-16*b*c*e^3*...
```

3.106.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2)^{3/2}}{d + ex + fx^2} \, dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2} \, dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.106.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.106.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx=\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

3.107 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx$

3.107.1 Optimal result

Integrand size = 27, antiderivative size = 704

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f(e^2-4df)}$$

$$-\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)}+\frac{c^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f^2}$$

$$-\frac{\left((ce-bf)(f(be-2af)+2c(e^2-5df))\left(e-\sqrt{e^2-4df}\right)-2f(2c^2d(e^2-4df)+f(2b^2df+4af(cd+af)\right.}{2\sqrt{2}f^2(e^2-4df)^{3/2}\sqrt{ce^2-2cdf-bef+2af^2-}}$$

$$+\frac{\left((ce-bf)(f(be-2af)+2c(e^2-5df))\left(e+\sqrt{e^2-4df}\right)-2f(2c^2d(e^2-4df)+f(2b^2df+4af(cd+af)\right.}{2\sqrt{2}f^2(e^2-4df)^{3/2}\sqrt{ce^2-2cdf-bef+2af^2+}}$$

output
```
-(2*f*x+e)*(c*x^2+b*x+a)^(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)+c^(3/2)*arctanh(
1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2-(-2*c*f*x-2*b*f+c*e)*(c*x^2
+b*x+a)^(1/2)/f/(-4*d*f+e^2)-1/4*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+
e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-
2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d
*(-4*d*f+e^2)+f*(2*b^2*d*f+4*a*f*(a*f+c*d)-b*e*(3*a*f+c*d)))+(-b*f+c*e)*(f
*(-2*a*f+b*e)+2*c*(-5*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^2/(-4*d*f+e^2)^(
3/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(
1/2)+1/4*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f
+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-
b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d*(-4*d*f+e^2)+f*(2*b^2*d
*f+4*a*f*(a*f+c*d)-b*e*(3*a*f+c*d)))+(-b*f+c*e)*(f*(-2*a*f+b*e)+2*c*(-5*d*
f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^2/(-4*d*f+e^2)^(3/2)*2^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.107.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.61 (sec) , antiderivative size = 2854, normalized size of antiderivative = 4.05

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

output
```
((-2*f*Sqrt[a + x*(b + c*x)]*(c*e^2*x - b*f*(2*d + e*x) + c*d*(e - 2*f*x)
+ a*f*(e + 2*f*x)))/((e^2 - 4*d*f)*(d + x*(e + f*x))) + 4*c^(3/2)*ArcTanh[
(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])] + (2*RootSum[c^2*d - b*c*e
 + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4
*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^3*d^4*Log[x]) + b*c^2*d^3*e
*Log[x] + 4*a*c^2*d^3*f*Log[x] - 6*a*b^2*d^2*f^2*Log[x] - 7*a^2*c*d^2*f^2*
Log[x] + 9*a^2*b*d*e*f^2*Log[x] - 4*a^3*e^2*f^2*Log[x] + 4*a^3*d*f^3*Log[x
] + c^3*d^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - b*c^2*d^3*e*Log
[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a*c^2*d^3*f*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1] + 6*a*b^2*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x
 + c*x^2] - x*#1] + 7*a^2*c*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1] - 9*a^2*b*d*e*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 4*
a^3*e^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 4*a^3*d*f^3*Log
[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c^2*d^3*e*Log[x]*#1
+ 4*a^(3/2)*b*d^2*f^2*Log[x]*#1 - 2*a^(5/2)*d*e*f^2*Log[x]*#1 + 2*Sqrt[a]*
c^2*d^3*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 4*a^(3/2)*b*d^
2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*a^(5/2)*d*e*f^2*
Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c^2*d^4*Log[x]*#1^2 - a^
2*d^2*f^2*Log[x]*#1^2 - c^2*d^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
1]*#1^2 + a^2*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2...
```

3.107.3 Rubi [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {1302, 27, 2138, 27, 2143, 27, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{\int \frac{\sqrt{cx^2+bx+a}(8cfx^2+2(3ce+bf)x+3be-4af)}{2(fx^2+ex+d)}dx}{e^2-4df}-\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{cx^2+bx+a}(8cfx^2+2(3ce+bf)x+3be-4af)}{fx^2+ex+d}dx}{2(e^2-4df)}-\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{(e^2-4df)(d+ex+fx^2)}$$

$\downarrow$ 2138

$$-\frac{\frac{\int \frac{2\left(-2f\left(e^2-4df\right)x^2c^3+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)c-f\left(2dec^2+2aefc+b\left(e^2-10df\right)c+bf(be-2af)\right)xc\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{2cf^2}-\frac{2\sqrt{a+bx+cx^2}(-2bf+ce-2cfx)}{f}-}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 27

$$-\frac{\frac{\int \frac{-2f\left(e^2-4df\right)x^2c^3+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)c-f\left(2dec^2+2aefc+b\left(e^2-10df\right)c+bf(be-2af)\right)xc}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{cf^2}-\frac{2\sqrt{a+bx+cx^2}(-2bf+ce-2cfx)}{f}-}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 2143

$$-\frac{\frac{\frac{\int \frac{cf\left(2d\left(e^2-4df\right)c^2+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)+(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)x\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-2c^3\left(e^2-4df\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{cf^2}-\frac{2\sqrt{a+bx+cx^2}(-}{f}}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 27

$$-\frac{\frac{c\int \frac{2d\left(e^2-4df\right)c^2+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)+(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx-2c^3\left(e^2-4df\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{cf^2}-\frac{2\sqrt{a+bx+cx^2}(-2b}{f}}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 1092

$$-\frac{\frac{c\int \frac{2d\left(e^2-4df\right)c^2+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)+(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx-4c^3\left(e^2-4df\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{cf^2}-2\sqrt{a+}}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 219

3.107. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^2}dx$

$$-\frac{\frac{c\int \frac{2d\left(e^2-4df\right)c^2+f\left(2dfb^2-e(cd+3af)b+4af(cd+af)\right)+(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx-2c^{5/2}\left(e^2-4df\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{cf^2}-\frac{2\sqrt{a}}{}}{2\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 1365

$$-\frac{c\left(\frac{\left(\left(\sqrt{e^2-4df}+e\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2-4df\right)\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\right.}{}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 1154

$$-\frac{c\left(\frac{2\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2-4df\right)\right)\right)\int \frac{}{4\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{\right.\right.}}}{\sqrt{e^2-4df}}\right.}{}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

$\downarrow$ 219

$$-\frac{c\left(\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2-4df\right)\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e\right.\right.}}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right.}{}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

3.107. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^2}\,dx$

output
```
-(((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)))
 + ((-2*(c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/f - (-2*c^(5/2)*(e^
2 - 4*d*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])] + c*((((
c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) -
 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d
+ 3*a*f))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
(c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2
- 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4
*d*f]]) - (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^
2 - 4*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f
) - b*e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt
[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)
*Sqrt[e^2 - 4*d*f]])))/(c*f^2))/(2*(e^2 - 4*d*f))
```

3.107.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.107. $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx$

3.107.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7855 vs. 2(640) = 1280.

Time = 1.09 (sec) , antiderivative size = 7856, normalized size of antiderivative = 11.16

method	result	size
default	Expression too large to display	7856

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.107.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**2,x)
```

output
```
Timed out
```

3.107.7 Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \int \frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2+ex+d)^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^2, x)
```

3.107.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{(f\,x^2+e\,x+d)^2}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2, x)
```

3.108 $\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx$

3.108.1 Optimal result

Integrand size = 27, antiderivative size = 671

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = -\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2}$$
$$+\frac{3(4cde+4aef-b(e^2+4df)+2(ce^2-2bef+4af^2)x)\sqrt{a+bx+cx^2}}{4(e^2-4df)^2(d+ex+fx^2)}$$
$$-\frac{3\left(2(2cd-be+2af)(ce-bf)\left(e-\sqrt{e^2-4df}\right)-f(4be(cd+3af)-b^2(e^2+4df)-4a(ce^2+4af^2))\right)\text{arct}}{4\sqrt{2}(e^2-4df)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e}}}$$
$$+\frac{3\left(2(2cd-be+2af)(ce-bf)\left(e+\sqrt{e^2-4df}\right)-f(4be(cd+3af)-b^2(e^2+4df)-4a(ce^2+4af^2))\right)\text{arct}}{4\sqrt{2}(e^2-4df)^{5/2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e}}}$$

output
```
-1/2*(2*f*x+e)*(c*x^2+b*x+a)^(3/2)/(-4*d*f+e^2)/(f*x^2+e*x+d)^2+3/4*(4*c*d
*e+4*a*e*f-b*(4*d*f+e^2)+2*(4*a*f^2-2*b*e*f+c*e^2)*x)*(c*x^2+b*x+a)^(1/2)/
(-4*d*f+e^2)^2/(f*x^2+e*x+d)-3/8*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+
e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-
2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-f*(4*b*e*(3*
a*f+c*d)-b^2*(4*d*f+e^2)-4*a*(4*a*f^2+c*e^2))+2*(2*a*f-b*e+2*c*d)*(-b*f+c*
e)*(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(5/2)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+3/8*arctanh(1/4*(4*a*f-b*(e+
(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x
+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2
))*(-f*(4*b*e*(3*a*f+c*d)-b^2*(4*d*f+e^2)-4*a*(4*a*f^2+c*e^2))+2*(2*a*f-b*
e+2*c*d)*(-b*f+c*e)*(e+(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(5/2)*2^(1/2)/(c*
e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.108.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4727 vs. 2(671) = 1342.

Time = 17.36 (sec) , antiderivative size = 4727, normalized size of antiderivative = 7.04

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

output
```
(-2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^(3/2)*(e - Sqrt[e^2 - 4*d*
f] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^2*(e - Sqr
t[e^2 - 4*d*f] + 2*f*x)) + (2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^
(3/2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))
/((e^2 - 4*d*f)^2*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + (9*f^2*(a + x*(b + c*
x))^(3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])) - 4*c^2*f
*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((2*Sqrt[c]*(b^2*f^2 + 4*c^2*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f])))*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f + (2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2
- 4*d*f]]*(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - Sqrt[e^2 - 4*d*f]) - 4*a*c*f*(e
 - Sqrt[e^2 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])) + 4*c*(
-e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b*(-e + S
qrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2
- 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^
2*(a + b*x + c*x^2)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((-4*c*f*(4*a
*f - b*(e - Sqrt[e^2 - 4*d*f])) - 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(...
```

3.108.3 Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1302, 27, 1346, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{(d+ex+fx^2)^3}\,dx$$

$$\Big\downarrow \text{1302}$$

$$\frac{\int \frac{3(be-4af+2(ce-bf)x)\sqrt{cx^2+bx+a}}{2(fx^2+ex+d)^2}dx}{2(e^2-4df)}-\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{(be-4af+2(ce-bf)x)\sqrt{cx^2+bx+a}}{(fx^2+ex+d)^2}dx}{4(e^2-4df)}-\frac{(e+2fx)(a+bx+cx^2)^{3/2}}{2(e^2-4df)(d+ex+fx^2)^2}$$

$\downarrow$ 1346

$$\frac{3\left(\frac{\int -\frac{-\left(\left(e^2+4df\right)b^2\right)+4e(cd+3af)b-4a\left(ce^2+4af^2\right)+4(2cd-be+2af)(ce-bf)x}{2\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{e^2-4df}+\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}\right)}{4\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

$\downarrow$ 27

$$\frac{3\left(\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}-\frac{\int \frac{-\left(\left(e^2+4df\right)b^2\right)+4e(cd+3af)b-4a\left(ce^2+4af^2\right)+4(2cd-be+2af)(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{2\left(e^2-4df\right)}\right)}{4\left(e^2-4df\right)}$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

$\downarrow$ 1365

$$3\left(\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}-\frac{\frac{2\left(2\left(\sqrt{e^2-4df}+e\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)-\left(b^2\left(4\cdots\right.\right.\right.\right.}{\cdots}}{\sqrt{e^2-4df}}\cdots\right.$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

$\downarrow$ 1154

$$3\left(\frac{\sqrt{a+bx+cx^2}\left(2x\left(4af^2-2bef+ce^2\right)+4aef-b\left(4df+e^2\right)+4cde\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)}-\frac{\frac{4\left(2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)-\left(b^2\left(4\cdots\right.\right.\right.\right.}{\cdots}}{\cdots}\cdots\right.$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

3.108. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx$

$\downarrow$ 219

$$3\left(\frac{\sqrt{a+bx+cx^2}\,(2x(4af^2-2bef+ce^2)+4aef-b(4df+e^2)+4cde)}{(e^2-4df)(d+ex+fx^2)}-\frac{\sqrt{2}\left(2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)-\left(b^2\left(\right.\right.\right.\right.}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-}}}\right.$$

$$\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

output
```
-1/2*((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2
)^2) + (3*(((4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*
a*f^2)*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2)) - ((Sqr
t[2]*(2*(2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]) - f*(4*b
*e*(c*d + 3*a*f) - b^2*(e^2 + 4*d*f) - 4*a*(c*e^2 + 4*a*f^2)))*ArcTanh[(4*
a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(
2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (Sqrt[2]*(2*(2*c*d -
b*e + 2*a*f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - f*(4*b*e*(c*d + 3*a*f)
- b^2*(e^2 + 4*d*f) - 4*a*(c*e^2 + 4*a*f^2)))*ArcTanh[(4*a*f - b*(e + Sqrt
[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a +
b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
+ (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/(2*(e^2 - 4*d*f))))/(4*(e^2 - 4*d*f))
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1302
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*((d + e
*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))
   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p
+ 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 1346
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(g*b - 2*a*h - (b*h - 2*g*
c)*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/((b^2 - 4*a*c)*(p + 1)
)), x] - Simp[1/((b^2 - 4*a*c)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d
+ e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
 (2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*
(2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.108.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 16308 vs. $2(611) = 1222$.

Time = 1.61 (sec) , antiderivative size = 16309, normalized size of antiderivative = 24.31

method	result	size
default	Expression too large to display	16309

input
```
int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.108.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^3} dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^3} dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**3,x)
```

output
```
Timed out
```

3.108.7 Maxima [F]

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^3} dx = \int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^3} dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^3, x)
```

3.108.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx + cx^2)^{3/2}}{(d + ex + fx^2)^3} \, dx = \int \frac{(c\,x^2 + b\,x + a)^{3/2}}{(f\,x^2 + e\,x + d)^3} \, dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3, x)
```

3.109 $\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$

3.109.1 Optimal result

Integrand size = 27, antiderivative size = 717

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{(23040c^5d^2e - 3465b^5f^3 + 420b^3cf^2(27be+34af) - 504bc^2f(70abef+22a^2f^2+25b^2(e^2+df)) - 640c^4(\ldots}{7680\ldots}$$

$$+ \frac{(1155b^4f^3 - 252b^2cf^2(15be+14af) + 5760c^4d(e^2+df) + 24c^2f(322abef+50a^2f^2+175b^2(e^2+df))\ldots}{3840c^5}$$

$$- \frac{(231b^3f^3 - 36bcf^2(21be+13af) - 320c^3(e^3+6def) + 24c^2f(32aef+35b(e^2+df)))\,x^2\sqrt{a+bx+cx^2}\ldots}{960c^4}$$

$$+ \frac{f(99b^2f^2 - 4cf(81be+25af) + 360c^2(e^2+df))\,x^3\sqrt{a+bx+cx^2}}{480c^3}$$

$$+ \frac{f^2(36ce-11bf)x^4\sqrt{a+bx+cx^2}}{60c^2} + \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$+ \frac{(1024c^6d^3 + 231b^6f^3 - 252b^4cf^2(3be+5af) - 1536c^5d(bde+a(e^2+df)) + 840b^2c^2f(4abef+2a^2f^2+\ldots}{\ldots}$$

output
```
1/1024*(1024*c^6*d^3+231*b^6*f^3-252*b^4*c*f^2*(5*a*f+3*b*e)-1536*c^5*d*(b
*d*e+a*(d*f+e^2))+840*b^2*c^2*f*(4*a*b*e*f+2*a^2*f^2+b^2*(d*f+e^2))+384*c^
4*(3*b^2*d*(d*f+e^2)+3*a^2*f*(d*f+e^2)+2*a*b*e*(6*d*f+e^2))-320*c^3*(9*a^2
*b*e*f^2+a^3*f^3+9*a*b^2*f*(d*f+e^2)+b^3*(6*d*e*f+e^3)))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)+1/7680*(23040*c^5*d^2*e-3465*b^
5*f^3+420*b^3*c*f^2*(34*a*f+27*b*e)-504*b*c^2*f*(70*a*b*e*f+22*a^2*f^2+25*
b^2*(d*f+e^2))-640*c^4*(27*b*d*(d*f+e^2)+8*a*e*(6*d*f+e^2))+96*c^3*(128*a^
2*e*f^2+275*a*b*f*(d*f+e^2)+50*b^2*(6*d*e*f+e^3)))*(c*x^2+b*x+a)^(1/2)/c^6
+1/3840*(1155*b^4*f^3-252*b^2*c*f^2*(14*a*f+15*b*e)+5760*c^4*d*(d*f+e^2)+2
4*c^2*f*(322*a*b*e*f+50*a^2*f^2+175*b^2*(d*f+e^2))-160*c^3*(27*a*f*(d*f+e^
2)+10*b*(6*d*e*f+e^3)))*x*(c*x^2+b*x+a)^(1/2)/c^5-1/960*(231*b^3*f^3-36*b*
c*f^2*(13*a*f+21*b*e)-320*c^3*(6*d*e*f+e^3)+24*c^2*f*(32*a*e*f+35*b*(d*f+e
^2)))*x^2*(c*x^2+b*x+a)^(1/2)/c^4+1/480*f*(99*b^2*f^2-4*c*f*(25*a*f+81*b*e
)+360*c^2*(d*f+e^2))*x^3*(c*x^2+b*x+a)^(1/2)/c^3+1/60*f^2*(-11*b*f+36*c*e)
*x^4*(c*x^2+b*x+a)^(1/2)/c^2+1/6*f^3*x^5*(c*x^2+b*x+a)^(1/2)/c
```

3.109.2 Mathematica [A] (verified)

Time = 5.00 (sec) , antiderivative size = 618, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^5f^3+210b^3cf^2(54be+68af+11bfx)-168bc^2f(66a^2f^2+42abf(5e+fx)+\cdots}{\cdots}$$

input
```
Integrate[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^5*f^3 + 210*b^3*c*f^2*(54*b*e + 68
*a*f + 11*b*f*x) - 168*b*c^2*f*(66*a^2*f^2 + 42*a*b*f*(5*e + f*x) + b^2*(7
5*e^2 + 75*d*f + 45*e*f*x + 11*f^2*x^2)) + 128*c^5*(90*d^2*(2*e + f*x) + 1
5*d*x*(6*e^2 + 8*e*f*x + 3*f^2*x^2) + x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*
x^2 + 10*f^3*x^3)) + 48*c^3*(2*a^2*f^2*(128*e + 25*f*x) + b^2*(100*e^3 + 1
75*e^2*f*x + 6*e*f*(100*d + 21*f*x^2) + f^2*x*(175*d + 33*f*x^2)) + 2*a*b*
f*(275*e^2 + 161*e*f*x + f*(275*d + 39*f*x^2))) - 64*c^4*(a*(80*e^3 + 135*
e^2*f*x + 96*e*f*(5*d + f*x^2) + 5*f^2*x*(27*d + 5*f*x^2)) + b*(270*d^2*f
+ 15*d*(18*e^2 + 20*e*f*x + 7*f^2*x^2) + x*(50*e^3 + 105*e^2*f*x + 81*e*f^
2*x^2 + 22*f^3*x^3)))) + 15*(1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3
*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*
e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*
f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3
+ 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt
[a] + Sqrt[a + x*(b + c*x)]])])/(7680*c^(13/2))
```

3.109.3 Rubi [A] (verified)

Time = 2.35 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int \frac{f^2(36ce-11bf)x^5-2f(5af^2-18c(e^2+df))x^4+12ce(e^2+6df)x^3+36cd(e^2+df)x^2+36cd^2ex+12cd^3}{2\sqrt{cx^2+bx+a}}dx}{6c}+\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{f^2(36ce-11bf)x^5-2f(5af^2-18c(e^2+df))x^4+12ce(e^2+6df)x^3+36cd(e^2+df)x^2+36cd^2ex+12cd^3}{\sqrt{cx^2+bx+a}}dx}{12c}+\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{\int \frac{f\left(360\left(e^2+df\right)c^2-4f(81be+25af)c+99b^2f^2\right)x^4-8\left(-11abf^3+36acef^2-15c^2\left(e^3+6dfe\right)\right)x^3+360c^2d\left(e^2+df\right)x^2+360c^2d^2ex+120c^2d^3}{2\sqrt{cx^2+bx+a}}dx}{5c}+\frac{f^2x^4\sqrt{a+bx+c\ldots}}{5\ldots}}{12c}$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{f\left(360\left(e^2+df\right)c^2-4f(81be+25af)c+99b^2f^2\right)x^4-8\left(-11abf^3+36acef^2-15c^2\left(e^3+6dfe\right)\right)x^3+360c^2d\left(e^2+df\right)x^2+360c^2d^2ex+120c^2d^3}{\sqrt{cx^2+bx+a}}dx}{10c}+\frac{f^2x^4\sqrt{a+bx+c\ldots}}{5\ldots}}{12c}$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{3\left(320d^3c^3+960d^2exc^3-\left(-320\left(e^3+6dfe\right)c^3+24f\left(32aef+35b\left(e^2+df\right)\right)c^2-36bf^2(21be+13af)c+231b^3f^3\right)x^3-2\left(-480d\left(e^2+df\right)c^3+360af\left(e^2+df\right)c^2-4af^2(81\ldots\right.\right.}{2\sqrt{cx^2+bx+a}}}{4c}\ldots}{10c}\ldots}{12c}$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{320d^3c^3+960d^2exc^3-\left(-320\left(e^3+6dfe\right)c^3+24f\left(32aef+35b\left(e^2+df\right)\right)c^2-36bf^2(21be+13af)c+231b^3f^3\right)x^3-2\left(-480d\left(e^2+df\right)c^3+360af\left(e^2+df\right)c^2-4af^2(81\ldots\right.}{\sqrt{cx^2+bx+a}}}{8c}\ldots}{10c}\ldots}{12c}$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 2192

$$3\left(\frac{\int \frac{1920d^3c^4+\left(1155f^3b^4-252cf^2(15be+14af)b^2+5760c^4d\left(e^2+df\right)+24c^2f\left(175\left(e^2+df\right)b^2+322aefb+50a^2f^2\right)-160c^3\left(27af\left(e^2+df\right)+10b\left(e^3+6dfe\right)\right)\right)x^2+4\left(1\ldots\right.}{2\sqrt{cx^2+bx+a}}}{3c}\ldots\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\big\downarrow$ 27

3.109. $\int \frac{\left(d+ex+fx^2\right)^3}{\sqrt{a+bx+cx^2}}\,dx$

$$3\left(\frac{\frac{\int \frac{1920d^3c^4+\left(1155f^3b^4-252cf^2(15be+14af)b^2+5760c^4d\left(e^2+df\right)+24c^2f\left(175\left(e^2+df\right)b^2+322aefb+50a^2f^2\right)-160c^3\left(27af\left(e^2+df\right)+10b\left(e^3+6dfe\right)\right)\right)x^2+4(1}{\sqrt{cx^2+bx+a}}}{6c}}{}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\downarrow$ 2192

$$3\left(\int \frac{7680d^3c^5-11520ad\left(e^2+df\right)c^4+320a\left(27af\left(e^2+df\right)+10b\left(e^3+6dfe\right)\right)c^3-48af\left(175\left(e^2+df\right)b^2+322aefb+50a^2f^2\right)c^2+504ab^2f^2(15be+14af)c-2310ab^4f^3+(}{}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\downarrow$ 27

$$3\left(\int \frac{2\left(3840d^3c^5-5760ad\left(e^2+df\right)c^4+160a\left(27af\left(e^2+df\right)+10b\left(e^3+6dfe\right)\right)c^3-24af\left(175\left(e^2+df\right)b^2+322aefb+50a^2f^2\right)c^2+252ab^2f^2(15be+14af)c-1155ab^4f^3\right)}{}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\downarrow$ 1160

$$3\left(\frac{\frac{15\left(384c^4\left(3a^2f\left(df+e^2\right)+2abe\left(6df+e^2\right)+3b^2d\left(df+e^2\right)\right)+840b^2c^2f\left(2a^2f^2+4abef+b^2\left(df+e^2\right)\right)-320c^3\left(a^3f^3+9a^2bef^2+9ab^2f\left(df+e^2\right)+b^3\left(6def+e^3\right)\right)-25}{2c}}{}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\downarrow$ 1092

3.109. $\int \frac{\left(d+ex+fx^2\right)^3}{\sqrt{a+bx+cx^2}}\,dx$

$$3\left(\frac{\frac{15\left(384c^4\left(3a^2f\left(df+e^2\right)+2abe\left(6df+e^2\right)+3b^2d\left(df+e^2\right)\right)+840b^2c^2f\left(2a^2f^2+4abef+b^2\left(df+e^2\right)\right)-320c^3\left(a^3f^3+9a^2bef^2+9ab^2f\left(df+e^2\right)+b^3\left(6def+e^3\right)\right)-2\cdots}{c}}{\cdots}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

$\Big\downarrow$ 219

$$3\left(\frac{x\sqrt{a+bx+cx^2}\left(24c^2f\left(50a^2f^2+322abef+175b^2\left(df+e^2\right)\right)-252b^2cf^2(14af+15be)-160c^3\left(27af\left(df+e^2\right)+10b\left(6def+e^3\right)\right)+1155b^4f^3+5760c^4d\left(df+e^2\right)\right)}{2c}+\frac{\sqrt{a}\cdots}{\cdots}\right.$$

$$\frac{f^3x^5\sqrt{a+bx+cx^2}}{6c}$$

input
```
Int[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a
 + b*x + c*x^2])/(5*c) + ((f*(99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c
^2*(e^2 + d*f))*x^3*Sqrt[a + b*x + c*x^2])/(4*c) + (3*(-1/3*((231*b^3*f^3
- 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f*(32*a*
e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/c + (((1155*b^4*f^3 -
252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*f*(322*a
*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 + d*f) +
 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((23040*c^5*d^2*
e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*c^2*f*(70*a*b*e
*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2 + d*f) + 8*a*
e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d*f) + 50*b^2*
(e^3 + 6*d*e*f)))*Sqrt[a + b*x + c*x^2])/c + (15*(1024*c^6*d^3 + 231*b^6*f
^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) +
840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d
*(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a
^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)))/(4*c))/(6*
c)))/(8*c))/(10*c))/(12*c)
```

3.109. $\int \frac{\left(d+ex+fx^2\right)^3}{\sqrt{a+bx+cx^2}}\,dx$

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.109.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{\left(-1280 f^{3} c^{5} x^{5}+1408 b\, c^{4} f^{3} x^{4}-4608 c^{5} e\, f^{2} x^{4}+1600 a\, c^{4} f^{3} x^{3}-1584 b^{2} c^{3} f^{3} x^{3}+5184 b\, c^{4} e\, f^{2} x^{3}-5760 c^{5} d\, f^{2} x^{3}-5760 c^{5} e^{2} f\, x^{3}-374\right.}{}$
default	Expression too large to display

input
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(-1280*c^5*f^3*x^5+1408*b*c^4*f^3*x^4-4608*c^5*e*f^2*x^4+1600*a*c^
4*f^3*x^3-1584*b^2*c^3*f^3*x^3+5184*b*c^4*e*f^2*x^3-5760*c^5*d*f^2*x^3-576
0*c^5*e^2*f*x^3-3744*a*b*c^3*f^3*x^2+6144*a*c^4*e*f^2*x^2+1848*b^3*c^2*f^3
*x^2-6048*b^2*c^3*e*f^2*x^2+6720*b*c^4*d*f^2*x^2+6720*b*c^4*e^2*f*x^2-1536
0*c^5*d*e*f*x^2-2560*c^5*e^3*x^2-2400*a^2*c^3*f^3*x+7056*a*b^2*c^2*f^3*x-1
5456*a*b*c^3*e*f^2*x+8640*a*c^4*d*f^2*x+8640*a*c^4*e^2*f*x-2310*b^4*c*f^3*
x+7560*b^3*c^2*e*f^2*x-8400*b^2*c^3*d*f^2*x-8400*b^2*c^3*e^2*f*x+19200*b*c
^4*d*e*f*x+3200*b*c^4*e^3*x-11520*c^5*d^2*f*x-11520*c^5*d*e^2*x+11088*a^2*
b*c^2*f^3-12288*a^2*c^3*e*f^2-14280*a*b^3*c*f^3+35280*a*b^2*c^2*e*f^2-2640
0*a*b*c^3*d*f^2-26400*a*b*c^3*e^2*f+30720*a*c^4*d*e*f+5120*a*c^4*e^3+3465*
b^5*f^3-11340*b^4*c*e*f^2+12600*b^3*c^2*d*f^2+12600*b^3*c^2*e^2*f-28800*b^
2*c^3*d*e*f-4800*b^2*c^3*e^3+17280*b*c^4*d^2*f+17280*b*c^4*d*e^2-23040*c^5
*d^2*e)*(c*x^2+b*x+a)^(1/2)/c^6-1/1024*(320*a^3*c^3*f^3-1680*a^2*b^2*c^2*f
^3+2880*a^2*b*c^3*e*f^2-1152*a^2*c^4*d*f^2-1152*a^2*c^4*e^2*f+1260*a*b^4*c
*f^3-3360*a*b^3*c^2*e*f^2+2880*a*b^2*c^3*d*f^2+2880*a*b^2*c^3*e^2*f-4608*a
*b*c^4*d*e*f-768*a*b*c^4*e^3+1536*a*c^5*d^2*f+1536*a*c^5*d*e^2-231*b^6*f^3
+756*b^5*c*e*f^2-840*b^4*c^2*d*f^2-840*b^4*c^2*e^2*f+1920*b^3*c^3*d*e*f+32
0*b^3*c^3*e^3-1152*b^2*c^4*d^2*f-1152*b^2*c^4*d*e^2+1536*b*c^5*d^2*e-1024*
c^6*d^3)/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 1583, normalized size of antiderivative = 2.21

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*a*c^5)
*d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c + 1680*
a^2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 48*a^
2*c^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(3*b
^2*c^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 - 12
0*a*b^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 +
 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f^3*x^
5 + 23040*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3
)*x^4 + 320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 +
528*a^2*b*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36
*(10*c^6*d - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (9
45*b^4*c^2 - 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(7
7*b^3*c^3 - 156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)
*f^2 + 120*(16*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15
*b^2*c^4 - 16*a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^
6*d*e^2 - 1600*b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*
f^3 + 12*(10*(35*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^
2 + 120*(48*c^6*d^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sq
rt(c*x^2 + b*x + a))/c^7, -1/15360*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e +
384*(3*b^2*c^4 - 4*a*c^5)*d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (23...
```

3.109.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1787 vs. $2(750) = 1500$.

Time = 0.99 (sec) , antiderivative size = 1787, normalized size of antiderivative = 2.49

$$\int \frac{(d + ex + fx^2)^3}{\sqrt{a + bx + cx^2}}\, dx = \text{Too large to display}$$

input
```
integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f**3*x**5/(6*c) + x**4*(-11*b*f**3/(12*
c) + 3*e*f**2)/(5*c) + x**3*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*
e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(4*c) + x**2*(-4*a*(-11*b*f**3/(12*c
) + 3*e*f**2)/(5*c) - 7*b*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*e*
f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(8*c) + 6*d*e*f + e**3)/(3*c) + x*(-3*
a*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2
+ 3*e**2*f)/(4*c) - 5*b*(-4*a*(-11*b*f**3/(12*c) + 3*e*f**2)/(5*c) - 7*b*(
-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2 + 3
*e**2*f)/(8*c) + 6*d*e*f + e**3)/(6*c) + 3*d**2*f + 3*d*e**2)/(2*c) + (-2*
a*(-4*a*(-11*b*f**3/(12*c) + 3*e*f**2)/(5*c) - 7*b*(-5*a*f**3/(6*c) - 9*b*
(-11*b*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(8*c) + 6*d*e
*f + e**3)/(3*c) - 3*b*(-3*a*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3
*e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(4*c) - 5*b*(-4*a*(-11*b*f**3/(12*c
) + 3*e*f**2)/(5*c) - 7*b*(-5*a*f**3/(6*c) - 9*b*(-11*b*f**3/(12*c) + 3*e*
f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(8*c) + 6*d*e*f + e**3)/(6*c) + 3*d**2
*f + 3*d*e**2)/(4*c) + 3*d**2*e)/c) + (-a*(-3*a*(-5*a*f**3/(6*c) - 9*b*(-1
1*b*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(4*c) - 5*b*(-4*
a*(-11*b*f**3/(12*c) + 3*e*f**2)/(5*c) - 7*b*(-5*a*f**3/(6*c) - 9*b*(-11*b
*f**3/(12*c) + 3*e*f**2)/(10*c) + 3*d*f**2 + 3*e**2*f)/(8*c) + 6*d*e*f + e
**3)/(6*c) + 3*d**2*f + 3*d*e**2)/(2*c) - b*(-2*a*(-4*a*(-11*b*f**3/(12...
```

3.109.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(\frac{10\,f^3x}{c}+\frac{36\,c^5ef^2-11\,bc^4f^3}{c^6}\right)x+\frac{360\,c^5e^2f+360\,c^5df^2-324\,bc^4ef\right.\right.\right.\right.\right.$$
$$-\frac{(1024\,c^6d^3-1536\,bc^5d^2e+1152\,b^2c^4de^2-1536\,ac^5de^2-320\,b^3c^3e^3+768\,abc^4e^3+1152\,b^2c^4d^2f-153}{}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^3*x/c + (36*c^5*e*f^2 - 11*
b*c^4*f^3)/c^6)*x + (360*c^5*e^2*f + 360*c^5*d*f^2 - 324*b*c^4*e*f^2 + 99*
b^2*c^3*f^3 - 100*a*c^4*f^3)/c^6)*x + (320*c^5*e^3 + 1920*c^5*d*e*f - 840*
b*c^4*e^2*f - 840*b*c^4*d*f^2 + 756*b^2*c^3*e*f^2 - 768*a*c^4*e*f^2 - 231*
b^3*c^2*f^3 + 468*a*b*c^3*f^3)/c^6)*x + (5760*c^5*d*e^2 - 1600*b*c^4*e^3 +
 5760*c^5*d^2*f - 9600*b*c^4*d*e*f + 4200*b^2*c^3*e^2*f - 4320*a*c^4*e^2*f
 + 4200*b^2*c^3*d*f^2 - 4320*a*c^4*d*f^2 - 3780*b^3*c^2*e*f^2 + 7728*a*b*c
^3*e*f^2 + 1155*b^4*c*f^3 - 3528*a*b^2*c^2*f^3 + 1200*a^2*c^3*f^3)/c^6)*x
+ (23040*c^5*d^2*e - 17280*b*c^4*d*e^2 + 4800*b^2*c^3*e^3 - 5120*a*c^4*e^3
 - 17280*b*c^4*d^2*f + 28800*b^2*c^3*d*e*f - 30720*a*c^4*d*e*f - 12600*b^3
*c^2*e^2*f + 26400*a*b*c^3*e^2*f - 12600*b^3*c^2*d*f^2 + 26400*a*b*c^3*d*f
^2 + 11340*b^4*c*e*f^2 - 35280*a*b^2*c^2*e*f^2 + 12288*a^2*c^3*e*f^2 - 346
5*b^5*f^3 + 14280*a*b^3*c*f^3 - 11088*a^2*b*c^2*f^3)/c^6) - 1/1024*(1024*c
^6*d^3 - 1536*b*c^5*d^2*e + 1152*b^2*c^4*d*e^2 - 1536*a*c^5*d*e^2 - 320*b^
3*c^3*e^3 + 768*a*b*c^4*e^3 + 1152*b^2*c^4*d^2*f - 1536*a*c^5*d^2*f - 1920
*b^3*c^3*d*e*f + 4608*a*b*c^4*d*e*f + 840*b^4*c^2*e^2*f - 2880*a*b^2*c^3*e
^2*f + 1152*a^2*c^4*e^2*f + 840*b^4*c^2*d*f^2 - 2880*a*b^2*c^3*d*f^2 + 115
2*a^2*c^4*d*f^2 - 756*b^5*c*e*f^2 + 3360*a*b^3*c^2*e*f^2 - 2880*a^2*b*c^3*
e*f^2 + 231*b^6*f^3 - 1260*a*b^4*c*f^3 + 1680*a^2*b^2*c^2*f^3 - 320*a^3*c^
3*f^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(1...
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^3}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{\left(f\,x^2+e\,x+d\right)^3}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(1/2), x)
```

3.110 $\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$

3.110.1 Optimal result

Integrand size = 27, antiderivative size = 316

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2(16aef+9b(e^2+2df)))\sqrt{a+bx+cx^2}}{192c^4}$$
$$+\frac{(35b^2f^2-4cf(20be+9af)+48c^2(e^2+2df))\,x\sqrt{a+bx+cx^2}}{96c^3}$$
$$+\frac{f(16ce-7bf)x^2\sqrt{a+bx+cx^2}}{24c^2}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$
$$+\frac{(128c^4d^2+35b^4f^2-40b^2cf(2be+3af)-64c^3(2bde+a(e^2+2df))+48c^2(4abef+a^2f^2+b^2(e^2+2d}{128c^{9/2}}$$

output
```
1/128*(128*c^4*d^2+35*b^4*f^2-40*b^2*c*f*(3*a*f+2*b*e)-64*c^3*(2*b*d*e+a*(
2*d*f+e^2))+48*c^2*(4*a*b*e*f+a^2*f^2+b^2*(2*d*f+e^2)))*arctanh(1/2*(2*c*x
+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+1/192*(384*c^3*d*e-105*b^3*f^2+20
*b*c*f*(11*a*f+12*b*e)-16*c^2*(16*a*e*f+9*b*(2*d*f+e^2)))*(c*x^2+b*x+a)^(1
/2)/c^4+1/96*(35*b^2*f^2-4*c*f*(9*a*f+20*b*e)+48*c^2*(2*d*f+e^2))*x*(c*x^2
+b*x+a)^(1/2)/c^3+1/24*f*(-7*b*f+16*c*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*f
^2*x^3*(c*x^2+b*x+a)^(1/2)/c
```

3.110.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.80

$$\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}}\, dx$$
$$= \frac{\sqrt{c}\sqrt{a + x(b + cx)}(-105b^3f^2 + 10bcf(24be + 22af + 7bfx) + 16c^3(12d(2e + fx) + x(6e^2 + 8efx + 3f^2x^2)) \ldots}{\ldots}$$

input
```
Integrate[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f^2 + 10*b*c*f*(24*b*e + 22*a*f +
 7*b*f*x) + 16*c^3*(12*d*(2*e + f*x) + x*(6*e^2 + 8*e*f*x + 3*f^2*x^2)) -
8*c^2*(a*f*(32*e + 9*f*x) + b*(18*e^2 + 36*d*f + 20*e*f*x + 7*f^2*x^2))) +
 3*(128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*
e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*A
rcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(192*c^(9/2))
```

3.110.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d + ex + fx^2)^2}{\sqrt{a + bx + cx^2}}\, dx$$
$$\Big\downarrow \text{2192}$$
$$\frac{\int \frac{f(16ce-7bf)x^3-2(3af^2-4c(e^2+2df))x^2+16cdex+8cd^2}{2\sqrt{cx^2+bx+a}}dx}{4c} + \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{f(16ce-7bf)x^3-2(3af^2-4c(e^2+2df))x^2+16cdex+8cd^2}{\sqrt{cx^2+bx+a}}dx}{8c} + \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$
$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\frac{\int \frac{48c^2d^2+\left(48\left(e^2+2df\right)c^2-4f(20be+9af)c+35b^2f^2\right)x^2+4\left(24dec^2-16aefc+7abf^2\right)x}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{fx^2\sqrt{a+bx+cx^2}(16ce-7bf)}{3c}}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{48c^2d^2+\left(48\left(e^2+2df\right)c^2-4f(20be+9af)c+35b^2f^2\right)x^2+4\left(24dec^2-16aefc+7abf^2\right)x}{\sqrt{cx^2+bx+a}}dx}{6c}+\frac{fx^2\sqrt{a+bx+cx^2}(16ce-7bf)}{3c}}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 2192

$$\frac{\frac{\frac{\int \frac{192d^2c^3-96a\left(e^2+2df\right)c^2+8af(20be+9af)c-70ab^2f^2+\left(-105f^2b^3+20cf(12be+11af)b+384c^3de-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)x}{2\sqrt{cx^2+bx+a}}dx}{2c}+x\sqrt{a+bx+cx^2}\left(-4cf(9af+\cdots\right.}{6c}\cdots}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{2\left(96d^2c^3-48a\left(e^2+2df\right)c^2+4af(20be+9af)c-35ab^2f^2\right)+\left(-105f^2b^3+20cf(12be+11af)b+384c^3de-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)x}{\sqrt{cx^2+bx+a}}dx}{4c}+x\sqrt{a+bx+cx^2}\left(-4cf(9a\cdots\right.}{6c}\cdots}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\frac{\frac{3\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2+128c^4d^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}+\sqrt{a+bx+cx^2}\left(-16c^2\left(16aef+9b\left(2df+e^2\cdots\right.\right.\right.}{4c}\cdots}{6c}\cdots}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\frac{\frac{3\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2+128c^4d^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\sqrt{a+bx+cx^2}\left(-16c^2\left(16c\cdots\right.\right.}{4c}\cdots}{6c}\cdots}{8c}+\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

3.110. $\int \frac{\left(d+ex+fx^2\right)^2}{\sqrt{a+bx+cx^2}}\,dx$

$\big\downarrow$ 219

$$\frac{\frac{\frac{\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2+128c^4d^2\right)}{2c^{3/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16c^2\left(16a\right.\right.}{}}{4c}}{6c}}{8c}$$

$$\frac{f^2x^3\sqrt{a+bx+cx^2}}{4c}$$

input
```
Int[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f^2*x^3*Sqrt[a + b*x + c*x^2])/(4*c) + ((f*(16*c*e - 7*b*f)*x^2*Sqrt[a +
b*x + c*x^2])/(3*c) + (((35*b^2*f^2 - 4*c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2
 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((384*c^3*d*e - 105*b^3*f^2 +
 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f + 9*b*(e^2 + 2*d*f)))*Sqrt[
a + b*x + c*x^2])/c + (3*(128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3
*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 +
 b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/(2*c^(3/2)))/(4*c))/(6*c))/(8*c)
```

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.110.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(48f^2c^3x^3-56bc^2f^2x^2+128c^3efx^2-72ac^2f^2x+70b^2cf^2x-160bc^2efx+192c^3dfx+96c^3e^2x+220abcf^2-256ac^2ef-105b^3f^2+240}{192c^4}$
default	$\frac{d^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{\sqrt{c}}+f^2\left(\frac{x^3\sqrt{cx^2+bx+a}}{4c}-\frac{7b\left(\frac{x^2\sqrt{cx^2+bx+a}}{3c}-\frac{5b\left(\frac{x\sqrt{cx^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{cx^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{}\right.}{2c^{\frac{3}{2}}}\right.}{4c}\right.}{6c}\right.}{}\right.$

input
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*c^3*f^2*x^3-56*b*c^2*f^2*x^2+128*c^3*e*f*x^2-72*a*c^2*f^2*x+70*b
^2*c*f^2*x-160*b*c^2*e*f*x+192*c^3*d*f*x+96*c^3*e^2*x+220*a*b*c*f^2-256*a*
c^2*e*f-105*b^3*f^2+240*b^2*c*e*f-288*b*c^2*d*f-144*b*c^2*e^2+384*c^3*d*e)
*(c*x^2+b*x+a)^(1/2)/c^4+1/128*(48*a^2*c^2*f^2-120*a*b^2*c*f^2+192*a*b*c^2
*e*f-128*a*c^3*d*f-64*a*c^3*e^2+35*b^4*f^2-80*b^3*c*e*f+96*b^2*c^2*d*f+48*
b^2*c^2*e^2-128*b*c^3*d*e+128*c^4*d^2)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 637, normalized size of antiderivative = 2.02

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{3\,(128\,c^4d^2 - 128\,bc^3de + 16\,(3\,b^2c^2 - 4\,ac^3)e^2 + (35\,b^4 - 120\,ab^2c + 48\,a^2c^2)f^2 + 16\,(2\,(3\,b^2c^2 - 4\,ac^3)d\right.$$
$$- \frac{3\,(128\,c^4d^2 - 128\,bc^3de + 16\,(3\,b^2c^2 - 4\,ac^3)e^2 + (35\,b^4 - 120\,ab^2c + 48\,a^2c^2)f^2 + 16\,(2\,(3\,b^2c^2 - 4\,ac^3)c}{}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (3
5*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5
*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f^2*x^3 + 384*c
^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7
*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^
4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(
c*x^2 + b*x + a))/c^5, -1/384*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*
c^2 - 4*a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^
2*c^2 - 4*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(-c)*arctan(1/2*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^
4*f^2*x^3 + 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 +
8*(16*c^4*e*f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3
)*e)*f + 2*(48*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c
^3*e)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

3.110.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(318) = 636$.

Time = 0.70 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.03

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{f^2x^3}{4c} + \frac{x^2\left(-\frac{7bf^2}{8c}+2ef\right)}{3c} + \frac{x\left(-\frac{3af^2}{4c}-\frac{5b\left(-\frac{7bf^2}{8c}+2ef\right)}{6c}+2df+e^2\right)}{2c} + \frac{-\frac{2a\left(-\frac{7bf^2}{8c}+2ef\right)}{3c}-\frac{3b\left(-\frac{3af^2}{4c}-\frac{5b\left(-\frac{7bf^2}{8c} \right.}{6}\right.}{4c}}{c} \right. \\ \frac{2\left(\frac{f^2(a+bx)^{\frac{9}{2}}}{9b^4}+\frac{(a+bx)^{\frac{7}{2}}\left(-4af^2+2bef\right)}{7b^4}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(6a^2f^2-6abef+2b^2df+b^2e^2\right)}{5b^4}+\frac{(a+bx)^{\frac{3}{2}}\left(-4a^3f^2+6a^2bef-4ab^2df-2ab^2e^2+2b^3de\right)}{3b^4}+\sqrt{a+bx}\left(a^4f^2\right.\right.}{b} \\ \frac{d^2x+dex^2+\frac{efx^4}{2}+\frac{f^2x^5}{5}+\frac{x^3\cdot\left(2df+e^2\right)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f**2*x**3/(4*c) + x**2*(-7*b*f**2/(8*c)
 + 2*e*f)/(3*c) + x*(-3*a*f**2/(4*c) - 5*b*(-7*b*f**2/(8*c) + 2*e*f)/(6*c)
 + 2*d*f + e**2)/(2*c) + (-2*a*(-7*b*f**2/(8*c) + 2*e*f)/(3*c) - 3*b*(-3*a
*f**2/(4*c) - 5*b*(-7*b*f**2/(8*c) + 2*e*f)/(6*c) + 2*d*f + e**2)/(4*c) +
2*d*e)/c) + (-a*(-3*a*f**2/(4*c) - 5*b*(-7*b*f**2/(8*c) + 2*e*f)/(6*c) + 2
*d*f + e**2)/(2*c) - b*(-2*a*(-7*b*f**2/(8*c) + 2*e*f)/(3*c) - 3*b*(-3*a*f
**2/(4*c) - 5*b*(-7*b*f**2/(8*c) + 2*e*f)/(6*c) + 2*d*f + e**2)/(4*c) + 2*
d*e)/(2*c) + d**2)*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2
*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqr
t(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f**2*(a + b*x)**(9/2)/(9*b**
4) + (a + b*x)**(7/2)*(-4*a*f**2 + 2*b*e*f)/(7*b**4) + (a + b*x)**(5/2)*(6
*a**2*f**2 - 6*a*b*e*f + 2*b**2*d*f + b**2*e**2)/(5*b**4) + (a + b*x)**(3/
2)*(-4*a**3*f**2 + 6*a**2*b*e*f - 4*a*b**2*d*f - 2*a*b**2*e**2 + 2*b**3*d*
e)/(3*b**4) + sqrt(a + b*x)*(a**4*f**2 - 2*a**3*b*e*f + 2*a**2*b**2*d*f +
a**2*b**2*e**2 - 2*a*b**3*d*e + b**4*d**2)/b**4)/b, Ne(b, 0)), ((d**2*x +
d*e*x**2 + e*f*x**4/2 + f**2*x**5/5 + x**3*(2*d*f + e**2)/3)/sqrt(a), True
))
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6f^2x}{c}+\frac{16c^3ef-7bc^2f^2}{c^4}\right)x+\frac{48c^3e^2+96c^3df-80bc^2ef+35b^2cf^2-36c}{c^4}\right.\right.$$
$$-\frac{(128c^4d^2-128bc^3de+48b^2c^2e^2-64ac^3e^2+96b^2c^2df-128ac^3df-80b^3cef+192abc^2ef+35b^4f^2}{128c^{\frac{9}{2}}}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f^2*x/c + (16*c^3*e*f - 7*b*c^2*f^2)/
c^4)*x + (48*c^3*e^2 + 96*c^3*d*f - 80*b*c^2*e*f + 35*b^2*c*f^2 - 36*a*c^2
*f^2)/c^4)*x + (384*c^3*d*e - 144*b*c^2*e^2 - 288*b*c^2*d*f + 240*b^2*c*e*
f - 256*a*c^2*e*f - 105*b^3*f^2 + 220*a*b*c*f^2)/c^4) - 1/128*(128*c^4*d^2
 - 128*b*c^3*d*e + 48*b^2*c^2*e^2 - 64*a*c^3*e^2 + 96*b^2*c^2*d*f - 128*a*
c^3*d*f - 80*b^3*c*e*f + 192*a*b*c^2*e*f + 35*b^4*f^2 - 120*a*b^2*c*f^2 +
48*a^2*c^2*f^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b)
)/c^(9/2)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f\,x^2+e\,x+d)^2}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(1/2), x)
```

3.111 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

3.111.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}+\frac{(8c^2d+3b^2f-4c(be+af))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))/c^(5/2)+1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c
*x^2+b*x+a)^(1/2)/c
```

3.111.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{c}(4ce-3bf+2cfx)\sqrt{a+x(b+cx)}+(8c^2d+3b^2f-4c(be+af))\,\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{4c^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*(4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (8*c^2*d + 3*b^
2*f - 4*c*(b*e + a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x
)])])/(4*c^(5/2))
```

3.111.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \ 2192$$

$$\frac{\int \frac{4cd-2af+(4ce-3bf)x}{2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2(2cd-af)+(4ce-3bf)x}{\sqrt{cx^2+bx+a}}dx}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \ 1160$$

$$\frac{\frac{\left(-4c(af+be)+3b^2f+8c^2d\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \ 1092$$

$$\frac{\frac{\left(-4c(af+be)+3b^2f+8c^2d\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{2c^{3/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

input
```
Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2
])/c + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)))/(4*c)
```

3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.111.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-2cfx+3bf-4ce)\sqrt{c\,x^{2}+bx+a}}{4c^{2}}-\frac{(4acf-3b^{2}f+4bce-8c^{2}d)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{5}{2}}}$
default	$\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+f\left(\frac{x\sqrt{c\,x^{2}+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx-}\right.}{2c^{\frac{3}{2}}}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*c*f*x+3*b*f-4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2-1/8*(4*a*c*f-3*b^2*f+4
*b*c*e-8*c^2*d)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$

$$=\left[-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-}{16\,c^3}\right.$$

$$-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,c^2fx+4\,c^2e-3\,bcf)\sqrt{cx^2+b}}{8\,c^3}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8
*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2
*c^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d -
 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b
*c*f)*sqrt(c*x^2 + b*x + a))/c^3]
```

3.111.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 0.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.95

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \begin{cases} \left(\frac{fx}{2c}+\frac{-\frac{3bf}{4c}+e}{c}\right)\sqrt{a+bx+cx^2}+\left(-\frac{af}{2c}-\frac{b\left(-\frac{3bf}{4c}+e\right)}{2c}+d\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right) \\ \frac{2d\sqrt{a+bx}+\frac{2e\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2f\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} \\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((((f*x/(2*c) + (-3*b*f/(4*c) + e)/c)*sqrt(a + b*x + c*x**2) + (-a
*f/(2*c) - b*(-3*b*f/(4*c) + e)/(2*c) + d)*Piecewise((log(b + 2*sqrt(c)*sq
rt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) +
 x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), ((2*d*sq
rt(a + b*x) + 2*e*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)/b + 2*f*(a**2*sq
rt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2)/b, Ne(b,
0)), ((d*x + e*x**2/2 + f*x**3/3)/sqrt(a), True))
```

3.111.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}+\frac{4\,ce-3\,bf}{c^2}\right)$$
$$-\frac{(8\,c^2d-4\,bce+3\,b^2f-4\,acf)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c + (4*c*e - 3*b*f)/c^2) - 1/8*(8*c^2*d -
 4*b*c*e + 3*b^2*f - 4*a*c*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*sqrt(c) + b))/c^(5/2)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

3.112 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.112.1 Optimal result

Integrand size = 27, antiderivative size = 374

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{2}f\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}f\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
-f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)
^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*
e)*(-4*d*f+e^2)^(1/2))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*
e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+f*arctanh(1/4*(4*a*f-b*(e
+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*
x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/
2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4
*d*f+e^2)^(1/2))^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\text{RootSum}\Big[c^2d-bce+b^2f+2\sqrt{a}ce\#1-4\sqrt{a}bf\#1 -2cd\#1^2+be\#1^2+4af\#1^2-2\sqrt{a}e\#1^3 +d\#1^4\&, \frac{c\log(x)-c\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\#1\right)-\log(x)\#1^2+\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-\right.}{\sqrt{a}ce-2\sqrt{a}bf-2cd\#1+be\#1+4af\#1-3\sqrt{a}e\#1^2+2d\#1^3}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c
*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (c*Log[x]
- c*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - Log[x]*#1^2 + Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(Sqrt[a]*c*e - 2*Sqrt[a]*b*f - 2
*c*d*#1 + b*e*#1 + 4*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ]
```

3.112.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1314, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \; 1314$$

$$\frac{2f\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{2f\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \; 1154$$

$$\frac{4f\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\right.\right.}{\sqrt{cx^2+bx+a}}}{\sqrt{e^2-4df}}$$

$$\frac{4f\int \frac{1}{4\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\right.\right.}{\sqrt{cx^2+bx+a}}}{\sqrt{e^2-4df}}$$

$\downarrow$ 219

$$\frac{\sqrt{2}f\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}f\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e -
Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
 (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
) + (Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
+ Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d
*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]
])
```

3.112.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1314
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[1/(
(b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[2*(c/q)   Int[1/((b +
 q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ
[b^2 - 4*a*c]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(330) = 660$.

Time = 0.98 (sec) , antiderivative size = 761, normalized size of antiderivative = 2.03

method	result
default	$\frac{\sqrt{2}\,\ln\left(\frac{\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2af^2-bef-2cdf+ce^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+bf-ce\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\sqrt{2}\sqrt{\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df-}}{}}}{}\right.}{\sqrt{-4df+e^2}\sqrt{\frac{-bf\sqrt{}}{}}}$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2
)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d
*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*(
(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*
(-e+(-4*d*f+e^2)^(1/2))))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11287 vs. $2(328) = 656$.

Time = 4.01 (sec) , antiderivative size = 11287, normalized size of antiderivative = 30.18

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.112.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.112.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.112.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.113 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx$

3.113.1 Optimal result

Integrand size = 27, antiderivative size = 789

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx$$

$$= \frac{\left(f(be^2-2bdf-aef)-c(e^3-3def)+f(f(be-2af)-c(e^2-2df))\,x\right)\sqrt{a+bx+cx^2}}{(e^2-4df)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(d+ex+fx^2)}$$

$$+\frac{\left(f(2cd-be+2af)(ce-bf)\left(e-\sqrt{e^2-4df}\right)-2f(2c^2d(e^2-4df)+f(3abef-4a^2f^2+b^2(e^2-6df))}{2\sqrt{2}\,(e^2-4df)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{}}$$

$$-\frac{\left(f(2cd-be+2af)(ce-bf)\left(e+\sqrt{e^2-4df}\right)-2f(2c^2d(e^2-4df)+f(3abef-4a^2f^2+b^2(e^2-6df))}{2\sqrt{2}\,(e^2-4df)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{}}$$

output
```
(f*(-a*e*f-2*b*d*f+b*e^2)-c*(-3*d*e*f+e^3)+f*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2
))*x)*(c*x^2+b*x+a)^(1/2)/(-4*d*f+e^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e)
)/(f*x^2+e*x+d)+1/4*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-
b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(2*c^2*d*(-4*d*f+e^2)
+f*(3*a*b*e*f-4*a^2*f^2+b^2*(-6*d*f+e^2))-c*(4*a*f*(-3*d*f+e^2)+b*(-5*d*e*
f+e^3)))+f*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e-(-4*d*f+e^2)^(1/2)))/(-4*d*f+e^
2)^(3/2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/4*arctanh(1/4*(4*a*f-b*(e+
(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x
+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2
))*(-2*f*(2*c^2*d*(-4*d*f+e^2)+f*(3*a*b*e*f-4*a^2*f^2+b^2*(-6*d*f+e^2))-c*
(4*a*f*(-3*d*f+e^2)+b*(-5*d*e*f+e^3)))+f*(2*a*f-b*e+2*c*d)*(-b*f+c*e)*(e+(
-4*d*f+e^2)^(1/2)))/(-4*d*f+e^2)^(3/2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e)
)*2^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2
)
```

3.113. $\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)^2}\,dx$

3.113.2 Mathematica [A] (warning: unable to verify)

Time = 16.76 (sec) , antiderivative size = 1377, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx =$$

$$-\frac{8f^3(a+bx+cx^2)}{(e^2-4df)\left(4af^2-2bf\left(e-\sqrt{e^2-4df}\right)+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+x(b+cx}}$$

$$-\frac{8f^3(a+bx+cx^2)}{(e^2-4df)\left(4af^2-2bf\left(e+\sqrt{e^2-4df}\right)+c\left(e+\sqrt{e^2-4df}\right)^2\right)\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+x(b+cx}}$$

$$+\frac{2\sqrt{2}f^2\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\sqrt{a+bx+cx^2}}\right)}{(e^2-4df)^{3/2}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\sqrt{a+x(b+cx)}}$$

$$-\frac{8\sqrt{2}f^2\sqrt{ce^2-2cdf-bef+2af^2-ce\sqrt{e^2-4df}+bf\sqrt{e^2-4df}}\left(2bf+2c\left(-e+\sqrt{e^2-4df}\right)\right)\sqrt{a+bx}}{(e^2-4df)\left(4af^2+2bf\left(-e+\sqrt{e^2-4df}\right)+c\left(-e+\sqrt{e^2-4df}\right)^2\right)\left(16af^2+8bf\left(-e+\right.\right.}$$

$$-\frac{2\sqrt{2}f^2\sqrt{a+bx+cx^2}\,\mathrm{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}\sqrt{a+bx+cx^2}}\right)}{(e^2-4df)^{3/2}\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}\sqrt{a+x(b+cx)}}$$

$$-\frac{8\sqrt{2}f^2\sqrt{ce^2-2cdf-bef+2af^2+ce\sqrt{e^2-4df}-bf\sqrt{e^2-4df}}\left(-2bf+2c\left(e+\sqrt{e^2-4df}\right)\right)\sqrt{a+bx}}{(e^2-4df)\left(4af^2-2bf\left(e+\sqrt{e^2-4df}\right)+c\left(e+\sqrt{e^2-4df}\right)^2\right)\left(16af^2-8bf\left(e+\sqrt{\right.\right.}}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

output
```
(-8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 -
 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sq
rt[a + x*(b + c*x)]) - (8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 -
 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^
2 - 4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) + (2*Sqrt[2]*f^2*Sqrt[a + b*x +
 c*x^2]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[
e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*
d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e -
Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]) - (8*Sqrt[2]*f^2*Sqrt[c*e^2 -
2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]
*(2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + b*x + c*x^2]*ArcTanh[(-4*
a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*
f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*d*f)*(4*a*f
^2 + 2*b*f*(-e + Sqrt[e^2 - 4*d*f]) + c*(-e + Sqrt[e^2 - 4*d*f])^2)*(16*a*
f^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + x*(b + c*x)]) - (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*f
- b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sq...
```

3.113.3 Rubi [A] (warning: unable to verify)

Time = 1.41 (sec) , antiderivative size = 764, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1305, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}dx$$

$$\Big\downarrow \; 1305$$

$$\frac{\int \frac{-4a^2f^3+3abef^2+b^2\left(e^2-6df\right)f-4ac\left(e^2-3df\right)f+(2cd-be+2af)(ce-bf)xf+2c^2d\left(e^2-4df\right)-bc\left(e^3-5def\right)}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{\left(e^2-4df\right)\left(d+ex+fx^2\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-4a^2f^3+3abef^2+b^2(e^2-6df)f-4ac(e^2-3df)f+(2cd-be+2af)(ce-bf)xf+2c^2d(e^2-4df)-bc(e^3-5def)}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{2(e^2-4df)((cd-af)^2-(bd-ae)(ce-bf))}+$$
$$\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{(e^2-4df)(d+ex+fx^2)((cd-af)^2-(bd-ae)(ce-bf))}$$

↓ 1365

$$\frac{\frac{f\left(\left(\sqrt{e^2-4df}+e\right)(ce-bf)(2af-be+2cd)-2\left(-4a^2f^3+3abef^2-4acf(e^2-3df)+b^2f(e^2-6df)-bc(e^3-5def)+2c^2d(e^2-4df)\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4d\right.}}}{\sqrt{e^2-4df}}}{2(e^2-4}$$
$$\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{(e^2-4df)(d+ex+fx^2)((cd-af)^2-(bd-ae)(ce-bf))}$$

↓ 1154

$$\frac{\frac{2f\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2\left(-4a^2f^3+3abef^2-4acf(e^2-3df)+b^2f(e^2-6df)-bc(e^3-5def)+2c^2d(e^2-4df)\right)\right)\int \frac{}{4\left(4af^2-2b\left(e-\right.\right.}}{\sqrt{e^2-4df}}}{}$$
$$\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{(e^2-4df)(d+ex+fx^2)((cd-af)^2-(bd-ae)(ce-bf))}$$

↓ 219

$$\frac{\frac{f\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2\left(-4a^2f^3+3abef^2-4acf(e^2-3df)+b^2f(e^2-6df)-bc(e^3-5def)+2c^2d(e^2-4df)\right)\right)\text{arctanh}\left(\frac{4a}{2\sqrt{2}\sqrt{a}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}}{}$$
$$\frac{\sqrt{a+bx+cx^2}\left(fx\left(f(be-2af)-c\left(e^2-2df\right)\right)+f\left(-aef-2bdf+be^2\right)-c\left(e^3-3def\right)\right)}{(e^2-4df)(d+ex+fx^2)((cd-af)^2-(bd-ae)(ce-bf))}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

3.113. $\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)^2}\,dx$

output
```
((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c
*(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*((2*c*d - b*e + 2*a*f)*(
c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e
^2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5
*d*e*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 -
4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d
*f]]) - (f*((2*c*d - b*e + 2*a*f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*
(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4
*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[
e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/(2*(e^2 - 4*d*f)*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f)))
```

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2107 vs. $2(735) = 1470$.

Time = 1.13 (sec) , antiderivative size = 2108, normalized size of antiderivative = 2.67

method	result	size
default	Expression too large to display	2108

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(4*d*f-e^2)*(-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-b*e*f-2*c*d*f+c*e^2)*f^2/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*((x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*
f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*2
^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))-1/(4*d
*f-e^2)*(-2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)*f^2/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*((x-1/2/f*(-e+(-4*d*f+
e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2
)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+c*e^2)/f^2)^(1/2)+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*f/(b*f*(-4*d*f+e
^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*2^(1/2)/(...
```

3.113.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

output
```
Timed out
```

3.113.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(fx^2+ex+d)^2}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)^2), x)
```

3.113.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)^2}\,dx=\int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)^2}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^2), x)
```

3.114 $\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx$

3.114.1 Optimal result

Integrand size = 27, antiderivative size = 649

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(3ab^4cef^2-ab^5f^3+ab^3cf(5af^2-3c(e^2+df))-bc^2(c^3d^3+5a^3f^3+3ac^2d(e^2 -}{}$$

$$-\frac{(187b^3f^3-4bcf^2(114be+73af)-64c^3(e^3+6def)+16c^2f(20aef+21b(e^2+df)))\sqrt{a+bx+cx^2}}{64c^5}$$

$$+\frac{f(41b^2f^2-4cf(22be+7af)+48c^2(e^2+df))\,x\sqrt{a+bx+cx^2}}{32c^4}$$

$$+\frac{f^2(8ce-5bf)x^2\sqrt{a+bx+cx^2}}{8c^3}+\frac{f^3x^3\sqrt{a+bx+cx^2}}{4c^2}$$

$$+\frac{3(105b^4f^3-280b^2cf^2(be+af)+128c^4d(e^2+df)+80c^2f(6abef+a^2f^2+3b^2(e^2+df))-64c^3(3af(e^2 +}{128c^{11/2}}$$

output
```
3/128*(105*b^4*f^3-280*b^2*c*f^2*(a*f+b*e)+128*c^4*d*(d*f+e^2)+80*c^2*f*(6
*a*b*e*f+a^2*f^2+3*b^2*(d*f+e^2))-64*c^3*(3*a*f*(d*f+e^2)+b*(6*d*e*f+e^3))
)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+2*(3*a*b^4*c
*e*f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^3*d^3+5*a^3*f^
3+3*a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a*f^2-c*(6*d*f+
e^2))+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*c*f+b^2*f-b*c*
e+2*c^2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*d*f-3*a*c^3*e^
2+b^4*f^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*d^2)*x)/c^5/(-
4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)-1/64*(187*b^3*f^3-4*b*c*f^2*(73*a*f+114*b*e
)-64*c^3*(6*d*e*f+e^3)+16*c^2*f*(20*a*e*f+21*b*(d*f+e^2)))*(c*x^2+b*x+a)^(
1/2)/c^5+1/32*f*(41*b^2*f^2-4*c*f*(7*a*f+22*b*e)+48*c^2*(d*f+e^2))*x*(c*x^
2+b*x+a)^(1/2)/c^4+1/8*f^2*(-5*b*f+8*c*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1/4*
f^3*x^3*(c*x^2+b*x+a)^(1/2)/c^2
```

3.114.2 Mathematica [A] (verified)

Time = 5.80 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\sqrt{c}(315b^6f^3x+105b^5f^2(3af+cx(-8e+fx))-2b^4cf(105af(4e+9fx)+cx(-$$

input
```
Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[c]*(315*b^6*f^3*x + 105*b^5*f^2*(3*a*f + c*x*(-8*e + f*x)) - 2*b^4*c
*f*(105*a*f*(4*e + 9*f*x) + c*x*(-360*e^2 + 140*e*f*x + 3*f*(-120*d + 7*f*
x^2))) + 8*b^3*c*(-210*a^2*f^3 + a*c*f*(90*e^2 + 530*e*f*x + f*(90*d - 77*
f*x^2)) + c^2*x*(-24*e^3 + 30*e^2*f*x + 3*f^2*x*(10*d + f*x^2) + 2*e*f*(-7
2*d + 7*f*x^2))) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3
+ 186*e^2*f*x + 2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*
x*(-24*d^2*f + 6*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4
*e*f^2*x^2 + f^3*x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) +
a^2*c*(16*e^3 + 36*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2
)) + 2*a*c^2*(-12*d^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2
*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8
*c^3*d^2*(d - 3*e*x) + a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^
2)) + 2*a*c^2*(12*d^2*f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3
+ 30*e^2*f*x + 14*e*f^2*x^2 + 3*f^3*x^3)))) - 3*(b^2 - 4*a*c)*(105*b^4*f^3
 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6*a*b*e*f
 + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^3 + 6*d
*e*f)))*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(
b + c*x)])])/(64*c^(11/2)*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)])
```

3.114.3 Rubi [A] (verified)

Time = 2.88 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\right.\right.}{}$$

$$\frac{2\int -\frac{-\frac{\left(b^2-4ac\right)f^3x^4}{c}+\frac{\left(b^2-4ac\right)f^2(3ce-bf)x^3}{c^2}+\frac{\left(b^2-4ac\right)f\left(3\left(e^2+df\right)c^2-f(3be+af)c+b^2f^2\right)x^2}{c^3}-\frac{\left(b^2-4ac\right)\left(-\left(\left(e^3+6dfe\right)c^3\right)+3f\left(aef+b\left(e^2+df\right)\right)c^2-bf\right.}{c^4}}{2\sqrt{cx^2+bx+}}}{b^2-4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\frac{(b^2-4ac)f^3x^4}{c}+\frac{(b^2-4ac)f^2(3ce-bf)x^3}{c^2}+\frac{(b^2-4ac)f\left(3\left(e^2+df\right)c^2-f(3be+af)c+b^2f^2\right)x^2}{c^3}-\frac{(b^2-4ac)\left(-\left(\left(e^3+6dfe\right)c^3\right)+3f\left(aef+b\left(e^2+df\right)\right)c^2-bf^2(3be\cdots}{c^4}}{\sqrt{cx^2+bx+a}}$$

$$\frac{\cdots}{b^2-4ac}$$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\cdots\right.\right.$$

$\downarrow$ 2192

$$\int \frac{\frac{3(b^2-4ac)f^2(8ce-5bf)x^3}{c}+\frac{2(b^2-4ac)f\left(12\left(e^2+df\right)c^2-f(12be+7af)c+4b^2f^2\right)x^2}{c^2}-\frac{8(b^2-4ac)\left(-\left(\left(e^3+6dfe\right)c^3\right)+3f\left(aef+b\left(e^2+df\right)\right)c^2-bf^2(3be+2af)c+\cdots}{c^3}}{2\sqrt{cx^2+bx+a}}$$

$$\frac{\cdots}{4c}$$

$$\frac{\cdots}{b^2\cdots}$$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\cdots\right.\right.$$

$\downarrow$ 27

$$\int \frac{\frac{3(b^2-4ac)f^2(8ce-5bf)x^3}{c}+\frac{2(b^2-4ac)f\left(12\left(e^2+df\right)c^2-f(12be+7af)c+4b^2f^2\right)x^2}{c^2}-\frac{8(b^2-4ac)\left(-\left(\left(e^3+6dfe\right)c^3\right)+3f\left(aef+b\left(e^2+df\right)\right)c^2-bf^2(3be+2af)c+\cdots}{c^3}}{\sqrt{cx^2+bx+a}}$$

$$\frac{\cdots}{8c}$$

$$\frac{\cdots}{b^2\cdots}$$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\cdots\right.\right.$$

$\downarrow$ 2192

$$\int \frac{3\left(\frac{(b^2-4ac)f\left(48\left(e^2+df\right)c^2-4f(22be+7af)c+41b^2f^2\right)x^2}{c}-\frac{4(b^2-4ac)\left(-4\left(e^3+6dfe\right)c^3+4f\left(5aef+3b\left(e^2+df\right)\right)c^2-bf^2(12be+13af)c+4b^3f^3\right)x}{c^2}+\frac{16(b^2-4ac\cdots}{\cdots}\right.}{2\sqrt{cx^2+bx+a}}$$

$$\frac{\cdots}{3c}$$

$$\frac{\cdots}{8c}$$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\cdots\right.\right.$$

$\downarrow$ 27

$$\int \frac{\frac{(b^2-4ac)f\left(48\left(e^2+df\right)c^2-4f(22be+7af)c+41b^2f^2\right)x^2}{c}-\frac{4(b^2-4ac)\left(-4\left(e^3+6dfe\right)c^3+4f\left(5aef+3b\left(e^2+df\right)\right)c^2-bf^2(12be+13af)c+4b^3f^3\right)x}{c^2}+\frac{16(b^2-4ac)(\cdots}{\cdots}}{\sqrt{cx^2+bx+a}}$$

$$\frac{\cdots}{2c}$$

$$\frac{\cdots}{8c}$$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\cdots\right.\right.$$

$\downarrow$ 2192

3.114. $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

$$\int \frac{\left(b^2-4ac\right)\left(2\left(32f^3b^4-cf^2(96be+137af)b^2+96c^4d\left(e^2+df\right)+4c^2f\left(24\left(e^2+df\right)b^2+70aefb+15a^2f^2\right)-16c^3\left(9af\left(e^2+df\right)+2b\left(e^3+6dfe\right)\right)\right)-c\left(-64\left(e^3+6dfe\right)c^3+\right.\right.}{2c^2\sqrt{cx^2+bx+a}}$$

$2c$

$2c$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\right.\right.$$

$\downarrow$ 27

$$\left(b^2-4ac\right)\int \frac{2\left(32f^3b^4-cf^2(96be+137af)b^2+96c^4d\left(e^2+df\right)+4c^2f\left(24\left(e^2+df\right)b^2+70aefb+15a^2f^2\right)-16c^3\left(9af\left(e^2+df\right)+2b\left(e^3+6dfe\right)\right)\right)-c\left(-64\left(e^3+6dfe\right)c^3+\right.}{\sqrt{cx^2+bx+a}}$$

$4c^3$

$2c$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\right.\right.$$

$\downarrow$ 1160

$$\left(b^2-4ac\right)\left(\frac{3}{2}\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+b\left(6def+e^3\right)\right)+105b^4f^3+128c^4d\left(df+e^2\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx-\sqrt{a+bx-}\right.$$

$4c^3$

$2c$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\right.\right.$$

$\downarrow$ 1092

$$\left(b^2-4ac\right)\left(3\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+b\left(6def+e^3\right)\right)+105b^4f^3+128c^4d\left(df+e^2\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+b}}\right.$$

$4c^3$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\right.\right.$$

$\downarrow$ 219

$$\left(b^2-4ac\right)\left(\frac{3\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(80c^2f\left(a^2f^2+6abef+3b^2\left(df+e^2\right)\right)-280b^2cf^2(af+be)-64c^3\left(3af\left(df+e^2\right)+b\left(6def+e^3\right)\right)+105b^4f^3+128c^4d\left(df+e\right.\right.}{2\sqrt{c}}\right.$$

$4c^3$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\right.\right.$$

3.114. $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

input
```
Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) -
b*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f)
) - a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^
2*f^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2
- b*c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^
3*c*e*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(c^5*
(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((b^2 - 4*a*c)*f^3*x^3*Sqrt[a + b*
x + c*x^2])/(4*c^2) + (((b^2 - 4*a*c)*f^2*(8*c*e - 5*b*f)*x^2*Sqrt[a + b*x
 + c*x^2])/c^2 + (((b^2 - 4*a*c)*f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) +
48*c^2*(e^2 + d*f))*x*Sqrt[a + b*x + c*x^2])/(2*c^2) + ((b^2 - 4*a*c)*(-((
187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c
^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*Sqrt[a + b*x + c*x^2]) + (3*(105*b^4*f
^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6*a*b*e
*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^3 + 6
*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[
c])))/(4*c^3))/(2*c))/(8*c))/(b^2 - 4*a*c)
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.114.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 1139, normalized size of antiderivative = 1.76

method	result	size
risch	Expression too large to display	1139
default	Expression too large to display	2299

input
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*c^3*f^3*x^3-40*b*c^2*f^3*x^2+64*c^3*e*f^2*x^2-56*a*c^2*f^3*x+82*b
^2*c*f^3*x-176*b*c^2*e*f^2*x+96*c^3*d*f^2*x+96*c^3*e^2*f*x+292*a*b*c*f^3-3
20*a*c^2*e*f^2-187*b^3*f^3+456*b^2*c*e*f^2-336*b*c^2*d*f^2-336*b*c^2*e^2*f
+384*c^3*d*e*f+64*c^3*e^3)*(c*x^2+b*x+a)^(1/2)/c^5+1/128/c^5*(256*c^5*d^3*
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+224*a^3*c^2*f^3*(2*c*x+b)/(4*a*c
-b^2)/(c*x^2+b*x+a)^(1/2)+374*a*b^4*f^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)-128*a*b*c^3*e^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-912*a^2*
b^2*c*f^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-384*a^2*c^3*d*f^2*(2*c
*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-384*a^2*c^3*e^2*f*(2*c*x+b)/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)+1344*a^2*b*c^2*e*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2)-912*a*b^3*c*e*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+6
72*a*b^2*c^2*d*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+672*a*b^2*c^2
*e^2*f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-768*a*b*c^3*d*e*f*(2*c*x+
b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(240*a^2*c^3*f^3-840*a*b^2*c^2*f^3+1440
*a*b*c^3*e*f^2-576*a*c^4*d*f^2-576*a*c^4*e^2*f+315*b^4*c*f^3-840*b^3*c^2*e
*f^2+720*b^2*c^3*d*f^2+720*b^2*c^3*e^2*f-1152*b*c^4*d*e*f-192*b*c^4*e^3+38
4*c^5*d^2*f+384*c^5*d*e^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+
b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-144*a^2*b*c^2*f^3+384*a^2*c^3*e
*f^2-328*a*b^3*c*f^3+288*a*b^2*c^2*e*f^2+192*a*b*c^3*d*f^2+192*a*b*c^3*...
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1570 vs. $2(621) = 1242$.

Time = 1.21 (sec) , antiderivative size = 3143, normalized size of antiderivative = 4.84

$$\int \frac{(d + ex + fx^2)^3}{(a + bx + cx^2)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/256*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(a*b^3*c^3 - 4*a^2*b*c^4
)*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*f^3 +
8*(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d - 5*(7*a*b^5*c - 40*a^2
*b^3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*c^6)*d*e^2 - 64*(b^3
*c^4 - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64
*a^3*c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 5*(7*b^5*
c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^5 - 4*a*c^6)*d^2
- 24*(b^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)
*e^2)*f)*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^3*c^3 - 4*a^2*b
*c^4)*d*e + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^2)*f + (128*(b
^3*c^4 - 4*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 + 5*(21*b^7 - 1
40*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5*c^2 - 24*a*
b^3*c^3 + 16*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*b^2*c^3)*e)
*f^2 + 16*(8*(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c^4)*d*e +
3*(5*b^5*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(c)*log(-8*c^2*
x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c)
 - 4*(128*b*c^6*d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 -
4*a*c^6)*f^3*x^5 - 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5
)*f^3)*x^4 - 64*(3*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^
3*c^2 + 1808*a^3*b*c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4...
```

3.114.6 Sympy [F]

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)**3/(a + b*x + c*x**2)**(3/2), x)
```

3.114.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1093, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(4\left(\frac{2\,(b^2c^4f^3-4\,ac^5f^3)x}{b^2c^5-4\,ac^6}+\frac{8\,b^2c^4ef^2-32\,ac^5ef^2-3\,b^3c^3f^3+12\,abc^4f^3}{b^2c^5-4\,ac^6}\right)x+\frac{48\,b^2c^4e^2f-192\,ac^5e^2}{}\right.\right.\right.}{}$$
$$-\frac{3\,(128\,c^4de^2-64\,bc^3e^3+128\,c^4d^2f-384\,bc^3def+240\,b^2c^2e^2f-192\,ac^3e^2f+240\,b^2c^2df^2-192\,ac^3df^2-}{128}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/64*(((2*(4*(2*(b^2*c^4*f^3 - 4*a*c^5*f^3)*x/(b^2*c^5 - 4*a*c^6) + (8*b^2
*c^4*e*f^2 - 32*a*c^5*e*f^2 - 3*b^3*c^3*f^3 + 12*a*b*c^4*f^3)/(b^2*c^5 - 4
*a*c^6))*x + (48*b^2*c^4*e^2*f - 192*a*c^5*e^2*f + 48*b^2*c^4*d*f^2 - 192*
a*c^5*d*f^2 - 56*b^3*c^3*e*f^2 + 224*a*b*c^4*e*f^2 + 21*b^4*c^2*f^3 - 104*
a*b^2*c^3*f^3 + 80*a^2*c^4*f^3)/(b^2*c^5 - 4*a*c^6))*x + (64*b^2*c^4*e^3 -
 256*a*c^5*e^3 + 384*b^2*c^4*d*e*f - 1536*a*c^5*d*e*f - 240*b^3*c^3*e^2*f
+ 960*a*b*c^4*e^2*f - 240*b^3*c^3*d*f^2 + 960*a*b*c^4*d*f^2 + 280*b^4*c^2*
e*f^2 - 1376*a*b^2*c^3*e*f^2 + 1024*a^2*c^4*e*f^2 - 105*b^5*c*f^3 + 616*a*
b^3*c^2*f^3 - 784*a^2*b*c^3*f^3)/(b^2*c^5 - 4*a*c^6))*x - (256*c^6*d^3 - 3
84*b*c^5*d^2*e + 384*b^2*c^4*d*e^2 - 768*a*c^5*d*e^2 - 192*b^3*c^3*e^3 + 6
40*a*b*c^4*e^3 + 384*b^2*c^4*d^2*f - 768*a*c^5*d^2*f - 1152*b^3*c^3*d*e*f
+ 3840*a*b*c^4*d*e*f + 720*b^4*c^2*e^2*f - 2976*a*b^2*c^3*e^2*f + 1152*a^2
*c^4*e^2*f + 720*b^4*c^2*d*f^2 - 2976*a*b^2*c^3*d*f^2 + 1152*a^2*c^4*d*f^2
 - 840*b^5*c*e*f^2 + 4240*a*b^3*c^2*e*f^2 - 3904*a^2*b*c^3*e*f^2 + 315*b^6
*f^3 - 1890*a*b^4*c*f^3 + 2704*a^2*b^2*c^2*f^3 - 480*a^3*c^3*f^3)/(b^2*c^5
 - 4*a*c^6))*x - (128*b*c^5*d^3 - 768*a*c^5*d^2*e + 384*a*b*c^4*d*e^2 - 19
2*a*b^2*c^3*e^3 + 512*a^2*c^4*e^3 + 384*a*b*c^4*d^2*f - 1152*a*b^2*c^3*d*e
*f + 3072*a^2*c^4*d*e*f + 720*a*b^3*c^2*e^2*f - 2496*a^2*b*c^3*e^2*f + 720
*a*b^3*c^2*d*f^2 - 2496*a^2*b*c^3*d*f^2 - 840*a*b^4*c*e*f^2 + 3680*a^2*b^2
*c^2*e*f^2 - 2048*a^3*c^3*e*f^2 + 315*a*b^5*f^3 - 1680*a^2*b^3*c*f^3 + ...
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e\,x+d)^3}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2), x)
```

3.115 $\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx$

3.115.1 Optimal result

Integrand size = 27, antiderivative size = 309

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc(c^2d^2-3a^2f^2+ac(e^2+2df))-(2c^4d^2}{c^3(b^2-4}$$
$$+\frac{f(8ce-7bf)\sqrt{a+bx+cx^2}}{4c^3}+\frac{f^2x\sqrt{a+bx+cx^2}}{2c^2}$$
$$+\frac{(15b^2f^2-12cf(2be+af)+8c^2(e^2+2df))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}}$$

output
```
1/8*(15*b^2*f^2-12*c*f*(a*f+2*b*e)+8*c^2*(2*d*f+e^2))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+2*(2*a*b^2*c*e*f-a*b^3*f^2+4*a*c^2*
e*(-a*f+c*d)-b*c*(c^2*d^2-3*a^2*f^2+a*c*(2*d*f+e^2))-(2*c^4*d^2+b^4*f^2-2*
b^2*c*f*(2*a*f+b*e)-2*c^3*(b*d*e+a*(2*d*f+e^2))+c^2*(6*a*b*e*f+2*a^2*f^2+b
^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+1/4*f*(-7*b*f+8*c
*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/2*f^2*x*(c*x^2+b*x+a)^(1/2)/c^2
```

3.115.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-\frac{\sqrt{c}(15b^4f^2x+b^3f(15af+cx(-24e+5fx))+4bc(-13a^2f^2+2c^2d(d-2ex)+ac(2e^2+4df+20efx-5f^2x^2))-2b^2 \cdots}{(b^2-\cdots}}{\cdots}$$

input
```
Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-((Sqrt[c]*(15*b^4*f^2*x + b^3*f*(15*a*f + c*x*(-24*e + 5*f*x)) + 4*b*c*(
-13*a^2*f^2 + 2*c^2*d*(d - 2*e*x) + a*c*(2*e^2 + 4*d*f + 20*e*f*x - 5*f^2*
x^2)) - 2*b^2*c*(a*f*(12*e + 31*f*x) + c*x*(-4*e^2 - 8*d*f + 4*e*f*x + f^2
*x^2)) + 8*c^2*(2*c^2*d^2*x + a^2*f*(8*e + 3*f*x) + a*c*(-4*d*(e + f*x) +
x*(-2*e^2 + 4*e*f*x + f^2*x^2)))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]))
+ (15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(Sqrt[
c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(4*c^(7/2))
```

3.115.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2191, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3a\cdots\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$
$$\frac{2\int -\frac{\frac{\left(b^2-4ac\right)f^2x^2}{c}+\frac{\left(b^2-4ac\right)f(2ce-bf)x}{c^2}+\frac{\left(b^2-4ac\right)\left(\left(e^2+2df\right)c^2-f(2be+af)c+b^2f^2\right)}{c^3}}{2\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{\left(b^2-4ac\right)f^2x^2}{c}+\frac{\left(b^2-4ac\right)f(2ce-bf)x}{c^2}+\frac{\left(b^2-4ac\right)\left(\left(e^2+2df\right)c^2-f(2be+af)c+b^2f^2\right)}{c^3}}{\sqrt{cx^2+bx+a}}dx}{b^2-4ac}+$$
$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 2192

$$\frac{\frac{\int \frac{\left(b^2-4ac\right)\left(2\left(2\left(e^2+2df\right)c^2-f(4be+3af)c+2b^2f^2\right)+cf(8ce-7bf)x\right)}{2c^2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{f^2x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}+$$
$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 27

$$\frac{\frac{\left(b^2-4ac\right)\int \frac{2\left(2\left(e^2+2df\right)c^2-f(4be+3af)c+2b^2f^2\right)+cf(8ce-7bf)x}{\sqrt{cx^2+bx+a}}dx}{4c^3}+\frac{f^2x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}+$$
$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1160

$$\frac{\frac{\left(b^2-4ac\right)\left(\frac{1}{2}\left(-12cf(af+2be)+15b^2f^2+8c^2\left(2df+e^2\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx+f\sqrt{a+bx+cx^2}(8ce-7bf)\right)}{4c^3}+\frac{f^2x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}+$$
$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1092

$$\frac{\frac{\left(b^2-4ac\right)\left(\left(-12cf(af+2be)+15b^2f^2+8c^2\left(2df+e^2\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}+f\sqrt{a+bx+cx^2}(8ce-7bf)\right)}{4c^3}+\frac{f^2x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}+$$
$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 219

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$
$$\frac{\frac{\left(b^2-4ac\right)\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12cf(af+2be)+15b^2f^2+8c^2\left(2df+e^2\right)\right)}{2\sqrt{c}}+f\sqrt{a+bx+cx^2}(8ce-7bf)\right)}{4c^3}+\frac{f^2x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}{2c^2}}{b^2-4ac}$$

3.115. $\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

input
```
Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a
^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*
f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e
^2 + 2*d*f)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((b^2 - 4*a
*c)*f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) + ((b^2 - 4*a*c)*(f*(8*c*e - 7*b*
f)*Sqrt[a + b*x + c*x^2] + ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^
2 + 2*d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqr
t[c])))/(4*c^3))/(b^2 - 4*a*c)
```

3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.115.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{f(-2cfx+7bf-8ce)\sqrt{c\,x^2+bx+a}}{4c^3}-\frac{-\frac{16c^3d^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{8a^2c\,f^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{14a\,b^2f^2(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{16abefc(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+}}}{}$
default	$\frac{2d^2(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+f^2\left(\frac{x^3}{2c\sqrt{c\,x^2+bx+a}}-\frac{5b\left(\frac{x^2}{c\sqrt{c\,x^2+bx+a}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx}}\right)}{2c}\right)}{2c}\right)}{}\right)$

input
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*f*(-2*c*f*x+7*b*f-8*c*e)*(c*x^2+b*x+a)^(1/2)/c^3-1/8/c^3*(-16*c^3*d^2
*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+8*a^2*c*f^2*(2*c*x+b)/(4*a*c-b^
2)/(c*x^2+b*x+a)^(1/2)-14*a*b^2*f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2)+16*a*b*e*f*c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+(12*a*c^2*f^2-1
5*b^2*c*f^2+24*b*c^2*e*f-16*c^3*d*f-8*c^3*e^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1
/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-4*a*b*c*f^2
+16*a*c^2*e*f-7*b^3*f^2+8*b^2*c*e*f-16*c^3*d*e)*(-1/c/(c*x^2+b*x+a)^(1/2)-
b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)))
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(289) = 578$.

Time = 0.76 (sec) , antiderivative size = 1305, normalized size of antiderivative = 4.22

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^
3*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a
^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x
^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*
(b^3*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*
(2*(b^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(c)*log(
-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) -
 4*a*c) + 4*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a
*c^4)*f^2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 - (8*(b^2*c^3 - 4*a*c^4)*e
*f - 5*(b^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a*b*c^3*d - (3*a*b^2*c^2 - 8*
a^2*c^3)*e)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*(b^2*c^3 - 2*a*c^4)*e^2 + (
15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 2*a*c^4)*d - (
3*b^3*c^2 - 10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2
*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2
*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (8*(
b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2 + 8*(
2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 + 8*(2*(a*b^2*
c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3*c^2 - 4*a*b*
c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b^3*c^2 - 4*a
*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(-c)*arctan(1/2*sqrt(c...
```

3.115.6 Sympy [F]

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)**2/(a + b*x + c*x**2)**(3/2), x)
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,(b^2c^2f^2-4\,ac^3f^2)x}{b^2c^3-4\,ac^4} + \frac{8\,b^2c^2ef-32\,ac^3ef-5\,b^3cf^2+20\,abc^2f^2}{b^2c^3-4\,ac^4}\right)x - \frac{16\,c^4d^2-16\,bc^3de+8\,b^2c^2e^2-16\,a\cdots}{\cdots}\right.}{\cdots}$$
$$-\frac{(8\,c^2e^2+16\,c^2df-24\,bcef+15\,b^2f^2-12\,acf^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{7}{2}}}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*(((2*(b^2*c^2*f^2 - 4*a*c^3*f^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^2*e*
f - 32*a*c^3*e*f - 5*b^3*c*f^2 + 20*a*b*c^2*f^2)/(b^2*c^3 - 4*a*c^4))*x -
(16*c^4*d^2 - 16*b*c^3*d*e + 8*b^2*c^2*e^2 - 16*a*c^3*e^2 + 16*b^2*c^2*d*f
 - 32*a*c^3*d*f - 24*b^3*c*e*f + 80*a*b*c^2*e*f + 15*b^4*f^2 - 62*a*b^2*c*
f^2 + 24*a^2*c^2*f^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^2 - 32*a*c^3*d*e
 + 8*a*b*c^2*e^2 + 16*a*b*c^2*d*f - 24*a*b^2*c*e*f + 64*a^2*c^2*e*f + 15*a
*b^3*f^2 - 52*a^2*b*c*f^2)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 1/
8*(8*c^2*e^2 + 16*c^2*d*f - 24*b*c*e*f + 15*b^2*f^2 - 12*a*c*f^2)*log(abs(
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e\,x+d)^2}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2), x)
```

3.116 $\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx$

3.116.1 Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}} + \frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

output
```
f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)+2*(c*(2*a*e-b
*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)
^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(abf+2c^2dx+b^2fx+bc(d-ex)-2ac(e+fx))}{c\,(-b^2+4ac)\,\sqrt{a+x(b+cx)}} + \frac{2f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x)))/(c*(-b
^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (2*f*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] +
Sqrt[a + x*(b + c*x)])])/c^(3/2)
```

3.116.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2f\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{f\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/
(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.116.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.81

method	result
default	$\frac{2d(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+f\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right)$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f*(-x/c/(c*x^2+b*x+a)^(1/2)-
1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+e*(-1/c/(c*x
^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(99) = 198$.

Time = 0.47 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.86

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b\right.}{2\,(ab^2c^2}\right.$$

$$-\frac{((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(bc^2d-2\,ac^2e}{ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^2+(b^3c^2-4\,abc^3)}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f
)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^
2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^
3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^
2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b
*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x
)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2
+ (b^3*c^2 - 4*a*b*c^3)*x)]
```

3.116.6 Sympy [F]

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

3.116.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.06

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(2c^2d-bce+b^2f-2acf)x}{b^2c-4ac^2}+\frac{bcd-2ace+abf}{b^2c-4ac^2}\right)}{\sqrt{cx^2+bx+a}} - \frac{f\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x/(b^2*c - 4*a*c^2) + (b*c*d - 2*a
*c*e + a*b*f)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 12.88 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}} - \frac{e\,(4\,a+2\,b\,x)}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}} + \frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}} + \frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
(f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
 2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c -
 b^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c
 - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

3.117 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

3.117.1 Optimal result

Integrand size = 27, antiderivative size = 666

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2acf}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$-\frac{f\left(c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4d}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2}\right.\right.}}$$
$$+\frac{f\left(c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2}\right.\right.}}$$

output
```
2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d)-c*(-2*a*c*f+b^2*f-b*c*e+2*c^2*
d)*x)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2
)-1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f
+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b
*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(
2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1
/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*
(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2
))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-
2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e
-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-
a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d*f+e^
2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

3.117.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.28 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-2(b^3f+b^2c(-e+fx)+bc(-3af+c(d-ex))+2c^2(cdx+a(e}{}$$

input
```
Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-2*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x
+ a*(e - f*x))) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[c^2*d - b*c*
e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 +
4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*e^2*Log[x]) + c^2*d*f*Lo
g[x] + 2*b*c*e*f*Log[x] - b^2*f^2*Log[x] - a*c*f^2*Log[x] + c^2*e^2*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - c^2*d*f*Log[-Sqrt[a] + Sqrt[a + b
*x + c*x^2] - x*#1] - 2*b*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
1] + b^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*c*f^2*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c*e*f*Log[x]*#1 + 2*Sqr
t[a]*b*f^2*Log[x]*#1 + 2*Sqrt[a]*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2
] - x*#1]*#1 - 2*Sqrt[a]*b*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1
]*#1 + c*e^2*Log[x]*#1^2 - c*d*f*Log[x]*#1^2 - b*e*f*Log[x]*#1^2 + a*f^2*L
og[x]*#1^2 - c*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + c*d
*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + b*e*f*Log[-Sqrt[a]
+ Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a*f^2*Log[-Sqrt[a] + Sqrt[a + b*x +
 c*x^2] - x*#1]*#1^2)/(Sqrt[a]*c*e - 2*Sqrt[a]*b*f - 2*c*d*#1 + b*e*#1 + 4
*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ])/((b^2 - 4*a*c)*(c^2*d^2 - b*c*
d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)]
)
```

3.117.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \text{1305}$$

$$\frac{2\int -\frac{(b^2-4ac)(f(be-af)-c(e^2-df)-f(ce-bf)x)}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

3.117. $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

3.117. $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

3.117. $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

3.117. $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

input
```
Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
$Aborted
```

3.117.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

3.117.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1905 vs. $2(609) = 1218$.

Time = 0.86 (sec) , antiderivative size = 1906, normalized size of antiderivative = 2.86

method	result	size
default	Expression too large to display	1906

input
```
int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*d*f+e^2)^(1/2)*(2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*
(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f
*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^
2)^(1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2)^(
1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4
*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2)-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)*f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)
^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^...
```

3.117.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.117.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx=\int \frac{1}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.118 $\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx$

3.118.1 Optimal result

Integrand size = 27, antiderivative size = 891

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(3ab^4cef^2 - ab^5f^3 + ab^3cf(5af^2 - 3c(e^2+df)) - bc^2(c^3d^3 + 5a^3f^3 + 3ac^2d(e^2 -}{}$$

$$- \frac{2(3b^6cef^2 - b^7f^3 + 3b^5cf(6af^2 - c(e^2+df)) - 3b^3c^2(29a^2f^3 + c^2d(e^2+df) - 10acf(e^2+df)) - 4bc^3(2c}{}$$

$$+ \frac{f^2(12ce-11bf)\sqrt{a+bx+cx^2}}{4c^4} + \frac{f^3x\sqrt{a+bx+cx^2}}{2c^3} + \frac{f(35b^2f^2 - 20cf(3be+af) + 24c^2(e^2+df))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{9/2}}$$

output
```
2/3*(3*a*b^4*c*e*f^2-a*b^5*f^3+a*b^3*c*f*(5*a*f^2-3*c*(d*f+e^2))-b*c^2*(c^
3*d^3+5*a^3*f^3+3*a*c^2*d*(d*f+e^2)-9*a^2*c*f*(d*f+e^2))-a*b^2*c^2*e*(12*a
*f^2-c*(6*d*f+e^2))+2*a*c^3*e*(3*c^2*d^2+3*a^2*f^2-a*c*(6*d*f+e^2))-(-2*a*
c*f+b^2*f-b*c*e+2*c^2*d)*(a^2*c^2*f^2-4*a*b^2*c*f^2+7*a*b*c^2*e*f-2*a*c^3*
d*f-3*a*c^3*e^2+b^4*f^2-2*b^3*c*e*f+b^2*c^2*d*f+b^2*c^2*e^2-b*c^3*d*e+c^4*
d^2)*x)/c^5/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+1/8*f*(35*b^2*f^2-20*c*f*(a*f
+3*b*e)+24*c^2*(d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2
))/c^(9/2)-2/3*(3*b^6*c*e*f^2-b^7*f^3+3*b^5*c*f*(6*a*f^2-c*(d*f+e^2))-3*b^
3*c^2*(29*a^2*f^3+c^2*d*(d*f+e^2)-10*a*c*f*(d*f+e^2))-4*b*c^3*(2*c^3*d^3-2
9*a^3*f^3+3*a*c^2*d*(d*f+e^2)+24*a^2*c*f*(d*f+e^2))-24*a^2*c^4*e*(6*a*f^2-
c*(6*d*f+e^2))-b^4*c^2*e*(42*a*f^2-c*(6*d*f+e^2))+6*b^2*c^3*e*(2*c^2*d^2+2
8*a^2*f^2-a*c*(6*d*f+e^2))-c*(16*c^6*d^3-10*b^6*f^3+3*b^4*c*f^2*(26*a*f+7*
b*e)-24*c^5*d*(b*d*e-a*(d*f+e^2))-6*b^2*c^2*f*(25*a*b*e*f+27*a^2*f^2+2*b^2
*(d*f+e^2))+6*c^4*(b^2*d*(d*f+e^2)-16*a^2*f*(d*f+e^2)-2*a*b*e*(6*d*f+e^2))
+c^3*(240*a^2*b*e*f^2+56*a^3*f^3+84*a*b^2*f*(d*f+e^2)+b^3*(6*d*e*f+e^3)))*
x)/c^5/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)+1/4*f^2*(-11*b*f+12*c*e)*(c*x^2+
b*x+a)^(1/2)/c^4+1/2*f^3*x*(c*x^2+b*x+a)^(1/2)/c^3
```

3.118.2 Mathematica [A] (verified)

Time = 11.98 (sec) , antiderivative size = 872, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx = \frac{-105b^7f^3x^2 - 10b^6f^2x(21af + 2cx(-9e+7fx)) + 6b^4cf(5a^2f(6e+53fx) - 6a\cdots}{\cdots}$$

$$+ \frac{f(35b^2f^2 - 20cf(3be+af) + 24c^2(e^2+df))\log\left(b+2cx+2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{8c^{9/2}}$$

input
```
Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-105*b^7*f^3*x^2 - 10*b^6*f^2*x*(21*a*f + 2*c*x*(-9*e + 7*f*x)) + 6*b^4*c
*f*(5*a^2*f*(6*e + 53*f*x) - 6*a*c*x*(4*e^2 + 4*d*f + 30*e*f*x - 31*f^2*x^
2) + c^2*x^3*(-16*e^2 - 16*d*f + 6*e*f*x + f^2*x^2)) - 3*b^5*f*(35*a^2*f^2
 - 10*a*c*f*x*(12*e + 23*f*x) + c^2*x^2*(24*e^2 + 24*d*f - 80*e*f*x + 7*f^
2*x^2)) - 48*b*c^2*(27*a^4*f^3 - 4*c^4*d^2*x^2*(d - e*x) + a^2*c^2*(-4*d^2
*f + 4*e^3*x - 64*e*f^2*x^3 + 7*f^3*x^4 - 4*d*e*(e - 6*f*x)) - 2*a*c^3*(d^
3 - e^3*x^3 + 3*d*e*x^2*(e - 2*f*x) + 3*d^2*x*(-e + f*x)) - 2*a^3*c*f*(5*e
^2 + 39*e*f*x + f*(5*d - 14*f*x^2))) - 8*b^3*c*(-95*a^3*f^3 + c^3*(d^3 - e
^3*x^3 + 9*d^2*x*(e - f*x) - 3*d*e*x^2*(3*e + 2*f*x)) - 3*a*c^2*f*x^2*(18*
e^2 - 74*e*f*x + f*(18*d + 7*f*x^2)) + 3*a^2*c*f*(3*e^2 + 105*e*f*x + f*(3
*d + 29*f*x^2))) + 32*c^3*(4*c^4*d^3*x^3 + 3*a^4*f^2*(16*e + 5*f*x) + 6*a*
c^3*d*x*(d^2 + e^2*x^2 + d*f*x^2) - 2*a^3*c*(2*e^3 + 9*e^2*f*x + f^2*x*(9*
d - 10*f*x^2) + 12*e*f*(d - 3*f*x^2)) - 3*a^2*c^2*(2*d^2*e + 4*d*f*x^2*(3*
e + 2*f*x) + x^2*(2*e^3 + 8*e^2*f*x - 6*e*f^2*x^2 - f^3*x^3))) - 48*b^2*c^
2*(a^3*f^2*(25*e + 63*f*x) - c^3*d*x*(d^2 + e^2*x^2 + d*x*(-6*e + f*x)) +
a^2*c*f*x*(-21*e^2 - 12*e*f*x + 7*f*(-3*d + 7*f*x^2)) + a*c^2*(d^2*(e - 6*
f*x) - 2*d*x*(3*e^2 - 3*e*f*x + 7*f^2*x^2) + x^2*(e^3 - 14*e^2*f*x + 6*e*f
^2*x^2 + f^3*x^3))))/(12*c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f
*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) + 24*c^2*(e^2 + d*f))*Log[b + 2*c*x +
2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(8*c^(9/2))
```

3.118.3 Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 937, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2191, 27, 2191, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 2191$$

$$\frac{2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\right.\right.}{}$$

$$\frac{2\int \frac{3\left(4a-\frac{b^2}{c}\right)f^3x^4-\frac{3\left(b^2-4ac\right)f^2(3ce-bf)x^3}{c^2}-\frac{3\left(b^2-4ac\right)f\left(3\left(e^2+df\right)c^2-f(3be+af)c+b^2f^2\right)x^2}{c^3}+\frac{3\left(b^2-4ac\right)\left(-\left(\left(e^3+6dfe\right)c^3\right)+3f\left(aef+b\left(e^2+df\right)\right)c^2-\right.}{c^4}}{}}{}$$

$$\Big\downarrow \ 27$$

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^4f^2-2b^3cef+b^2c^2df+b^2\right.\right.$$

$$\int \frac{3\left(4a-\frac{b^2}{c}\right)f^3x^4-\frac{3\left(b^2-4ac\right)f^2(3ce-bf)x^3}{c^2}-\frac{3\left(b^2-4ac\right)f\left(3\left(e^2+df\right)c^2-f(3be+af)c+b^2f^2\right)x^2}{c^3}+\frac{3\left(b^2-4ac\right)\left(-\left(\left(e^3+6dfe\right)c^3\right)+3f\left(aef+b\left(e^2+df\right)\right)c^2-bf\right.}{c^4}}{}$$

$\downarrow$ 2191

$$2\left(-af^3b^5+3acef^2b^4+acf\left(5af^2-3c\left(e^2+df\right)\right)b^3-ac^2e\left(12af^2-c\left(e^2+6df\right)\right)b^2-c^2\left(c^3d^3+3ac^2\left(e^2+df\right)d\right.\right.$$

$$2\left(-f^3b^7+3cef^2b^6+3cf\left(6af^2-c\left(e^2+df\right)\right)b^5-c^2e\left(42af^2-c\left(e^2+6df\right)\right)b^4-3c^2\left(29a^2f^3-10ac\left(e^2+df\right)f+c^2d\left(e^2+df\right)\right)b^3+6c^3e\left(2c^2d^2+28a^2f^2-ac\right.\right.$$

$\downarrow$ 27

$$2\left(-af^3b^5+3acef^2b^4+acf\left(5af^2-3c\left(e^2+df\right)\right)b^3-ac^2e\left(12af^2-c\left(e^2+6df\right)\right)b^2-c^2\left(c^3d^3+3ac^2\left(e^2+df\right)d\right.\right.$$

$$2\left(-f^3b^7+3cef^2b^6+3cf\left(6af^2-c\left(e^2+df\right)\right)b^5-c^2e\left(42af^2-c\left(e^2+6df\right)\right)b^4-3c^2\left(29a^2f^3-10ac\left(e^2+df\right)f+c^2d\left(e^2+df\right)\right)b^3+6c^3e\left(2c^2d^2+28a^2f^2-ac\right.\right.$$

$\downarrow$ 2192

$$2\left(-af^3b^5+3acef^2b^4+acf\left(5af^2-3c\left(e^2+df\right)\right)b^3-ac^2e\left(12af^2-c\left(e^2+6df\right)\right)b^2-c^2\left(c^3d^3+3ac^2\left(e^2+df\right)d\right.\right.$$

$$2\left(-f^3b^7+3cef^2b^6+3cf\left(6af^2-c\left(e^2+df\right)\right)b^5-c^2e\left(42af^2-c\left(e^2+6df\right)\right)b^4-3c^2\left(29a^2f^3-10ac\left(e^2+df\right)f+c^2d\left(e^2+df\right)\right)b^3+6c^3e\left(2c^2d^2+28a^2f^2-ac\right.\right.$$

$\downarrow$ 27

$$2\left(-af^3b^5+3acef^2b^4+acf\left(5af^2-3c\left(e^2+df\right)\right)b^3-ac^2e\left(12af^2-c\left(e^2+6df\right)\right)b^2-c^2\left(c^3d^3+3ac^2\left(e^2+df\right)d\right.\right.$$

$$2\left(-f^3b^7+3cef^2b^6+3cf\left(6af^2-c\left(e^2+df\right)\right)b^5-c^2e\left(42af^2-c\left(e^2+6df\right)\right)b^4-3c^2\left(29a^2f^3-10ac\left(e^2+df\right)f+c^2d\left(e^2+df\right)\right)b^3+6c^3e\left(2c^2d^2+28a^2f^2-ac\right.\right.$$

3.118. $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

$\Big\downarrow$ 1160

$$\frac{2\left(-af^3b^5+3acef^2b^4+acf\left(5af^2-3c\left(e^2+df\right)\right)b^3-ac^2e\left(12af^2-c\left(e^2+6df\right)\right)b^2-c^2\left(c^3d^3+3ac^2\left(e^2+df\right)d\right.}{}$$

$$\frac{2\left(-f^3b^7+3cef^2b^6+3cf\left(6af^2-c\left(e^2+df\right)\right)b^5-c^2e\left(42af^2-c\left(e^2+6df\right)\right)b^4-3c^2\left(29a^2f^3-10ac\left(e^2+df\right)f+c^2d\left(e^2+df\right)\right)b^3+6c^3e\left(2c^2d^2+28a^2f^2-ac\right.}{}$$

$\Big\downarrow$ 1092

$$\frac{2\left(-af^3b^5+3acef^2b^4+acf\left(5af^2-3c\left(e^2+df\right)\right)b^3-ac^2e\left(12af^2-c\left(e^2+6df\right)\right)b^2-c^2\left(c^3d^3+3ac^2\left(e^2+df\right)d\right.}{}$$

$$\frac{2\left(-f^3b^7+3cef^2b^6+3cf\left(6af^2-c\left(e^2+df\right)\right)b^5-c^2e\left(42af^2-c\left(e^2+6df\right)\right)b^4-3c^2\left(29a^2f^3-10ac\left(e^2+df\right)f+c^2d\left(e^2+df\right)\right)b^3+6c^3e\left(2c^2d^2+28a^2f^2-ac\right.}{}$$

$\Big\downarrow$ 219

$$\frac{2\left(-af^3b^5+3acef^2b^4+acf\left(5af^2-3c\left(e^2+df\right)\right)b^3-ac^2e\left(12af^2-c\left(e^2+6df\right)\right)b^2-c^2\left(c^3d^3+3ac^2\left(e^2+df\right)d\right.}{}$$

$$\frac{2\left(-f^3b^7+3cef^2b^6+3cf\left(6af^2-c\left(e^2+df\right)\right)b^5-c^2e\left(42af^2-c\left(e^2+6df\right)\right)b^4-3c^2\left(29a^2f^3-10ac\left(e^2+df\right)f+c^2d\left(e^2+df\right)\right)b^3+6c^3e\left(2c^2d^2+28a^2f^2-ac\right.}{}$$

input
```
Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

3.118. $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

output
```
(2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) -
b*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f)
) - a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^
2*f^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2
- b*c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^
3*c*e*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(3*c^
5*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - ((2*(3*b^6*c*e*f^2 - b^7*f^3 +
3*b^5*c*f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 +
 d*f) - 10*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*
d*(e^2 + d*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 +
 6*d*f)) - b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2
 + 28*a^2*f^2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*
f^2*(7*b*e + 26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*
a*b*e*f + 27*a^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*
a^2*f*(e^2 + d*f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3
*f^3 + 84*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x))/(c^5*(b^2 - 4*a*
c)*Sqrt[a + b*x + c*x^2]) - (3*(((b^2 - 4*a*c)^2*f^3*x*Sqrt[a + b*x + c*x^
2])/(2*c^3) + ((b^2 - 4*a*c)^2*f*(f*(12*c*e - 11*b*f)*Sqrt[a + b*x + c*x^2
] + ((35*b^2*f^2 - 20*c*f*(3*b*e + a*f) + 24*c^2*(e^2 + d*f))*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c])))/(4*c^4)))/(b^...
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.118. $\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx$

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3250 vs. 2(865) = 1730.

Time = 1.57 (sec) , antiderivative size = 3251, normalized size of antiderivative = 3.65

method	result	size
default	Expression too large to display	3251
risch	Expression too large to display	19191

input
```
int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d^3*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2
*c*x+b)/(c*x^2+b*x+a)^(1/2))+f^3*(1/2*x^5/c/(c*x^2+b*x+a)^(3/2)-7/4*b/c*(x
^4/c/(c*x^2+b*x+a)^(3/2)-5/2*b/c*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(
-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(
-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(
c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+
b*x+a)^(1/2))))+1/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^
(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))-4*a/c*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b
/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b
/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*
c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x
+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3
/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3
/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))))-5/2*a/c*(-1/3*x
^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2
*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2...
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1996 vs. $2(865) = 1730$.

Time = 4.62 (sec) , antiderivative size = 3995, normalized size of antiderivative = 4.48

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5*(7*b^6*c^2 -
 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4*c^4 - 8*a*b
^2*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e)*f^2)*
x^4 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7*a^2*b^6 -
 60*a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b
^3*c^4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3
 - 64*a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d - 5*
(b^6*c^2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*e)*f^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^8 -
46*a*b^6*c + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(2*(
b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*b*
c^4)*e)*f^2)*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f
+ 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*
(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2
+ 16*a^3*b^2*c^3)*e)*f^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sq
rt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(192*a^2*b*c^5*d*e^2
- 128*a^3*c^5*e^3 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12
*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 1
6*a^2*b*c^5)*f^3)*x^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + ...
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d + ex + fx^2)^3}{(a + bx + cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.118.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1392, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/12*((((3*(2*(b^4*c^3*f^3 - 8*a*b^2*c^4*f^3 + 16*a^2*c^5*f^3)*x/(b^4*c^4
- 8*a*b^2*c^5 + 16*a^2*c^6) + (12*b^4*c^3*e*f^2 - 96*a*b^2*c^4*e*f^2 + 192
*a^2*c^5*e*f^2 - 7*b^5*c^2*f^3 + 56*a*b^3*c^3*f^3 - 112*a^2*b*c^4*f^3)/(b^
4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 4*(32*c^7*d^3 - 48*b*c^6*d^2*e + 12
*b^2*c^5*d*e^2 + 48*a*c^6*d*e^2 + 2*b^3*c^4*e^3 - 24*a*b*c^5*e^3 + 12*b^2*
c^5*d^2*f + 48*a*c^6*d^2*f + 12*b^3*c^4*d*e*f - 144*a*b*c^5*d*e*f - 24*b^4
*c^3*e^2*f + 168*a*b^2*c^4*e^2*f - 192*a^2*c^5*e^2*f - 24*b^4*c^3*d*f^2 +
168*a*b^2*c^4*d*f^2 - 192*a^2*c^5*d*f^2 + 60*b^5*c^2*e*f^2 - 444*a*b^3*c^3
*e*f^2 + 768*a^2*b*c^4*e*f^2 - 35*b^6*c*f^3 + 279*a*b^4*c^2*f^3 - 588*a^2*
b^2*c^3*f^3 + 160*a^3*c^4*f^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 3
*(64*b*c^6*d^3 - 96*b^2*c^5*d^2*e + 24*b^3*c^4*d*e^2 + 96*a*b*c^5*d*e^2 -
16*a*b^2*c^4*e^3 - 64*a^2*c^5*e^3 + 24*b^3*c^4*d^2*f + 96*a*b*c^5*d^2*f -
96*a*b^2*c^4*d*e*f - 384*a^2*c^5*d*e*f - 24*b^5*c^2*e^2*f + 144*a*b^3*c^3*
e^2*f - 24*b^5*c^2*d*f^2 + 144*a*b^3*c^3*d*f^2 + 60*b^6*c*e*f^2 - 360*a*b^
4*c^2*e*f^2 + 192*a^2*b^2*c^3*e*f^2 + 768*a^3*c^4*e*f^2 - 35*b^7*f^3 + 230
*a*b^5*c*f^3 - 232*a^2*b^3*c^2*f^3 - 448*a^3*b*c^3*f^3)/(b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6))*x + 6*(8*b^2*c^5*d^3 + 32*a*c^6*d^3 - 12*b^3*c^4*d^2*e
 - 48*a*b*c^5*d^2*e + 48*a*b^2*c^4*d*e^2 - 32*a^2*b*c^4*e^3 + 48*a*b^2*c^4
*d^2*f - 192*a^2*b*c^4*d*e*f - 24*a*b^4*c^2*e^2*f + 168*a^2*b^2*c^3*e^2*f
- 96*a^3*c^4*e^2*f - 24*a*b^4*c^2*d*f^2 + 168*a^2*b^2*c^3*d*f^2 - 96*a^...
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^3}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e\,x+d)^3}{(c\,x^2+b\,x+a)^{5/2}}\,dx$$

input
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2), x)
```

3.119 $\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx$

3.119.1 Optimal result

Integrand size = 27, antiderivative size = 444

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(2ab^2cef - ab^3f^2 + 4ac^2e(cd-af) - bc(c^2d^2 - 3a^2f^2 + ac(e^2+2df)) - (2c^4d^2}{3c^3(b^2-4} - \frac{2(2b^4cef + 48a^2c^3ef - b^5f^2 + 4b^2c^2e(2cd-3af) + b^3c(10af^2 - c(e^2+2df)) - 4bc^2(2c^2d^2 + 8a^2f^2 + ac(}{3c^3(b^2-} + \frac{f^2\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

output
```
2/3*(2*a*b^2*c*e*f-a*b^3*f^2+4*a*c^2*e*(-a*f+c*d)-b*c*(c^2*d^2-3*a^2*f^2+a
*c*(2*d*f+e^2))-(2*c^4*d^2+b^4*f^2-2*b^2*c*f*(2*a*f+b*e)-2*c^3*(b*d*e+a*(2
*d*f+e^2))+c^2*(6*a*b*e*f+2*a^2*f^2+b^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)/
(c*x^2+b*x+a)^(3/2)+f^2*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
/c^(5/2)-2/3*(2*b^4*c*e*f+48*a^2*c^3*e*f-b^5*f^2+4*b^2*c^2*e*(-3*a*f+2*c*d
)+b^3*c*(10*a*f^2-c*(2*d*f+e^2))-4*b*c^2*(2*c^2*d^2+8*a^2*f^2+a*c*(2*d*f+e
^2))-2*c*(8*c^4*d^2-2*b^4*f^2+b^2*c*f*(14*a*f+b*e)-c^3*(8*b*d*e-4*a*(2*d*f
+e^2))-c^2*(12*a*b*e*f+16*a^2*f^2-b^2*(2*d*f+e^2)))*x)/c^3/(-4*a*c+b^2)^2/
(c*x^2+b*x+a)^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(-3b^5f^2x^2-2b^4f^2x(3a+2cx^2)+4bc(5a^3f^2+2c^3dx^2(3d-2ex)+2a^2c(e^2+2c}{}$$
$$+\frac{2f^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(-3*b^5*f^2*x^2 - 2*b^4*f^2*x*(3*a + 2*c*x^2) + 4*b*c*(5*a^3*f^2 + 2*c^
3*d*x^2*(3*d - 2*e*x) + 2*a^2*c*(e^2 + 2*d*f - 6*e*f*x) + 3*a*c^2*(d - e*x
)*(d + x*(-e + 2*f*x))) + b^3*(-3*a^2*f^2 + 18*a*c*f^2*x^2 + c^2*(-d^2 + 6
*d*x*(-e + f*x) + e*x^2*(3*e + 2*f*x))) + 8*c^2*(2*c^3*d^2*x^3 - a^3*f*(4*
e + 3*f*x) + a*c^2*x*(3*d^2 + e^2*x^2 + 2*d*f*x^2) - 2*a^2*c*(d*e + f*x^2*
(3*e + 2*f*x))) + 2*b^2*c*(21*a^2*f^2*x + c^2*x*(3*d^2 + e^2*x^2 + 2*d*x*(
-6*e + f*x)) - 2*a*c*(d*(e - 6*f*x) + x*(-3*e^2 + 3*e*f*x - 7*f^2*x^2)))))
/(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (2*f^2*ArcTanh[(Sqrt[c]
*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/c^(5/2)
```

3.119.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2191, 27, 2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc(-3a}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$\frac{2\int \frac{3\left(4a-\frac{b^2}{c}\right)f^2x^2-\frac{3\left(b^2-4ac\right)f(2ce-bf)x}{c^2}+\frac{f^2b^4-cf(2be+af)b^2+8c^4d^2-c^3\left(8bde-4a\left(e^2+2df\right)\right)-c^2\left(4a^2f^2-b^2\left(e^2+2df\right)\right)}{c^3}}{2(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 27

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\int \frac{\frac{f^2b^4}{c^3}-\frac{f(2be+af)b^2}{c^2}-8deb+8cd^2+3\left(4a-\frac{b^2}{c}\right)f^2x^2+4a(e^2+2df)-\frac{4a^2f^2-b^2(e^2+2df)}{c}-\frac{3(b^2-4ac)f(2ce-bf)x}{c^2}}{(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}$$

$\Big\downarrow$ 2191

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-2cx(-c^2(16a^2f^2+12abef-(b^2(2df+e^2)))+b^2cf(14af+be)-c^3(8bde-4a(2df+e^2))-2b^4f^2+8c^4d^2)-4bc^2(8a^2f^2+ac(2df+e^2)+2c^2d^2)+48a}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\Big\downarrow$ 27

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-2cx(-c^2(16a^2f^2+12abef-(b^2(2df+e^2)))+b^2cf(14af+be)-c^3(8bde-4a(2df+e^2))-2b^4f^2+8c^4d^2)-4bc^2(8a^2f^2+ac(2df+e^2)+2c^2d^2)+48a}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\Big\downarrow$ 1092

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-2cx(-c^2(16a^2f^2+12abef-(b^2(2df+e^2)))+b^2cf(14af+be)-c^3(8bde-4a(2df+e^2))-2b^4f^2+8c^4d^2)-4bc^2(8a^2f^2+ac(2df+e^2)+2c^2d^2)+48a}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

$\Big\downarrow$ 219

$$\frac{2(-x(c^2(2a^2f^2+6abef+b^2(2df+e^2))-2b^2cf(2af+be)-2c^3(a(2df+e^2)+bde)+b^4f^2+2c^4d^2)-bc(-3a}{3c^3(b^2-4ac)(a+bx+cx^2)^{3/2}}$$

$$\frac{\frac{2(-2cx(-c^2(16a^2f^2+12abef-(b^2(2df+e^2)))+b^2cf(14af+be)-c^3(8bde-4a(2df+e^2))-2b^4f^2+8c^4d^2)-4bc^2(8a^2f^2+ac(2df+e^2)+2c^2d^2)+48a}{c^3(b^2-4ac)\sqrt{a+bx+cx^2}}}{3(b^2-4ac)}$$

input
```
Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

3.119. $\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx$

output
```
(2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a
^2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*
f) - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e
^2 + 2*d*f)))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - ((2*(2*b
^4*c*e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*
(10*a*f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 +
 2*d*f)) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*
d*e - 4*a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f
)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (3*(b^2 - 4*a*c)*f^2*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2))/(3*(b^2 -
4*a*c))
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.119.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1128 vs. 2(428) = 856.

Time = 1.00 (sec) , antiderivative size = 1129, normalized size of antiderivative = 2.54

method	result	size
default	Expression too large to display	1129

input
```
int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d^2*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2
*c*x+b)/(c*x^2+b*x+a)^(1/2))+f^2*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(
-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(
-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a
)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(
c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+
b*x+a)^(1/2))))+1/c*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^
(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c
*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+2*e*f*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b
/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b
/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*
c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x
+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3
/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3
/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2))))+2*d*e*(-1/3/c/(c
*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1
6/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+(2*d*f+e^2)*(-1/2*x/c/
(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2...
```

3.119.5 Fricas [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 1581, normalized size of antiderivative = 3.56

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a
^4*c^2)*f^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b
*c^5*d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)
*f^2 + (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*
c^3 - 12*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*
a^3*b*c^2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^
2 - (b^5*c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3
+ 4*a^2*c^4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3
*e^2 + (b^2*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*
a^2*b^2*c^2 + 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(
c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8
*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^
3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^
4 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^
4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3)*f^2*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^
4 - 8*a^3*b^2*c + 16*a^4*c^2)*f^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x...
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.119.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{f^2\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{5}{2}}} + \frac{2\left(\left(\left(\frac{2\,(8\,c^5d^2-8\,bc^4de+b^2c^3e^2+4\,ac^4e^2+2\,b^2c^3df+8\,ac^4df+b^3c^2ef-12\,abc^3ef-2\,b^4cf^2+14\,ab^2c^2f^2-16\,a^2c^3f^2)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4} + \frac{3\,(8\,bc^4d^2-8\,b^2c^3de+b}{}\right.\right.\right.}{}$$

input
```
integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
-f^2*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2) +
 2/3*(((2*(8*c^5*d^2 - 8*b*c^4*d*e + b^2*c^3*e^2 + 4*a*c^4*e^2 + 2*b^2*c^3
*d*f + 8*a*c^4*d*f + b^3*c^2*e*f - 12*a*b*c^3*e*f - 2*b^4*c*f^2 + 14*a*b^2
*c^2*f^2 - 16*a^2*c^3*f^2)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b
*c^4*d^2 - 8*b^2*c^3*d*e + b^3*c^2*e^2 + 4*a*b*c^3*e^2 + 2*b^3*c^2*d*f + 8
*a*b*c^3*d*f - 4*a*b^2*c^2*e*f - 16*a^2*c^3*e*f - b^5*f^2 + 6*a*b^3*c*f^2)
/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(b^2*c^3*d^2 + 4*a*c^4*d^2 -
b^3*c^2*d*e - 4*a*b*c^3*d*e + 2*a*b^2*c^2*e^2 + 4*a*b^2*c^2*d*f - 8*a^2*b*
c^2*e*f - a*b^4*f^2 + 7*a^2*b^2*c*f^2 - 4*a^3*c^2*f^2)/(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4))*x - (b^3*c^2*d^2 - 12*a*b*c^3*d^2 + 4*a*b^2*c^2*d*e + 1
6*a^2*c^3*d*e - 8*a^2*b*c^2*e^2 - 16*a^2*b*c^2*d*f + 32*a^3*c^2*e*f + 3*a^
2*b^3*f^2 - 20*a^3*b*c*f^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 +
 b*x + a)^(3/2)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex+fx^2)^2}{(a+bx+cx^2)^{5/2}}\,dx=\int \frac{{\left(f\,x^2+e\,x+d\right)}^2}{{\left(c\,x^2+b\,x+a\right)}^{5/2}}\,dx$$

input
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2), x)
```

3.120 $\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx$

3.120.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-(2c^2d-bce+b^2f-2acf)\,x\right)}{3c\,(b^2-4ac)\,(a+bx+cx^2)^{3/2}} + \frac{2\left(8cd-4be+4af+\frac{b^2f}{c}\right)(b+2cx)}{3\,(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$

output
```
2/3*(c*(2*a*e-b*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2
)/(c*x^2+b*x+a)^(3/2)+2/3*(8*c*d-4*b*e+4*a*f+b^2*f/c)*(2*c*x+b)/(-4*a*c+b^
2)^2/(c*x^2+b*x+a)^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{-2b^3(d+3x(e-fx))+16c(-a^2e+2c^2dx^3+acx(3d+fx^2))-4b^2(a(e-6fx)}{3\,(b^2-4ac)^2\,(a+}$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*b^3*(d + 3*x*(e - f*x)) + 16*c*(-(a^2*e) + 2*c^2*d*x^3 + a*c*x*(3*d +
f*x^2)) - 4*b^2*(a*(e - 6*f*x) - c*x*(3*d - 6*e*x + f*x^2)) + 8*b*(2*a^2*f
 - 2*c^2*x^2*(-3*d + e*x) + 3*a*c*(d - e*x + f*x^2)))/(3*(b^2 - 4*a*c)^2*(
a + x*(b + c*x))^(3/2))
```

3.120.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2191, 27, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}-\frac{2\int \frac{\frac{fb^2}{c}-4eb+8cd+4af}{2(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}-\frac{\left(4af+\frac{b^2f}{c}-4be+8cd\right)\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c(b^2-4ac)(a+bx+cx^2)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/
(3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f +
(b^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

3.120.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.34

method	result
trager	$\frac{\frac{16}{3}a\,c^2 f\,x^3+\frac{4}{3}b^2 c f\,x^3-\frac{16}{3}b\,c^2 e\,x^3+\frac{32}{3}c^3 d\,x^3+8abcf\,x^2+2b^3 f\,x^2-8b^2 ce\,x^2+16b\,c^2 d\,x^2+8a\,b^2 fx-8abcex+16xa\,c^2 d-2b^3 ex+4b^2 cdx}{\left(4ac-b^2\right)^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}$
gosper	$\frac{\frac{16}{3}a\,c^2 f\,x^3+\frac{4}{3}b^2 c f\,x^3-\frac{16}{3}b\,c^2 e\,x^3+\frac{32}{3}c^3 d\,x^3+8abcf\,x^2+2b^3 f\,x^2-8b^2 ce\,x^2+16b\,c^2 d\,x^2+8a\,b^2 fx-8abcex+16xa\,c^2 d-2b^3 ex+4b^2 cdx}{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}\left(16a^2c^2-8a\,b^2c+b^4\right)}$
default	$d\left(\frac{\frac{4cx}{3}+\frac{2b}{3}}{(4ac-b^2)(c\,x^2+bx+a)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3(4ac-b^2)^2\sqrt{c\,x^2+bx+a}}\right)+f\left(-\frac{x}{2c(c\,x^2+bx+a)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{4cx}{3}}{\left(4ac-b^2\right)\left(c\right.}\right.}{}\right)}{}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(8*a*c^2*f*x^3+2*b^2*c*f*x^3-8*b*c^2*e*x^3+16*c^3*d*x^3+12*a*b*c*f*x^2
+3*b^3*f*x^2-12*b^2*c*e*x^2+24*b*c^2*d*x^2+12*a*b^2*f*x-12*a*b*c*e*x+24*a*
c^2*d*x-3*b^3*e*x+6*b^2*c*d*x+8*a^2*b*f-8*a^2*c*e-2*a*b^2*e+12*a*b*c*d-b^3
*d)/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(3/2)
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(121) = 242$.

Time = 1.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.18

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\,(8\,a^2bf + 2\,(8\,c^3d - 4\,bc^2e + (b^2c + 4\,ac^2)f)x^3 + 3\,(8\,bc^2d - 4\,b^2ce + (b^3 + 4\,abc}{3\,(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^4 + 2\,(b}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(8*a^2*b*f + 2*(8*c^3*d - 4*b*c^2*e + (b^2*c + 4*a*c^2)*f)*x^3 + 3*(8*
b*c^2*d - 4*b^2*c*e + (b^3 + 4*a*b*c)*f)*x^2 - (b^3 - 12*a*b*c)*d - 2*(a*b
^2 + 4*a^2*c)*e + 3*(4*a*b^2*f + 2*(b^2*c + 4*a*c^2)*d - (b^3 + 4*a*b*c)*e
)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*x)
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.120.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.79

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\bigg(\bigg(\bigg(\frac{2(8c^3d-4bc^2e+b^2cf+4ac^2f)x}{b^4-8ab^2c+16a^2c^2}+\frac{3(8bc^2d-4b^2ce+b^3f+4abcf)}{b^4-8ab^2c+16a^2c^2}\bigg)x+\frac{3(2b^2cd+8ac^2d-b^3e-4ab}{b^4-8ab^2c+16a^2c}}{3(cx^2+bx+a)^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

output
```
2/3*(((2*(8*c^3*d - 4*b*c^2*e + b^2*c*f + 4*a*c^2*f)*x/(b^4 - 8*a*b^2*c +
16*a^2*c^2) + 3*(8*b*c^2*d - 4*b^2*c*e + b^3*f + 4*a*b*c*f)/(b^4 - 8*a*b^2
*c + 16*a^2*c^2))*x + 3*(2*b^2*c*d + 8*a*c^2*d - b^3*e - 4*a*b*c*e + 4*a*b
^2*f)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x - (b^3*d - 12*a*b*c*d + 2*a*b^2*e
+ 8*a^2*c*e - 8*a^2*b*f)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)
^(3/2)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.34

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\,(8\,f\,a^2\,b-8\,e\,a^2\,c+12\,f\,a\,b^2\,x-2\,e\,a\,b^2+12\,f\,a\,b\,c\,x^2-12\,e\,a\,b\,c\,x+12\,d\,a\,b\,c\,\cdots}{}$$

input

```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

output

```
(2*(16*c^3*d*x^3 - b^3*d + 3*b^3*f*x^2 - 2*a*b^2*e + 8*a^2*b*f - 8*a^2*c*e
 - 3*b^3*e*x + 24*a*c^2*d*x + 12*a*b^2*f*x + 6*b^2*c*d*x + 24*b*c^2*d*x^2
- 12*b^2*c*e*x^2 + 8*a*c^2*f*x^3 - 8*b*c^2*e*x^3 + 2*b^2*c*f*x^3 + 12*a*b*
c*d - 12*a*b*c*e*x + 12*a*b*c*f*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)
^(3/2))
```

3.121 $\int \frac{1}{\sqrt{-7+2x+5x^2}(8+12x+5x^2)}\,dx$

3.121.1 Optimal result

Integrand size = 27, antiderivative size = 51

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \frac{1}{10}\arctan\left(\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right) + \frac{1}{5}\operatorname{arctanh}\left(\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right)$$

output
```
1/10*arctan(5/2*(2+x)/(5*x^2+2*x-7)^(1/2))+1/5*arctanh(5*(1+x)/(5*x^2+2*x-
7)^(1/2))
```

3.121.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \frac{1}{10}\arctan\left(\frac{5+\frac{5x}{2}}{\sqrt{-7+2x+5x^2}}\right) + \frac{1}{5}\operatorname{arctanh}\left(\frac{5+5x}{\sqrt{-7+2x+5x^2}}\right)$$

input
```
Integrate[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

output
```
ArcTan[(5 + (5*x)/2)/Sqrt[-7 + 2*x + 5*x^2]]/10 + ArcTanh[(5 + 5*x)/Sqrt[-
7 + 2*x + 5*x^2]]/5
```

3.121.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1317, 27, 1362, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx$$

$$\Big\downarrow \text{1317}$$

$$\frac{1}{50}\int -\frac{50(x+1)}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx-\frac{1}{50}\int -\frac{50(x+2)}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x+2}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx-\int \frac{x+1}{\sqrt{5x^2+2x-7}\,(5x^2+12x+8)}dx$$

$$\Big\downarrow \text{1362}$$

$$-32\int \frac{1}{\frac{6400(x+1)^2}{5x^2+2x-7}-256}d\frac{8(x+1)}{\sqrt{5x^2+2x-7}}-8\int \frac{1}{\frac{400(x+2)^2}{5x^2+2x-7}+64}d\left(-\frac{2(x+2)}{\sqrt{5x^2+2x-7}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{10}\arctan\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)-32\int \frac{1}{\frac{6400(x+1)^2}{5x^2+2x-7}-256}d\frac{8(x+1)}{\sqrt{5x^2+2x-7}}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{10}\arctan\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\text{arctanh}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

input
```
Int[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

output
```
ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sq
rt[-7 + 2*x + 5*x^2]]/5
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

rule 1362
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g*(g*b - 2*a*h)   Subst[I
nt[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[
g*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b
, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && Ne
Q[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f
), 0]
```

3.121.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(41) = 82$.

Time = 0.77 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.82

method	result
default	$-\frac{\sqrt{-\frac{4(2+x)^2}{(-1-x)^2}+9}\left(2\operatorname{arctanh}\left(\frac{\sqrt{-\frac{4(2+x)^2}{(-1-x)^2}+9}}{5}\right)+\arctan\left(\frac{5\sqrt{-\frac{4(2+x)^2}{(-1-x)^2}+9}\,(2+x)}{2\left(\frac{4(2+x)^2}{(-1-x)^2}-9\right)(-1-x)}\right)\right)}{10\sqrt{-\frac{\frac{4(2+x)^2}{(-1-x)^2}-9}{\left(1+\frac{2+x}{-1-x}\right)^2}}\left(1+\frac{2+x}{-1-x}\right)}$
trager	$\operatorname{RootOf}\left(80\_Z^2-16\_Z+1\right)\ln\left(-\frac{-129600\operatorname{RootOf}\left(80\_Z^2-16\_Z+1\right)^2x+8750\sqrt{5x^2+2x-7}\operatorname{RootOf}\left(80\_Z^2-1\right.}{2}\right.$

input
```
int(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-4*(2+x)^2/(-1-x)^2+9)^(1/2)*(2*arctanh(1/5*(-4*(2+x)^2/(-1-x)^2+9)
^(1/2))+arctan(5/2*(-4*(2+x)^2/(-1-x)^2+9)^(1/2)/(4*(2+x)^2/(-1-x)^2-9)*(2
+x)/(-1-x)))/(-(4*(2+x)^2/(-1-x)^2-9)/(1+(2+x)/(-1-x))^2)^(1/2)/(1+(2+x)/(
-1-x))
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.02

$$\begin{aligned}\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx &= \frac{1}{20}\arctan\left(\frac{27\,x^2+20\,\sqrt{5\,x^2+2\,x-7}(x+2)+36\,x}{31\,x^2+16\,x-56}\right)\\ &\quad+\frac{1}{20}\arctan\left(-\frac{27\,x^2-20\,\sqrt{5\,x^2+2\,x-7}(x+2)+36\,x}{31\,x^2+16\,x-56}\right)\\ &\quad+\frac{1}{20}\log\left(\frac{15\,x^2+5\,\sqrt{5\,x^2+2\,x-7}(x+1)+26\,x+9}{x^2}\right)\\ &\quad-\frac{1}{20}\log\left(\frac{15\,x^2-5\,\sqrt{5\,x^2+2\,x-7}(x+1)+26\,x+9}{x^2}\right)\end{aligned}$$

input
```
integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="fricas")
```

output
```
1/20*arctan((27*x^2 + 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)/(31*x^2 + 1
6*x - 56)) + 1/20*arctan(-(27*x^2 - 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*
x)/(31*x^2 + 16*x - 56)) + 1/20*log((15*x^2 + 5*sqrt(5*x^2 + 2*x - 7)*(x +
 1) + 26*x + 9)/x^2) - 1/20*log((15*x^2 - 5*sqrt(5*x^2 + 2*x - 7)*(x + 1)
+ 26*x + 9)/x^2)
```

3.121.6 Sympy [F]

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \int \frac{1}{\sqrt{(x-1)\,(5x+7)}\,(5x^2+12x+8)}\,dx$$

input
```
integrate(1/(5*x**2+12*x+8)/(5*x**2+2*x-7)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(5*x + 7))*(5*x**2 + 12*x + 8)), x)
```

3.121.7 Maxima [F]

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \int \frac{1}{(5\,x^2+12\,x+8)\sqrt{5\,x^2+2\,x-7}}\,dx$$

input
```
integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((5*x^2 + 12*x + 8)*sqrt(5*x^2 + 2*x - 7)), x)
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(41) = 82$.

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.02

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = -\frac{1}{10}\arctan\left(-\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5\,x^2+2\,x-7}+5}{2\,(\sqrt{5}+5)}\right) - \frac{1}{10}\arctan\left(\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5\,x^2+2\,x-7}-5}{2\,(\sqrt{5}-5)}\right) + \frac{1}{10}\log\left(5\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x-7}\right)^2+2\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x-7}\right)\left(6\sqrt{5}+5\right) + 20\sqrt{5}+65\right) - \frac{1}{10}\log\left(5\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x-7}\right)^2 + 2\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x-7}\right)\left(6\sqrt{5}-5\right)-20\sqrt{5}+65\right)$$

input
```
integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="giac")
```

output
```
-1/10*arctan(-1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2*x - 7) + 5)/
(sqrt(5) + 5)) - 1/10*arctan(1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 +
 2*x - 7) - 5)/(sqrt(5) - 5)) + 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x -
 7))^2 + 2*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) + 5) + 20*sqrt(5
) + 65) - 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))^2 + 2*(sqrt(5)*x
- sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) - 5) - 20*sqrt(5) + 65)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\,(8+12x+5x^2)}\,dx = \int \frac{1}{\sqrt{5\,x^2+2\,x-7}\,(5\,x^2+12\,x+8)}\,dx$$

input
```
int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)),x)
```

output
```
int(1/((2*x + 5*x^2 - 7)^(1/2)*(12*x + 5*x^2 + 8)), x)
```

3.122 $\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx$

3.122.1 Optimal result

Integrand size = 29, antiderivative size = 1432

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx =$$

$$-\frac{\sqrt[4]{b^2d+b\left(\sqrt{b^2-4ac}\,d-ae\right)-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{\right.}}}{}$$

output
```
-(cos(2*arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))
^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^
(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(
1/2))^(1/2)))^2)^(1/2)/cos(2*arctan((2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e
)*(-4*a*c+b^2)^(1/2))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b^2*d+b*
(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/4)/(b
+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2)))*EllipticF(sin(2*arctan((2*c^2*d-b*c*e+b
^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(
1/2)))^(1/2)/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c
+b^2)^(1/2)))^(1/4)/(b+2*c*x+(-4*a*c+b^2)^(1/2))^(1/2))),1/2*(2+(2*a*f-b*e
+2*c*d)*(b+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c
*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/2)/(2*c^2*d+b*f*(b+(-4*a*c+b^2)^(1/2))-
c*(b*e+2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/2))^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1
/2))^(3/2)*(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b
^2)^(1/2)))^(1/4)*(2*a+x*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*((f*x^2+e*x+d)*(4*a
*c-(b+(-4*a*c+b^2)^(1/2))^2)^2/(b+2*c*x+(-4*a*c+b^2)^(1/2))^2/(4*a^2*f-2*a
*e*(b+(-4*a*c+b^2)^(1/2))+d*(b+(-4*a*c+b^2)^(1/2))^2))^(1/2)*(1+(2*a+x*(b+
(-4*a*c+b^2)^(1/2)))*(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^
(1/2))^(1/2)/(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b^2*d+b*(-a*e+d*(-4*a*c+b^2)^(1
/2))-a*(2*c*d-2*a*f+e*(-4*a*c+b^2)^(1/2)))^(1/2))*((1+(2*a+x*(b+(-4*a*c...
```

3.122.2 Mathematica [A] (warning: unable to verify)

Time = 4.80 (sec) , antiderivative size = 670, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = -\frac{\left(-b+\sqrt{b^2-4ac}-2cx\right)\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(e+\sqrt{e^2-4df}+2fx\right)}{\left(\left(b+\sqrt{b^2-4ac}\right)f-c\left(e+\sqrt{e^2-4df}\right)\right)\left(-b+\sqrt{b^2-4ac}-2cx\right)}}\sqrt{-\frac{c}{}}}{\left(\left(-b+\sqrt{b^2-4ac}\right)f+c\,(}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

output
```
-(((-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[-
((c*Sqrt[b^2 - 4*a*c]*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(((b + Sqrt[b^2 - 4
*a*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*
Sqrt[-((c*(4*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 - 4*d*f] - 2*Sqrt[b^2 - 4*a*
c]*f*x + 2*c*Sqrt[e^2 - 4*d*f]*x - e*(Sqrt[b^2 - 4*a*c] + 2*c*x) + b*(-e +
 Sqrt[e^2 - 4*d*f] + 2*f*x)))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e
^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*EllipticF[ArcSin[Sqrt[((
(-b + Sqrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))*(
-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]], (2*c*d - b*e + 2*a*f - Sqrt[b^2 - 4*a*
c]*Sqrt[e^2 - 4*d*f])/(2*c*d - b*e + 2*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 -
4*d*f])])/(((-b + Sqrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*Sqrt[(
c*Sqrt[b^2 - 4*a*c]*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(((b + Sqrt[b^2 - 4*
a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*S
qrt[a + x*(b + c*x)]*Sqrt[d + x*(e + f*x)]))
```

3.122.3 Rubi [A] (warning: unable to verify)

Time = 1.72 (sec) , antiderivative size = 1432, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1323, 1280, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}dx$$

$$\Big\downarrow \text{1323}$$

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{x\left(\sqrt{b^2-4ac}+b\right)+2a}\int \frac{1}{\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{fx^2+ex+d}}dx}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1280}$$

$$-\frac{2\left(\sqrt{b^2-4ac}+b+2cx\right)^{3/2}\sqrt{x\left(\sqrt{b^2-4ac}+b\right)+2a}\sqrt{\frac{\left(4ac-\left(\sqrt{b^2-4ac}+b\right)^2\right)^2(d+ex+fx^2)}{\left(\sqrt{b^2-4ac}+b+2cx\right)^2\left(4a^2f+d\left(\sqrt{b^2-4ac}+b\right)^2-2ae\left(\sqrt{b^2-4ac}+b\right)\right)}}}{\left(4ac-\left(\sqrt{b^2-4ac}\right.\right.}$$

$\downarrow$ 1416

$$-\frac{\sqrt[4]{db^2+\left(\sqrt{b^2-4ac}d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}e-2af\right)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right.}}{}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

output
```
-(((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e
 - 2*a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqr
t[b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^2*(d + e*x +
f*x^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*
a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - b*c*e + b^
2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*
c])*x))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2
 - 4*a*c]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - ((b + Sqr
t[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(
(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e -
2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 -
 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x
)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2
*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f
 - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])
*x))/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4
*a*c]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[
((2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)
*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x])/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d -
 a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*Sqrt[b + Sqrt[b^...
```

3.122. $\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx$

3.122.3.1 Defintions of rubi rules used

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b + r + 2
*c*x]*(Sqrt[2*a + (b + r)*x]/Sqrt[a + b*x + c*x^2])   Int[1/(Sqrt[b + r + 2
*c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.122.4 Maple [A] (warning: unable to verify)

Time = 5.22 (sec) , antiderivative size = 905, normalized size of antiderivative = 0.63

method	result
elliptic	$\frac{2\sqrt{(c\,x^2+bx+a)(f\,x^2+ex+d)}\left(\frac{-b+\sqrt{-4ac+b^2}}{2c}+\frac{e+\sqrt{-4df+e^2}}{2f}\right)\sqrt{\frac{\left(-\frac{e+\sqrt{-4df+e^2}}{2f}+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\left(x-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)}{\left(-\frac{e+\sqrt{-4df+e^2}}{2f}-\frac{-b+\sqrt{-4ac+b^2}}{2c}\right)\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)}}\left(x+\frac{b+\sqrt{-}}{}\right.}{\sqrt{c\,x^2+bx+a}\,\sqrt{f\,x^2+ex+d}\,\left(\right.}$
default	$\frac{8\left(2bcf\,x^2-2e\,x^2c^2-2c^2x^2\sqrt{-4df+e^2}-2cf\,x^2\sqrt{-4ac+b^2}+8acfx-2bcex-2bcx\sqrt{-4df+e^2}-2cex\sqrt{-4ac+b^2}-2cx\sqrt{-4df+e^2}\,\sqrt{-4a}\right.}{}$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((c*x^2+b*x+a)*(f*x^2+e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1
/2)*(1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*((-1/2*(e
+(-4*d*f+e^2)^(1/2))/f+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(-1/2*(e+(-4*d*f+e^2)^(1/2))/f-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^
2*((-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x-1/2/f*
(-e+(-4*d*f+e^2)^(1/2)))/(1/2/f*(-e+(-4*d*f+e^2)^(1/2))-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((-1/2*(b+(-4*a*c+b^2
)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
/(-1/2*(e+(-4*d*f+e^2)^(1/2))/f-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(x+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(-1/2*(e+(-4*d*f+e^2)^(1/2))/f+1/2*(b+(-4*a*c
+b^2)^(1/2))/c)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))/(c*f*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))^(1/2
)*EllipticF(((-1/2*(e+(-4*d*f+e^2)^(1/2))/f+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*
(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(e+(-4*d*f+e^2)^(1/2))/f-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-1/2*(b+(-
4*a*c+b^2)^(1/2))/c-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))*(1/2/c*(-b+(-4*a*c+b^2)
^(1/2))+1/2*(e+(-4*d*f+e^2)^(1/2))/f)/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))-1/2/f
*(-e+(-4*d*f+e^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+1/2*(e+(-4*d*f...
```

3.122.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)/(c*f*x^4 + (c*e + b*f
)*x^3 + (c*d + b*e + a*f)*x^2 + a*d + (b*d + a*e)*x), x)
```

3.122.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

3.122.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

3.122.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)^(1/2)), x)
```

3.123 $\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx$

3.123.1 Optimal result

Integrand size = 29, antiderivative size = 652

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx$$

$$= \frac{\sqrt{\frac{23}{11}}\left(1-i\sqrt{23}-4x\right)\sqrt{-1+i\sqrt{23}+4x}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(11i-\sqrt{23}\right)(2+3x+5x^2)}{\left(7i+\sqrt{23}\right)\left(1-i\sqrt{23}-4x\right)^2}}\left(1-\frac{\sqrt{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\left(6-\left(1-\cdots\right.\right.}{1-i\sqrt{23}-4x\cdots}\cdots\right.}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\sqrt{3-x+2x^2}\sqrt{\cdots}}$$

output
```
1/11*(cos(2*arctan(((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)*(6-x*(1-I*23^(1/
2)))^(1/2)/(-1+4*x+I*23^(1/2))^(1/2)))^2)^(1/2)/cos(2*arctan(((-3*I+23^(1/
2))/(7*I+23^(1/2)))^(1/4)*(6-x*(1-I*23^(1/2)))^(1/2)/(-1+4*x+I*23^(1/2))^(
1/2)))*EllipticF(sin(2*arctan(((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)*(6-x*
(1-I*23^(1/2)))^(1/2)/(-1+4*x+I*23^(1/2))^(1/2))),1/22*11^(1/2)*((66*I-22*
23^(1/2)+41*(-23*(3*I-23^(1/2))/(7*I+23^(1/2)))^(1/2)+41*I*((-3*I+23^(1/2)
)/(7*I+23^(1/2)))^(1/2))/(3*I-23^(1/2)))^(1/2))*253^(1/2)*(1-4*x-I*23^(1/2
))*(6-x*(1-I*23^(1/2)))^(1/2)*(-1+4*x+I*23^(1/2))^(1/2)*(1-(6-x*(1-I*23^(1
/2)))*((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/2)/(1-4*x-I*23^(1/2)))*((5*x^2+3
*x+2)*(11*I-23^(1/2))/(1-4*x-I*23^(1/2))^2/(7*I+23^(1/2)))^(1/2)*((11-11*(
6-x*(1-I*23^(1/2)))^2*(3*I-23^(1/2))/(1-4*x-I*23^(1/2))^2/(7*I+23^(1/2))-4
1*(6-x*(1-I*23^(1/2)))*(23^(1/2)+I)/(1-4*x-I*23^(1/2))/(7*I+23^(1/2)))/(1-
(6-x*(1-I*23^(1/2)))*((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/2)/(1-4*x-I*23^(1
/2)))^2)^(1/2)/(23+I*23^(1/2))/((-3*I+23^(1/2))/(7*I+23^(1/2)))^(1/4)/(2*x
^2-x+3)^(1/2)/(5*x^2+3*x+2)^(1/2)/(11-11*(6-x*(1-I*23^(1/2)))^2*(3*I-23^(1
/2))/(1-4*x-I*23^(1/2))^2/(7*I+23^(1/2))-41*(6-x*(1-I*23^(1/2)))*(23^(1/2)
+I)/(1-4*x-I*23^(1/2))/(7*I+23^(1/2)))^(1/2)
```

3.123.2 Mathematica [A] (warning: unable to verify)

Time = 2.25 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx$$

$$= \frac{\left(1+i\sqrt{23}-4x\right)\left(3i+\sqrt{31}+10ix\right)\sqrt{\frac{6i-2\sqrt{31}+20ix}{\left(11i+5\sqrt{23}-2\sqrt{31}\right)\left(-i+\sqrt{23}+4ix\right)}}\sqrt{\frac{63-3i\sqrt{23}-i\sqrt{31}-\sqrt{713}+\left(-22-10i\sqrt{23}+4i\sqrt{31}\cdots\right.}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(-i+\sqrt{23}+4ix\right)}}\cdots}{\left(-11i+5\sqrt{23}-2\sqrt{31}\right)\sqrt{\frac{3i+\sqrt{31}+10ix}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(-i+\cdots\right.}}\cdots}$$

input
```
Integrate[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
((1 + I*Sqrt[23] - 4*x)*(3*I + Sqrt[31] + (10*I)*x)*Sqrt[(6*I - 2*Sqrt[31]
 + (20*I)*x)/((11*I + 5*Sqrt[23] - 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]
*Sqrt[(63 - (3*I)*Sqrt[23] - I*Sqrt[31] - Sqrt[713] + (-22 - (10*I)*Sqrt[2
3] + (4*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] +
 (4*I)*x))]*EllipticF[ArcSin[Sqrt[2]*Sqrt[-((-63 + (3*I)*Sqrt[23] + I*Sqrt
[31] + Sqrt[713] + 2*(11 + (5*I)*Sqrt[23] - (2*I)*Sqrt[31])*x)/((11*I + 5*
Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x)))]], (1197 + 41*Sqrt[713]
)/484])/((-11*I + 5*Sqrt[23] - 2*Sqrt[31])*Sqrt[(3*I + Sqrt[31] + (10*I)*x
)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*Sqrt[3 - x
 + 2*x^2]*Sqrt[2 + 3*x + 5*x^2])
```

3.123.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1323, 1280, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}}dx$$

$$\Big\downarrow \text{1323}$$

$$\frac{\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\int \frac{1}{\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{5x^2+3x+2}}dx}{\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1280}$$

$$\frac{2\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(-\sqrt{23}+11i\right)\left(5x^2+3x+2\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\int \frac{1}{\sqrt{-\frac{\left(3i-\sqrt{23}\right)\left(6-\left(1-i\sqrt{23}\right)x\right)^2}{\left(7i+\sqrt{23}\right)\left(4x+i\sqrt{23}-1\right)^2}\cdots}}}{\left(23+i\sqrt{23}\right)\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(-\sqrt{23}+11i\right)(5x^2+3x+2)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\left(1+\frac{\sqrt{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\left(6-\left(1-i\sqrt{23}\right.\right.}{4x+i\sqrt{23}-1}\right.}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\sqrt{2x^2-x+3}\sqrt{5x^2}}$$

input
```
Int[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

output
```
(Sqrt[23/11]*(1 - I*Sqrt[23] - 4*x)*Sqrt[-1 + I*Sqrt[23] + 4*x]*Sqrt[6 - (
1 - I*Sqrt[23])*x]*Sqrt[((11*I - Sqrt[23])*(2 + 3*x + 5*x^2))/((7*I + Sqrt
[23])*(1 - I*Sqrt[23] - 4*x)^2)]*(1 + (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt
[23]))]*(6 - (1 - I*Sqrt[23])*x))/(-1 + I*Sqrt[23] + 4*x))*Sqrt[(11 + (41*
(I + Sqrt[23])*(6 - (1 - I*Sqrt[23])*x))/((7*I + Sqrt[23])*(-1 + I*Sqrt[23
] + 4*x)) - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[
23])*(-1 + I*Sqrt[23] + 4*x)^2))/(1 + (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt
[23]))]*(6 - (1 - I*Sqrt[23])*x))/(-1 + I*Sqrt[23] + 4*x))^2]*EllipticF[2*
ArcTan[((-((3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[6 - (1 - I*Sqrt[
23])*x])/Sqrt[-1 + I*Sqrt[23] + 4*x]], (44 - (41*(I + Sqrt[23]))/Sqrt[11 +
 I*Sqrt[23]])/88])/((23 + I*Sqrt[23])*(-((3*I - Sqrt[23])/(7*I + Sqrt[23])
))^(1/4)*Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]*Sqrt[11 + (41*(I + Sqrt
[23])*(6 - (1 - I*Sqrt[23])*x))/((7*I + Sqrt[23])*(-1 + I*Sqrt[23] + 4*x))
 - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(-1
+ I*Sqrt[23] + 4*x)^2)])
```

3.123.3.1 Defintions of rubi rules used

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b + r + 2
*c*x]*(Sqrt[2*a + (b + r)*x]/Sqrt[a + b*x + c*x^2])   Int[1/(Sqrt[b + r + 2
*c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.123.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.61

method	result
elliptic	$-\frac{i\sqrt{(2x^2-x+3)(5x^2+3x+2)}\left(\frac{11}{20}-\frac{i\sqrt{23}}{4}-\frac{i\sqrt{31}}{10}\right)\sqrt{\frac{\left(-\frac{11}{20}+\frac{i\sqrt{31}}{10}-\frac{i\sqrt{23}}{4}\right)\left(x-\frac{1}{4}+\frac{i\sqrt{23}}{4}\right)}{\left(-\frac{11}{20}+\frac{i\sqrt{31}}{10}+\frac{i\sqrt{23}}{4}\right)\left(x-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)}}\left(x-\frac{1}{4}-\frac{i\sqrt{23}}{4}\right)^2\sqrt{\frac{i\sqrt{23}\left(x+\frac{3}{10}+\frac{i\sqrt{31}}{10}\right.}{\left(-\frac{11}{20}-\frac{i\sqrt{31}}{10}+\frac{i\sqrt{23}}{4}\right)\left(x-\right.}}}{115\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}\left(-\frac{11}{20}+\frac{i\sqrt{31}}{10}-\frac{i\sqrt{23}}{4}\right)}$
default	$\frac{4i\sqrt{5x^2+3x+2}\sqrt{2x^2-x+3}\left(2i\sqrt{31}+5i\sqrt{23}-11\right)\sqrt{-\frac{\left(2i\sqrt{31}-5i\sqrt{23}-11\right)\left(-1+4x+i\sqrt{23}\right)}{\left(2i\sqrt{31}+5i\sqrt{23}-11\right)\left(i\sqrt{23}-4x+1\right)}}\left(i\sqrt{23}-4x+1\right)^2\sqrt{\frac{i\sqrt{23}\left(i\sqrt{31}+10x+3\right)}{\left(2i\sqrt{31}-5i\sqrt{23}+11\right)\left(i\sqrt{23}-\right.}}}{23\sqrt{10x^4+x^3+16x^2+7x+6}\left(2i\sqrt{31}-5i\sqrt{23}-11\right)\sqrt{\left(-1\right.}}$

input
```
int(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/115*I*((2*x^2-x+3)*(5*x^2+3*x+2))^(1/2)/(2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)
^(1/2)*(11/20-1/4*I*23^(1/2)-1/10*I*31^(1/2))*((-11/20+1/10*I*31^(1/2)-1/4
*I*23^(1/2))*(x-1/4+1/4*I*23^(1/2))/(-11/20+1/10*I*31^(1/2)+1/4*I*23^(1/2)
)/(x-1/4-1/4*I*23^(1/2)))^(1/2)*(x-1/4-1/4*I*23^(1/2))^2*(I*23^(1/2)*(x+3/
10+1/10*I*31^(1/2))/(-11/20-1/10*I*31^(1/2)+1/4*I*23^(1/2))/(x-1/4-1/4*I*2
3^(1/2)))^(1/2)*(I*23^(1/2)*(x+3/10-1/10*I*31^(1/2))/(-11/20+1/10*I*31^(1/
2)+1/4*I*23^(1/2))/(x-1/4-1/4*I*23^(1/2)))^(1/2)/(-11/20+1/10*I*31^(1/2)-1
/4*I*23^(1/2))*23^(1/2)*10^(1/2)/((x-1/4+1/4*I*23^(1/2))*(x-1/4-1/4*I*23^(
1/2))*(x+3/10+1/10*I*31^(1/2))*(x+3/10-1/10*I*31^(1/2)))^(1/2)*EllipticF((
(-11/20+1/10*I*31^(1/2)-1/4*I*23^(1/2))*(x-1/4+1/4*I*23^(1/2))/(-11/20+1/1
0*I*31^(1/2)+1/4*I*23^(1/2))/(x-1/4-1/4*I*23^(1/2)))^(1/2),((11/20+1/4*I*2
3^(1/2)+1/10*I*31^(1/2))*(11/20-1/4*I*23^(1/2)-1/10*I*31^(1/2))/(11/20-1/4
*I*23^(1/2)+1/10*I*31^(1/2))/(11/20+1/4*I*23^(1/2)-1/10*I*31^(1/2)))^(1/2)
)
```

3.123.5 Fricas [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3\,x+2}\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)/(10*x^4 + x^3 + 16*x^2
+ 7*x + 6), x)
```

3.123.6 Sympy [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}}\,dx$$

input
```
integrate(1/(5*x**2+3*x+2)**(1/2)/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral(1/(sqrt(2*x**2 - x + 3)*sqrt(5*x**2 + 3*x + 2)), x)
```

3.123.7 Maxima [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3\,x+2}\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

3.123.8 Giac [F]

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+3\,x+2}\sqrt{2\,x^2-x+3}}\,dx$$

input
```
integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-x+3}\,\sqrt{5\,x^2+3\,x+2}}\,dx$$

input
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^(1/2)),x)
```

output
```
int(1/((2*x^2 - x + 3)^(1/2)*(3*x + 5*x^2 + 2)^(1/2)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])  #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```